Ilkogretim Online - Elementary Education Online, 2019; Vol 18 (Issue 2): pp. 1019-1024
http://ilkogretim-online.org
doi: 10.17051/ilkonline.2019.562091

Uncertainty Based Information On Fuzzy Sets

Mr. Kumud Borgohain Department of Mathematics Dhemaji College E-mail: kumudborgohain@yahoo.in

Abstract:

This paper deals with a study on the Fuzzy theory with the probability theory which is also used to express uncertainty. The concept of fuzziness is introduced in order to measure the uncertainty of fuzzy set. We will discuss various forms of uncertainty and possibility, fuzzy measures such as belief, plausibility, possibility and probability measures , possibility distribution and fuzziness.
Key Words: Fuzzy set, Uncertainty, cardinality

1. Introduction

Information designed to reduce uncertainty is called uncertainty-based information. The nature of uncertainty-based information depends on a mathematical theory in which the uncertainty associated with different problem-solving situations is formalized. Each form of uncertainty in a problem-solving situation is a mathematical model of the situation. Information based on uncertainty was first formulated by classical set theory and then by probability theory. The term information theory is almost always used to denote a theory based on the well-known measure of probability uncertainty established by Claude Shannon in 1948.

The topological structures of soft set theories dealing with uncertainties were first studied by Cheang [2] .He introduces the notion of fuzzy topology and also studied some basic properties. [3]G. J. Klir in 2006 studied Uncertainty and Information in details. [4, 5, 6]G. J. Klir and A. G. Bronevich elaborate more details Measures of uncertainty and uncertainty. Some other researchers [7],[8],[9],[10] studied various forms of uncertainty and information's about fuzzy sets. Our main aim in this research paper is to develop the basic properties of uncertainty fuzzy soft sets and establish several equivalent forms of fuzzy soft possibility measures.

2. PRELIMINARIES

Definition2.1 [9] A pair (F, A) is called a fuzzy soft set over U where F: $A \rightarrow \breve{P}(U)$ is a mapping from a into P(u)

Definition2.2 [9] A pair (F, E) is called a soft set (over U) if and only if F is a mapping of E into the set of all subset of the set U. In other words, the soft set is a parameterized family of subsets of the set U. Every set F(ε), $\varepsilon \in$ F from this family may be considered as the set of ε -elements of the soft set(F,E) or as the set of ε -approximate elements of the soft set.

Definition2.3 [9] Let U be an initial universe set and E be the set of parameters. Let $A \subseteq E$, a pair (F,A) is called a fuzzy soft set over U . Where F is a mapping given by $F : A \rightarrow I^U$, Where I^U denotes the collection of all fuzzy subset of U.

Definition 2.4[9] If T is a fuzzy soft topology on (U, E) , then (U,E,T) is said to be a fuzzy soft topological space . Also each member of T is called a fuzzy soft open set in (U, E, T)

Definition 2.5[9] The complement of a fuzzy soft set (F, A) is denoted by $(F,A)^C$ and is defined by

$(F, A)^C = (F^C, A)$. Where $F^C: A \rightarrow \breve{P}(U)$ is mapping given by

$F^C(\alpha) = U - F(\alpha) = [F(\alpha)]^C \ \forall \propto \in A$

Definition 2.6[9]

Union of two fuzzy soft sets (F, A) and (G, B) over a common universe U is a fuzzy soft set (H,C) where C= AUB $\forall$ ε Є C

H (ε) = F(ε) if ε Є A-B, G (B) if ε Є B-A, F (ε) ∪ G (ε) if ε Є A∩ B

And is written as(F,A) Û (G,B) = (H,C)

3. Main results

3.1 : Type of Uncertainty

There are only three types of uncertainty recognized in the five different theories up to now.

These are

(i) Non specificity or imprecision ;Which is connected with sizes (Cardinalities) of relevant sets of alternatives.
(ii) Fuzziness of vagueness: Which results from imprecise boundaries of fuzzy sets.
(iii) Strife or discord: Which express conflicts among the varies sets of alternatives.

3.2 : Non Specificity of Crisp sets

Measurement of uncertainty in terms of classical set theory was first formulated by Hartly in 1928, by using a function from the class of functions defined by

$U(A)=c. \log_b |A|$, where $|A|$ denotes the cardinality of a finite non empty set A and b>1, c>0 are positive constants. The choice of values of the constants b and c determines the unit in which uncertainty is measured.

3.3 Some standard meaning of the measure of Uncertainty

The meaning of uncertainty measured by the Hartley function depends on the meaning of the set A.

(i) Predictive Uncertainty: When A is a set of predicted states of a variable ,U(A) is a measure of predictive uncertainty.

(ii) Diagnostic uncertainty: When A is a set of possible diseases of a patient determined from relevant medical evidence , U(A) is a measure of diagnostic uncertainty.

(iii) Retrodictive Uncertainty: When A is a set of possible answers to an unsettled historical question, U(A) is a measure of retrodictive uncertainty.

(iv) Prescriptive Uncertainty: When A is a set of possible policies ,U(A) is a measure of prescriptive uncertainty.

3.4 Amount of Fuzzy soft Uncertainty –Based Information

Consider a set A of possible alternatives (Predictive, Prescriptive etc.) and let this fuzzy soft set is reduced to its subset B by some action. Then, the amount of uncertainty based information I(A,B) produced by the action, which is relevant to the situation is equal to the amount of reduced fuzzy soft uncertainty given by the fuzzy soft difference U(A)- U(B)

That is $I(A,B)= log_2 \frac{|A|}{|B|}$

When the action eliminates all alternatives except one (i.e when $|B|=1$), we obtain

$$I(A,B)= log_2 |A| = U(A)$$

This means that U(A) may also be viewed as the amount of information needed to characterize one element of Fuzzy soft set A.

3.5 : Simple , Joint and conditional uncertainty

Consider two universal sets X and Y ,let $R \tilde{\subseteq} X \times Y$ be a relation which describes a set of possible joint alternatives in some situations of interest and let $R_X \tilde{\subseteq} X \times Y$ and $R_Y \tilde{\subseteq} Y$ are the domain and range of R , respectively. Then three distinct Hartley functions can be defined on the power set of X,Y and X x Y represented by U(X), U(Y) and U(X,Y) instead of $U(R_X)$, $U(R_Y)$, U(R) respectively. Functions

$$U(x) = log_2 |R_x|$$

$$U(Y) = log_2 |R_Y|$$

Are called the simple uncertainties, while function $U(x,Y) = log_2 |R|$ is called joint uncertainty.

Two additional Hartly functions are defined by the formula $U(X|Y)=log_2\frac{|R|}{|R_Y|}$ and $U(Y|X)=log_2\frac{|R|}{|R_X|}$ is called conditional uncertainty.

3.6 : Non-Specificity of Fuzzy sets

The generalization of Hartley function was proposed from classical set theory to fuzzy set theory in 1980 by the name U- uncertainty. Let A be non empty fuzzy soft set defined on a finite universal set X, then the generalized Hartley function has the form $U(A)=\frac{1}{h(A)}\int_0^{h(A)} log_2\left|{}^{\alpha}A\right|d\alpha$, where $\left|{}^{\alpha}A\right|$ denote the cardinality of the α-cut of A and h(A) is the height of A, Here U(A) measures the non-specificity of A is also called weighted average of values of the Hartley function for all distinct α-cut of the normalized counterparts of A defined by A(x)/h(A) for all $x\in X$. Each weight is a difference between the values of α of given α-cut and the immediately preceding α-cut.

Example 1.1 : Let A be a fuzzy set on N whose membership function ids defined by

$$A=\frac{0.1}{1}+\frac{0.1}{2}+\frac{0.3}{3}+\frac{0.4}{4}+\frac{0.6}{5}+\frac{0.7}{6}+\frac{0.9}{7}+\frac{1.0}{8}+\frac{1.0}{9}+\frac{1.0}{10}+\frac{0.9}{11}+\frac{0.7}{12}+\frac{0.6}{13}+\frac{0.4}{14}+\frac{0.1}{15}$$

And A(x)=0 for all x >15 . Calculate the non specificity of A

We know that the non specificity of a fuzzy set A is denoted by U(A) and defined as

$$U(A)=\frac{1}{h(A)}\int_0^{h(A)} log_2\left|{}^{\alpha}A\right|d\alpha$$

Then we have

$$\int_0^{h(A)} log_2\left|{}^{\alpha}A\right|d\alpha=\int_0^{0.1} log_2 15d\alpha+\int_{0.1}^{0.3} log_2 12d\alpha+\int_{0.3}^{0.4} log_2 11d\alpha+\int_{0.4}^{0.6} log_2 9d\alpha+\int_{0.6}^{0.7} log_2 7d\alpha+\int_{0.7}^{0.9} log_2 5d\alpha+\int_{0.9}^{1.0} log_2 3d\alpha$$

$=0.1log_2 15+0.2log_2 12+0.1log_2 11+0.2log_2 9+0.1log_2 7+0.2log_2 5+0.1log_2 3$

$=0.1(3.91)+0.2(3.58)+0.1(3.46)+0.2(3.17)+0.1(2.81)+0.2(2.32)+0.1(1.58)$

Since h(A)=1 the amount of non specificity associated with the given fuzzy soft set is thus approximately three bits.

3.7 : General about the non-specificity of fuzzy soft sets

Let A be a non empty fuzzy soft set defined on R such that the α-cut of A are infinite sets. Then U(A) can be generalized as $U(A)=\frac{1}{h(A)}\int_0^{h(A)} log_2\left[1+\mu({}^{\alpha}A)\right]d\alpha$

Where ${}^{\alpha}A$ is a measurable and Lebesgue integrable function $\mu({}^{\alpha}A)$ is the measure of ${}^{\alpha}A$ defined by the lebesgue integral of the characteristic function of ${}^{\alpha}A$.

3.8 : Specificity for Possibility Distribution

The U- uncertainty was investigated more thoroughly within possibility theory, utilizing ordered possibility distributions.

Then possibility distribution function is defined as U: Ʀ →R⁺ where Ʀ denotes the set of all finite and ordered possibility distributions, each of which represents a normal fuzzy set.

Fuzziness of Fuzzy soft set:

Fuzziness is a type of uncertainty that involves only in fuzzy sets but not in crisp sets. The fuzziness function is defined as $f : f(x) \rightarrow R^{+}$

Where f(X) denotes the set of all fuzzy subsets of X.

3.9 :Measure of Fuzziness

The sensible measure of fuzziness is defined as $f : f(x) \rightarrow R^{+}$

And it satisfies the following conditions

i) f(A)=0 iff A is a crisp set.

ii) f(A) attains its maximum iff A(x)=0.5 $\forall x \in X$, which is intuitively conceived as the highest fuzziness.

iii) $f(A) \leq f(B)$ when set A is undoubtedly sharper than set B, which means that $A(x) \leq B(x)$ when $B(x) \leq 0.5$ and $A(x) \geq B(x)$ when $B(x) \geq 0.5, \forall x \in X$

The measure of fuzziness, f(A) is defined as $f(A) = \sum_{x \in X} \left[1 - \left|2A(x) - 1\right|\right]$..................(1)

The of function f is $\left[0, |x|\right]$,f(A)=0 iff A is a crisp set.

$f(A) = |x|$ when A(x)=0.5, $\forall x \in X$

The equation is applicable only to fuzzy set defined on finite universal set.

It can be modified to fuzzy sets defined on infinite but bounded subset of R.

Example 1.2 : Calculate the degree of fuzziness of the set A where

$$A = \frac{0.1}{1} + \frac{0.1}{2} + \frac{0.3}{3} + \frac{0.4}{4} + \frac{0.6}{5} + \frac{0.7}{6} + \frac{0.9}{7} + \frac{1.0}{8} + \frac{1.0}{9} + \frac{1.0}{10} + \frac{0.9}{11} + \frac{0.7}{12} + \frac{0.6}{13} + \frac{0.4}{14} + \frac{0.1}{15}$$

We have

$$f(A) = \sum_{i=1}^{15} \left[1 - \left|2A(x) - 1\right|\right]$$

$$= \left[1 - \left|2 \times 0.1 - 1\right|\right] + \left[1 - \left|2 \times 0.1 - 1\right|\right] + \left[1 - \left|2 \times 0.3 - 1\right|\right] + \ldots\ldots\ldots\ldots \left[1 - \left|2 \times 0.4 - 1\right|\right] + \left[1 - \left|2 \times 0.1 - 1\right|\right]$$

=[1-0.8]+[1-0.8]+[1-0.4]+.......[1-0.2]+[1-0.8]

=15-[0.8+0.8+0.4+0.2+0.2+0.4+0.8+1.0+1.0+1.0+0.8+0.4+0.2+0.2+0.8]

=15-9

=6

CONCLUSION

Today, fuzzy mathematics research is considered attractive and very useful due to its growing role in mathematics and applied sciences. This field of mathematics is increasingly recognized as a convenient and very powerful way to study the behavior of various mathematical models in the field of application. In that regard, we continued to investigate the nature of the uncertainty in our studies. Next, we will introduce various definitions and examples one after another and explain them in detail. We hope that the results of this study will help researchers improve and foster the study of fathers of uncertainty-based information and provide a general framework for its application in real life. increase.

References:

[1] Abellan, J. and Moral, S. "A non-specificity measure for convex sets of probability distributions." Intern, J. of Uncertainty, Fuzziness and Knowledge-Based Systems, 8(3), pp. 357-367.2000

[2] Chang C.L., “Fuzzy Topological Space” J. Math Anal. Appli Vol .(24) PP182-190, 1968

[3]G. J. Klir, “Uncertainty and Information: Foundations of Generalized Information Theory”, Hoboken, NJ: Wiley-Interscience, 2006

[4]G. J. Klir and A. G. Bronevich, "Measures of uncertainty and uncertainty-based information in GIT: A historical overview," Proc. of NAFIPS'08, Piscataway, NJ: IEEE, 2008

[5]G. I. Klir, "Generalized information theory," Fuzzy Sets and Systems, vol. 40, pp. 127-142, 1991.

[6]Kalir, G. J. , "Generalized information theory: aims, results, and open problems." Reliability Engineering and System Safety, 85(1-3), pp. 21-38, 2004

[7]Klir, G. J. and Wierman M.J, “Uncertainty-Based Information: Elements of Generalized Information Theory” (Second Edition). Physica-Verlag/Springer-Verlag, Heidelberg and New York.1999

[8]Kapur, J. N., Baciu, G. and Kesavan, H. K. , "The MinMax information measure." Intern. J. of Systems Science, 26(1), pp. 1-12.1995

[9] L.A. Zadeh “ Fuzzy Sets’ Information and Control, 8,338-353,1965

[10]Ramer, A. "Uniqueness of information measure in the theory of evidence." Fuzzy Sets and Systems, 24(2), pp. 183-196.1987