Ilkogretim Online - Elementary Education Online, 2021; 20 (1): pp. 948-957
http://ilkogretim-online.org
doi: 10.17051/ilkonline.2021.01.91

Measuring Spatial Ability and Geometric Thinking Level of Prospective Elementary School Teachers Using the Rasch Model

Trimutini Trimurtini, *Doctoral Student of Graduate School Universitas Negeri Semarang, Indonesia, trimurtini3unnes@students.unnes.ac.id, ORCID 0000-0001-8152-7071, Lecturer of Elementary School Teacher Education Education Faculty Universitas Negeri Semarang, Indonesia,*
Stevanus Budi Waluya, *Universitas Negeri Semarang, Indonesia, s.b.waluya@mail.unnes.ac.id, ORCID 0000-0002-8834-1138*
Walid Walid, *Universitas Negeri Semarang, Indonesia, walid.mat@mail.unnes.ac.id, ORCID 0000-0003-3548-0457*
Nur Karomah Dwidayati, *Universitas Negeri Semarang, Indonesia, nurkaromah.mat@mail.unnes.ac.id.*
Iqbal Kharisudin, *Univesitas Negeri Semarang, Indonesia, iqbalkharisudin@mail.unnes.ac.id, ORCID 0000-0002-1156-4974*

Abstract. The objectives: (1) measuring spatial ability and geometric thinking level of Prospective Elementary School Teachers (PGSD students), (2) analyzing difficulties faced on a certain spatial ability. The study used a mixed-method quantitative and qualitative. Researchers developed test items to measure the spatial ability and geometric thinking of 139 Prospective Elementary School Teachers (PGSD students), Universitas Negeri Semarang. The sampling method employed was cluster random sampling, while three students selected for the interview were based on the 'poor' and 'very poor' category. Data analysis with the Rasch Model showed that the item reliability, item validity, item separation, and unidimensionality were classified as 'good'. The results revealed that spatial ability and geometric thinking of Prospective Elementary School Teachers were classified as 'fair' (62.5%) and 'good' (31.7%). The difficulties in completing tasks of spatial ability primarily on the perception and relation, which happened due to their lacking of imagination.

Keywords: Spatial ability, geometric thinking, rasch model, Prospective Elementary School Teachers

Received: 22.11.2020 Accepted: 05.12.2020 Published: 16.01.2021

INTRODUCTION

Spatial ability is beneficial for many life aspects, from daily needs to academic matters (science, technology, engineer, and mathematics) (Hegarty & Waller, 2005; Uttal et al., 2013). It is an ability to represent, transform, and recall nonlinguistic information, visual images, or symbols (Linn, Petersen, Linn, & Petersen, 1985). Spatial ability refers to an individual's ability to see the visual world accurately and to predict the relationship between various geometric entities (Akayuure, Asiedu-Addo, & Alebna, 2016). Due to the wide range of benefits, the study of spatial ability is spread across various fields of science, for example, science, engineering, mathematics, surgery, interior design to art.

Geometry is one of the subjects of mathematics which covers characteristics and the connection between points, lines, fields, and spaces (Akinci & Genç, 2020; Bergstrom & Zhang, 2016). It is learned from the primary to higher education level. In several countries like Ghana (Akayuure et al., 2016; Armah & Kissi, 2019), United States (Bergstrom & Zhang, 2016), Malaysia (Mdyunus & Hock, 2019), and Indonesia (Prayitno, Suryadi, & Mulyana, 2019; Syamsuddin, 2019), geometry learning is essential as it is related to other disciplines. In addition to its importance, geometry is internationally acclaimed as difficult, seen from the many students who faced problems. The PISA mathematics section includes four content categories: (1) Quantity, (2) Uncertainty and Data, (3) Change and Relationship, and (4) Space and Shape. The PISA's score standard is an average of 500 and a standard deviation of 100. The 2014 PISA results asserted that of the four assessment categories, Shape and Space obtained the lowest score of 490, where the average of Quantity was 495, the same went for the Uncertainty and Data. On the other hand, Change and Relationship scored 493. These outcomes revealed that there is no significant difference in the scores in four categories. Nevertheless, some countries do not put geometry as a compulsory domain of mathematics while in fact, it scored the lowest compared to other math domains (Mammarella, Giofrè, & Caviola, 2017).

Spatial ability can be taught to learners through geometry learning (Clements & Battista, 1992; Maier, 1996; Yilmaz, 2009). It refers to individual skill to see the visual world accurately and presume the relationship between various geometrical entities (Akayuure et al., 2016). For that reason, spatial ability as the core skill in human life is combined in formal education and enhanced through tools, technology, and a curriculum that is regulated appropriately (Yurt & Tünkler, 2016).

The success of geometry learning is influenced by the geometric thinking level. Van Hiele designed and described five levels of geometric thinking consisting of visualization, analysis, abstraction, deduction, and rigor. Further, Van Hiele said that what affects the level of geometric thinking of each individual is learning rather than age, biological age, or grade level at school (Škrbec, 2015). Moreover, the intended learning must be in accordance with the level that has been achieved by students, so that they can experience improvement to reach the next level. Therefore, the teacher needs to be able to understand a student reaching a certain level in geometric thinking (Mammarella et al., 2017). On the contrary, fewer geometry learning experiences lead to an inadequate experience of spatial ability (Armah et al., 2018). That being said, geometry learning that pays attention to the level of geometric thinking is the best suggestion for increasing the level of geometric thinking and is expected to provide a meaningful learning experience to achieve spatial ability.

At the higher education level, geometry course is studied in two ways, namely content and methodology (Akayuure et al., 2016), hence, they not only equip prospective teachers with the subject matter but more specifically to open up to the science of education (pedagogy) on how to teach effectively in elementary-level education. If these two things have not been mastered in higher education, there can be misconceptions in learning geometry at the school level (Devichi & Munier, 2013). PGSD students as prospective elementary school educators, for instance, need to master all the geometry materials of various geometry learning strategies in elementary schools. Therefore, it is important to study or measure the spatial ability and geometric thinking level as preliminary research to determine strategic steps in geometry lectures at PGSD.

Objectives: (1) measuring spatial ability and geometric thinking level of Prospective Elementary School Teachers (PGSD students), (2) analyzing difficulties on a certain spatial ability.

METHODS

This quantitative and qualitative research (mixed method) discusses the profile of spatial ability and the thinking level of prospective elementary school teachers (PGSD students) at Universitas Negeri Semarang which took place in natural conditions where researchers did not manipulate learning in the classroom. This study seeks to determine the profile of the spatial and geometric thinking skills of prospective elementary school teachers and to discuss their strengths and weaknesses.

Research Context

This study was carried out to the 4th-semester PGSD students who joined the geometry course and had passed two subjects; basic mathematical concepts and advanced mathematical concepts. Earlier in the 1st and 2nd semester, they had obtained a study of geometry including definitions, proofs, and examples of basic geometric concepts (Bulut & Bulut, 2012), especially in flat, space, and transformation geometry. Then, in the 4th semester, they had learned geometry learning strategies in elementary schools starting from curriculum review, learning steps, and learning media to assessments of elementary geometry learning.

Subject and Data

A total of 139 4th-semester PGSD student year 2019/2020 consisting of 21 males and 118 females. There were 31 question items, so, the total was 139 x 31 = 4309 data and no data was lost. Meanwhile, the qualitative data were obtained by interviewing three students in the 'poor' and 'very poor' category who were selected as research subjects based on the analysis results of spatial ability and geometric thinking using the Rasch model.

Instrument

Researchers developed a test instrument to measure the types of spatial ability including spatial orientation, visualization, perception, rotation and relation (Clements & Battista, 1992; Maier, 1996; Uttal et al., 2013) and levels of geometric thinking (Pujawan, Suryawan, & Prabawati, 2020). The instrument comprises 31 questions with 5 types of spatial ability and 4 levels of geometric thinking. All items were arranged based on literature studies related to spatial ability and geometric thinking, as described in table 1.

Table 1. *The Instrument Description of Spatial Ability and Geometric Thinking*

Aspect	Component	Description	Item code
Spatial ability (Pujawan et al., 2020)	Spatial perception	Being able to see an object with the horizontal and vertical reference	29-SpPe
	Spatial visualization	Being able to visualize things	10-SpVi
	Spatial rotation	Being able to determine object position after rotated in a particular direction	27-SpRo
	Spatial relation	Being able to connect parts with objects	15-SpRe
	Spatial orientation	Being able to see objects from different points of view	4-SpOr
Geometric thinking (Fuys, Geddes, & Tischler, 1988)	0- Visualization	Being able to identify, name, compare and operate geometric images according to their appearance (for example triangle, angles, intersecting lines, or parallel lines)	3-L0, 8-L0, 12-L0, 14-L0, 20-L0, 26-L0
	1- Analysis	Being able to analyze images concerning component, the relationship between components, and found out the characteristics/rules of each group empirically (for example: by folding or measuring using grids of the diagram)	1-L1, 2-L1, 6-L1, 7-L1, 9-L1, 11-L1, 13-L1, 16-L1, 17-L1, 25-L1
	2- Abstraction	Students give or follow informal arguments logically related to previously found properties/rules.	5-L2, 19-L2, 22-L2, 31-L2
	3- Deduction	Students prove theorems deductively and build reciprocal relationships between theorem networks.	23-L3, 28-L3, 30-L3
	4-Rigor	Students set theorems in different postulate systems and analyze/compare these systems	21-L4, 24-L4

The Quantitative Data Analysis

The collected data were analyzed using the Rasch Model. Firstly, the raw data were processed using excel and prepared for processing. After that, special coding was prepared to analyze the data that would be calculated by WinStep. The Rasch modeling combines the algorithm from the item "I" and the respondent "n" which is mathematically expressed as follows (Bond & Fox, 2015):

$$P_{ni}(X_{ni} = 1/\beta_n, \delta_i) = \frac{e^{(\beta_n - \delta_i)}}{1 + e^{(\beta_n - \delta_i)}} \quad (1)$$

Where $P_{ni}(X_{ni} = 1/\beta_n, \delta_i)$ is the probability of respondent n in item I to produce the correct answer, with the respondent's ability β_n and the item difficulty level δ_i

The Qualitative Data Analysis

The qualitative data were analyzed using an interactive model including data collection, data reduction, data presentation, and conclusion/verification (Miles, Huberman, & Saldana, 2014)

RESULTS

The Quality of Spatial Ability and Geometric Thinking Test

The Rasch model analysis in the following Table 2 shows the statistic summary of the item reliability and separation index. The item reliability, which scored 0.97, was categorized as 'excellent' for an instrument (Bond & Fox, 2015; Sumintono & Widhiarso, 2014). This high score indicated that all items define a latent variable well, which means that a total of 31 items are reliable and usable for diverse groups of the respondent. On the other hand, the separation index shows the range of item difficulty. In this study, the separation scored 5.91 and the standard deviation was 1.40 which clearly pointed out a favorable distribution of item difficulty level. These criteria indicated that the instrument was appropriate and reliable to identify the spatial ability and geometric thinking level of Prospective Elementary School Teachers. The logit mean score of the students was 0.67, meaning that all students tended to be able to answer all question items.

Table 2. *The reliability and separation as a result of the rasch model analysis*

	Logit Mean	Standard Deviation	Separation Index	Reliability	Standard Error
Student	0.67	0.85	1.48	0.69	0.07
Item	0.00	1.40	5.91	0.97	0.26

Table 3. *Psychometrics attributes of ıtems*

Item	Measure	Model S.E.	Infit MNSQ	ZSTD	Outfit MNSQ	ZSTD	Correlation PT Measure
24	3.20	.30	1.05	.28	1.62	1.58	.09
29	2.95	.27	1.00	.05	1.13	.52	.20
21	2.68	.25	.97	-.12	1.04	.24	.25
9	1.35	.19	.85	-2.29	.76	-2.26	.52
22	1.28	.19	1.17	2.38	1.18	1.66	.16
3	1.18	.19	1.09	1.44	1.11	1.07	.24
23	.87	.18	1.05	.89	1.09	1.04	.29
15	.77	.18	.96	-.73	.98	-.20	.40
19	.77	.18	1.07	1.12	1.09	1.08	.28
31	.71	.18	1.02	.37	1.00	-.02	.34
2	.67	.18	1.20	3.23	1.25	2.85	.13
8	.34	.18	.98	-.32	.99	-.05	.39
1	.27	.19	1.06	.96	1.13	1.43	.29
18	.27	.19	.92	-1.23	.89	-1.26	.46
27	.20	.19	1.09	1.30	1.06	.68	.28
25	.17	.19	1.01	.16	1.00	.06	.36
6	.06	.19	1.01	.18	.99	-0.5	.36
16	-.16	.19	1.04	.52	1.03	.26	.32
10	-.65	.21	1.00	.05	.95	-.25	.36
14	-.70	.22	.78	-2.06	.62	-2.61	.61
30	-.79	.22	.89	-.85	.86	-.75	.46
20	-1.00	.23	.94	-.43	.90	-.44	.40
13	-1.11	.24	.97	-.14	1.06	.35	.33
4	-1.42	.26	.98	-.06	1.13	.57	.31
5	-1.42	.26	.94	-.31	.78	-.85	.40
28	-1.49	.27	1.02	.16	.88	-.37	.31
7	-1.64	.28	1.03	.23	.87	-.37	.29
12	-1.72	.29	.84	-.77	1.06	.29	.43
11	-1.89	.31	.82	-.76	.72	-.79	.46
17	-1.89	.31	1.07	.38	1.02	.17	.22
26	-1.89	.31	.98	.00	.94	-.08	.29
Mean	0.00	.23	.99	.1	1.00	.1	
SD	1.40	.04	.09	1.1	.18	1.1	

Table 3 found that the logit values for question 24 (level 4, three-dimensional geometry) and 29 (spatial perception) were outside the value range + 2SD and -2SD or between 2.80 and -2.80. These results identified two questions that were outliers; nevertheless, the measure correlation point values were between 0.09 <x <0.61 (all positive) indicating that all items corresponded to the agreed latent variables.

The value of the MNSQ outfit should be between the values 0.5 to 1.5. In table 3, only item 24 (level 4, three-dimensional geometry) scored above the range, which was considered not good for measurement but the ZSTD value and point measurement correlation (correlation point measure) were acceptable.

Item 16 (level 1, three-dimensional geometry) had a probability of 0.0366 which was less than 5%; hence, this question was biased for gender category while the other 30 items were not. This is confirmed by the Person DIF (Differential Item Function) plot.

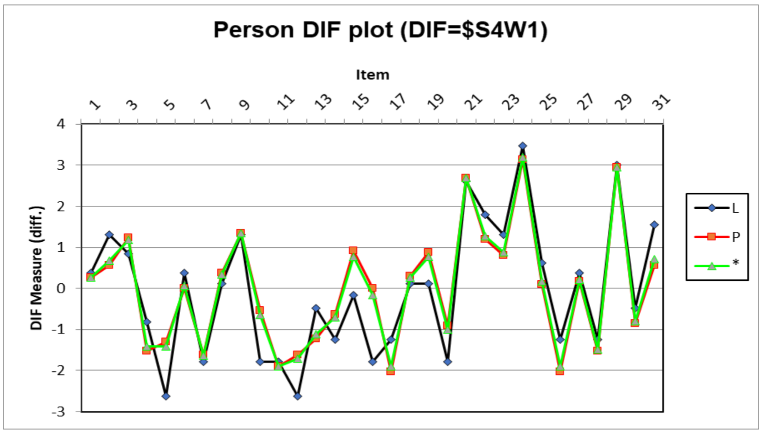

Figure 1. *The DIF measurement (code L: male, code P: female)*

Furthermore, Figure 1 asserts that question number 5, 10, 12, 14, 15, 16, 19, 20 were relatively easy for male students while question number 2, 17, 22, 23, 24, 26, and 31 were fairly difficult for them. On the contrary, female students (P) were always in an average position in all question numbers. This is possible because the sample size of female students was greater, namely, 118 people compared to male students that consisted of only 21 people.

The following Figure 2 is a wright map that describes the categorization of the students' spatial ability and geometric thinking level. Figure 2 shows that female students were spread across all categories from very good, good, fair, poor, and very poor. In contrast, male students were got around good, fair, and poor category.

Table 4. *The description of spatial ability and geometric thinking percentage among PGSD Students*

NO	Spatial Ability and Geometric Thinking	Female Student		Male Student		Total	
		Number	Percentage	Number	Percentage	Number	Percentage
1	Very good	4	3.4%	0	0%	4	2.9%
2	Good	32	27.1%	12	57.1%	44	31.7%
3	Fair	79	66.9%	8	38.1%	87	62.5%
4	Poor	2	1.7%	1	4.8%	3	2.2%
5	Very poor	1	0.9%	0	0%	1	0.7%
	Total	118	100%	21	100%	139	100%

Table 4 contains a description of spatial ability and geometric thinking percentage among 139 PGSD students. The majority of PGSD students were in the 'fair' (62.5%) and good (31.7%) category and only 2.9% were in the 'poor' and 'very poor'. This shows that a combined form of deductive and inductive approach can be applied to several subjects in PGSD, especially mathematics classes.

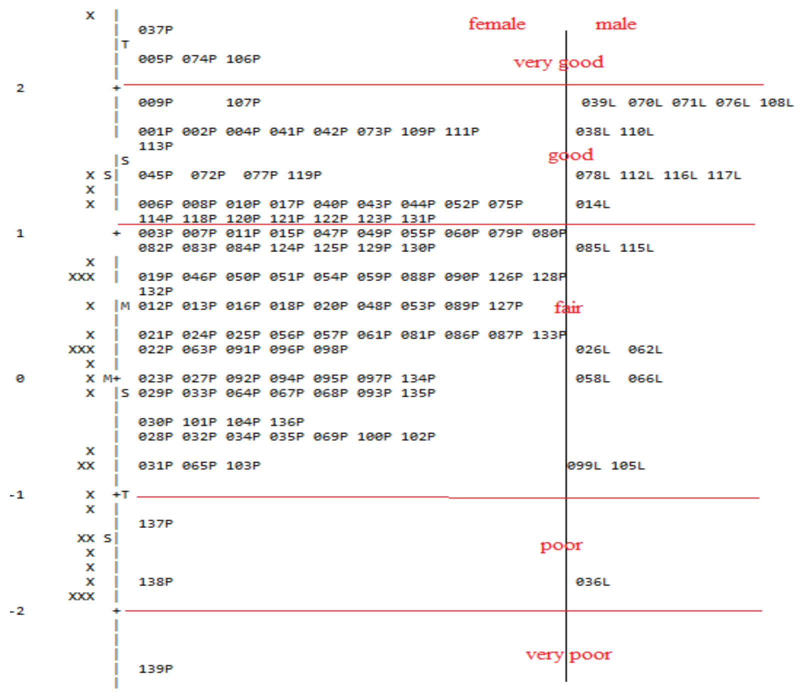

Figure 2. *Distribution of spatial and geometric thinking ability of 139 PGSD students*

The test consisting of 31 items was analyzed and grouped based on their distribution on the wright map (Figure 3). The distribution of the difficulty level was almost ideal in which the questions with the 'fair' difficulty level reached 48.4%, the 'difficult' and 'very difficult' level scored 19.4% and the 'easy' category obtained 32.3%. Meanwhile, if further analyzed related to spatial ability, the item was most difficult to work on was the perception. Moreover, the students considered level 4 (rigor) as a hard level in terms of geometric thinking.

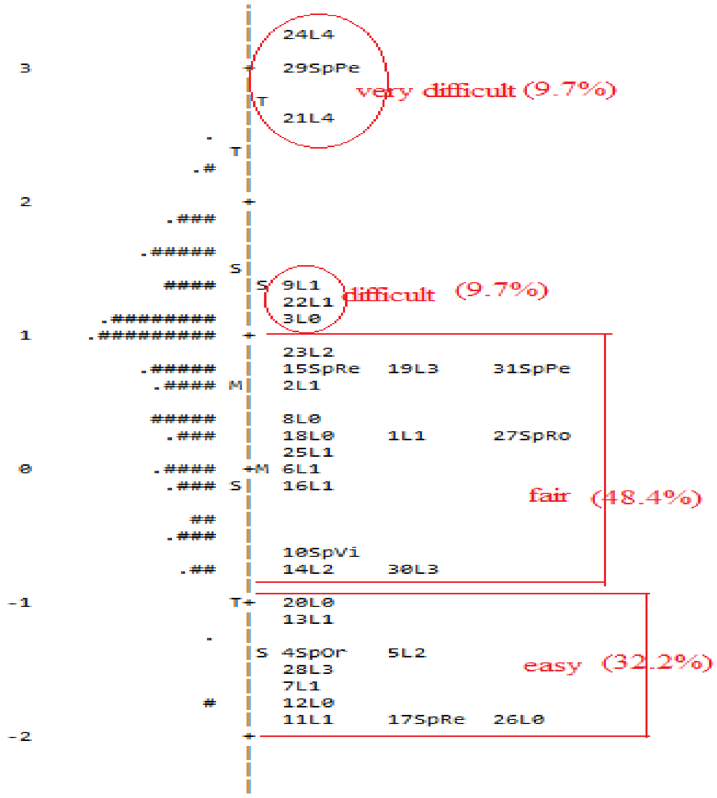

Figure 3. *Grouping the difficulty level of the question items*

Qualitative Data Analysis

The interview was carried out with 3 students who were selected based on the test results and analysis using the Rasch model in which the three of them were in the 'poor' and 'very poor' category. They are students with code 137P, 036L, and 139P. The interview questions about difficulties in spatial ability indicated that they did not see all types of spatial ability as difficult but only perception and relation.

Student 137P found it hard to master the spatial perception as she had not ever experienced geometry learning using vertical and horizontal reference. Perception requires the subject to determine the spatial relationship regarding their body orientation, despite distracting information. Student 139P and 036L faced difficulties in mastering the spatial relationships as they were lost in imagining the connection of parts of objects. In this case, a grid of patterned cubes that correspond to an existing cube image. The difficulty factor is that the motif formed in the cube image can only be seen from the side and upper. They thought it will be easier for them if there is a real cube object than just a picture. They also felt that they could answer correctly if they were allowed to dismantle the patterned cube into a grid of cubes as in the correct answer choice.

DISCUSSION AND CONCLUSIONS

The level of geometric thinking, which was categorized as 'very difficult', was at level 4 (rigor) (Kaleli & Koparan, 2016). It includes geometric thinking where students can perform various tasks of axiomatic deductive structures, find differences in two deductive structures (Suwito, Yuwono, Parta, Irawati, & Oktavianingtyas, 2016), use all types of proof, and describe the influence of adding or subtraction of an axiom in geometric systems (Vojkuvkova, 2012).

The questions given were about two-dimensional and three-dimensional geometry. This is the highest level in geometric thinking according to Van Heile which can only be achieved continuously after students master level 0 (visualization), level 1 (analysis), level 2 (abstraction or informal deduction), level 3 deduction (formal deductive) (Fuys et al., 1988; Kaleli & Koparan, 2016; Vojkuvkova, 2012). Even though according to Piaget, in terms of age, PGSD students have entered the formal operational stage, in which they can use logical operations well, link existing data and information, and use cognitive schemes to build understanding in the problem structure (Syamsuddin, 2019).

Geometric thinking levels 0, 1, 2, and 3 were distributed, but not consecutively, in the 'difficult', 'fair' and 'easy' category. This is interesting as PGSD students are positioned as elementary school teachers who need to master and understand the level of geometric thinking sequentially to provide a good learning experience related to the geometric thinking level of elementary students (Mayberry, 1983). Given the learning experience of PGSD students will be brought when they practice teaching in elementary schools. This is in line with Van Hieles who asserted that students must develop masterfully at each level before they can progress to the next (Howse & Howse, 2015).

There were five spatial abilities measured which included visualization, rotation, orientation, perception, and relation (Pujawan et al., 2020). However, abilities discussed in the interview only comprised of those considered to be difficult by the students. The interview questions were based on research instruments in the form of the test item and the spatial abilities related to these questions. The followings are hard spatial abilities according to the students (only two were interviewed).

Student 137P found it hard to master the spatial perception as she had not ever experienced geometry learning using vertical and horizontal reference. Perception requires the subject to determine the spatial relationship regarding their body orientation, despite distracting information. Several analysis processes on perception problems suggest that students use vertical gravity to find the right orientation (Linn et al., 1985; Yilmaz, 2009). The question indicator was "given a picture of a glass containing water put in a flat place, students are asked to describe the water surface if the glass is tilted". This indicator can be found or proven in real-life experiences(Hegarty & Waller, 2005; Uttal et al., 2013). Even so, the students were confused in determining the right picture. Yet they realized the mistake in the selected picture soon as the live demonstration was shown(Hardman, 2019). This experience showed that the spatial position of the students is in the static mental process (Maier, 1996; Uttal et al., 2013).

Student 139P and 036L faced difficulties in mastering the spatial relationships as they were lost in imagining the connection of parts of objects (Maier, 1996; Pujawan et al., 2020), in this case, a grid of patterned cubes that correspond to an existing cube image. The difficulty factor is that the motif formed in the cube image can only be seen from the side and upper. They thought it will be easier for them if there is a real cube object than just a picture. They also felt that they could answer correctly if they were allowed to dismantle the patterned cube into a grid of cubes as in the correct answer choice.

Furthermore, the analysis of the interview results with 3 students indicated that the items that were considered difficult to work on were the ability of perception and relation. The difficulties were caused by spatial ability, in which they should imagine what the answer was, as well as learning experiences with minimal use of media related to spatial ability in a geometry course. Studying spatial ability requires cognition, psychometry, and strategy(Linn et al., 1985), in addition to supporting multimedia (Ariel, Lembeck, Mo, & Hertzog, 2018; Chang, Sung, & Lin, 2007; Prasad, Maag, Redestowicz, & Hoe, 2018). In addition, the use of multimedia can affect spatial abilities (Heo & Toomey, 2019). Some of the media that are often used in learning geometry are tangram (Bergstrom & Zhang, 2016; Moursund, 2007), geoboard (Mudaly & Sibiya, 2018; Trimurtini, Safitri, Sari, & Nugraheni, 2020), android-based media (Bektiningsih, Nugraheni, & Sari, 2020), geogebra (Başara, 2020; Klemer & Rapoport, 2020; Topuz & Birgin, 2020) electronic module (Suastika & Wahyuningtyas, 2020), augmented(İbili, Çat, Resnyansky, Şahin, & Billinghurst, 2019) and virtual reality-based media (Gecu-parmaksiz & Delialioglu, 2019).

Rasch Model analysis produces complete information related to the quality of instrument items, item difficulty level, spatial ability, and geometric thinking of PGSD students. Information about each type of spatial ability and level of geometric thinking is important to determine the next step in geometry learning strategies in the PGSD Department. The analysis outtakes showed that the majority of PGSD students' spatial ability and geometric thinking was 62.5% and classified as 'fair' category while 31.7% of students were in the 'good' category. In terms of spatial ability mastery, perception skill requires improvement, which they thought was the hardest one. Further, the level of geometric thinking was spread evenly from level 0, level 1, level 2, level 3, and level 4 (rigor). This depends on the geometry materials being studied, either two-dimensional or three-dimensional geometry. Difficulties in completing the task of perception and relation were caused by their lacking spatial ability, where the students should imagine to find the right solution; also, their minimum experience of spatial ability in geometry learning.

The authors suggested that it is pivotal to provide a learning experience that facilitates students to determine the level of geometric thinking, how to improve it and master various spatial abilities through multimedia.

REFERENCES

Akayuure, P., Asiedu-Addo, S. ., & Alebna, V. (2016). Investigating the Effect of Origami Instruction on Pre-service Teachers ' Spatial Ability and Geometric Knowledge for Teaching. *International Journal of Education in Mathematics Science and Technology (IJEMST)*, *4*(3), 198–209. https://doi.org/10.18404/ijemst.78424

Akinci, M., & Genç, M. (2020). Pre-service teachers ' explanations of primitive terms : point , line and plane. *Ilkogretim Online - Elementary Education Online*, *19*(4), 1925–1947. https://doi.org/10.17051/ilkonline.2020.762588

Ariel, R., Lembeck, N. A., Mo, S., & Hertzog, C. (2018). Are there sex differences in con fi dence and metacognitive monitoring accuracy for everyday , academic , and psychometrically measured spatial ability ? ☆. *Intelligence*, *70*(April), 42–51. https://doi.org/10.1016/j.intell.2018.08.001

Armah, R. B., Cofie, P. O., Investigating, C. A., Armah, R. B., Cofie, P. O., & Okpoti, C. A. (2018). Investigating the Effect of van Hiele Phase- based Instruction on Pre- service Teachers ' Geometric Thinking. *International Journal of Research in Education and Sciece (IJRES)*, *4*(1), 3r14-330. https://doi.org/10.21890/ijres.383201

Armah, R. B., & Kissi, P. S. (2019). Use of the van Hiele Theory in Investigating Teaching Strategies used by College of Education Geometry Tutors. *EURASIA Journal of Mathematics, Science and Technology Education*, *15*(4), 1–13.

Başara, B. (2020). Investigating science and mathematics teacher candidate ' s perceptions of TPACK-21 based on 21st century skills. *Ilkogretim Online - Elementary Education Online*, *19*(4), 2212–2226. https://doi.org/10.17051/ilkonline.2020.763851

Bektiningsih, K., Nugraheni, N., & Sari, E. F. (2020). *Android-Based Educational Game Of Triangular And Rectangular Area As A Measurement Learning Media For Elementary Students*. *9*(02).

Bergstrom, C., & Zhang, D. (2016). Geometry interventions for K-12 students with and without disabilities : A research synthesis. *International Journal of Educational Research*, *1142*. https://doi.org/10.1016/j.ijer.2016.04.004

Bond, T. G., & Fox, C. M. (2015). *Applying the rasch model: Fundamental measurement in the human* (3rd ed.). New York: Routledge.

Bulut, N., & Bulut, M. (2012). Development of pre- service elementary mathematics teachers ' geometric thinking levels through an undergraduate geometry course. *Procedia - Social and Behavioral Sciences*, *46*(1986), 760–763. https://doi.org/10.1016/j.sbspro.2012.05.194

Chang, K., Sung, Y.-T., & Lin, S.-Y. (2007). Developing geometry thinking through multimedia learning activities. *Computers in HUman*, *23*, 2212–2229. https://doi.org/10.1016/j.chb.2006.03.007

Clements, D. H., & Battista, M. T. (1992). *Geometry and Spatial Reasoning*. In D . A . Grouws (Ed .), Handbook of research on mathematics teaching and learning (pp . 420-464). New York: Macmillan.

Devichi, C., & Munier, V. (2013). About the concept of angle in elementary school : Misconceptions and teaching sequences. *Journal of Mathematical Behavior*, *32*(1), 1–19. https://doi.org/10.1016/j.jmathb.2012.10.001

Fuys, D., Geddes, D., & Tischler, R. (1988). The Van Hiele Model of Thinking in Geometry among Adolescents. In *Natioanl Council of Teachers of Mathematics* (Vol. 3). New York.

Gecu-parmaksiz, Z., & Delialioglu, O. (2019). Augmented reality-based virtual manipulatives versus physical manipulatives for teaching geometric shapes to preschool children. *British Journal of Education Technology*, *0*(0), 1–15. https://doi.org/10.1111/bjet.12740

Hardman, J. (2019). Towards a pedagogical model of teaching with ICTs for mathematics attainment in primary school: A review of studies 2008–2018. *Heliyon*, *5*(5), e01726. https://doi.org/10.1016/j.heliyon.2019.e01726

Hegarty, M., & Waller, D. A. (2005). Individual Differences in Spatial Abilities. In *In P. Shah, & A. Miyake (Eds.). The Cambridge handbook of visuospatial thinking (pp. 121– 169)* (p. 556). New York: Cambridge University Press.

Heo, M., & Toomey, N. (2019). Learning with multimedia: The effects of gender, type of multimedia learning resources, and spatial ability. *Computers & Education*, *146*(March), 103747. https://doi.org/10.1016/j.compedu.2019.103747

Howse, T. D., & Howse, M. E. (2015). Linking The Van Hiele Theory to Instruction. *Teaching Children Mathematics*, *21*(5), 304–313.

İbili, E., Çat, M., Resnyansky, D., Şahin, S., & Billinghurst, M. (2019). An assessment of geometry teaching supported with augmented reality teaching materials to enhance students ' 3D geometry thinking skills. *International Journal of Mathematical Education in Science and Technology*, *5211*. https://doi.org/10.1080/0020739X.2019.1583382

Kaleli, G., & Koparan, T. (2016). The Effect of Designed Geometry Teaching Lesson to the Candidate Teachers ' Van Hiele Geometric Thinking Level. *Journal of Education and Training Studies*, *4*(1), 129–141. https://doi.org/10.11114/jets.v4i1.1067

Klemer, A., & Rapoport, S. (2020). Origami and GeoGebra Activities Contribute to Geometric Thinking in Second Graders. *Eurasia Journal of Mathematics, Science and Technology Education*, *16*(11), 3–12.

Linn, M. C., Petersen, A. C., Linn, M. C., & Petersen, A. C. (1985). Emergence and Characterization of Sex Differences in Spatial Ability : A Meta-Analysis. *Child Development*, *56*(6), 1479–1498.

Maier, P. H. (1996). Spatial Geometry and Spatial Ability - How to Make Solid Geometri Solid? *Maier, Pete Herbert*, 69–81.

Mammarella, I. C., Giofrè, D., & Caviola, S. (2017). Learning Geometry : the Development of Geometrical Concepts and the Role of Cognitive Processes. In *Acquisition of Complex Arithmetic Skills and Higher-Order Mathematics Concepts* (pp. 221–246). https://doi.org/10.1016/B978-0-12-805086-6/00010-2

Mayberry, J. (1983). THE VAN HIELE LEVELS OF GEOMETRIC THOUGHT IN UNDERGRADUATE PRESERVICE. *Journal for Research in Mathematics Education*, *14*(1), 58–69.

Mdyunus, A. S., & Hock, T. T. (2019). Geometric Thinking of Malaysian Elementary School Students. *International Journal of Instruction*, *12*(1), 1095–1112.

Miles, M. B., Huberman, A. M., & Saldana, J. (2014). *Qualitattive Data Analysis AMethods Sourcebook* (3rd ed.). California: SAGE Publications.

Moursund, D. (2007). *Introduction to Using Games in Education : A Guide for Teachers and Parents* (pp. 1–155). pp. 1–155. University of Oregon.

Mudaly, V., & Sibiya, M. R. (2018). The Effects of The Geoboard on Learner Understanding of Geometry Theorems. *International Journal of Science and Research*, *11*(1), 90–98. https://doi.org/10.21506/j.ponte.2018.11.8

Prasad, P. W. C., Maag, A., Redestowicz, M., & Hoe, L. S. (2018). Unfamiliar Technology: Reaction of International Students to Blended Learning. *Computers & Education*, *122*(July), 92–103. https://doi.org/10.1016/j.compedu.2018.03.016

Prayitno, M., Suryadi, D., & Mulyana, E. (2019). Geometric thinking level of the Indonesian seventh grade students of junior high school. *Journal of Physics:Conference Series*, *1188*. https://doi.org/10.1088/1742-6596/1188/1/012036

Pujawan, I. G. N., Suryawan, I. P. P., & Prabawati, D. A. A. (2020). The Effect of Van Hiele Learning Model on Students ' Spatial Abilities. *International Journal of Instruction*, *13*(3), 461–474.

Škrbec, M. (2015). Identifying and Fostering Higher Levels of Geometric Thinking. *Eurasia Journal of Mathematics, Science and Technology Education*, *11*(3), 601–617. https://doi.org/10.12973/eurasia.2015.1339a

Suastika, I. K., & Wahyuningtyas, D. T. (2020). Inquiry-based E-module for Geometry Learning Subject. *Universal Journal of Educational Research*, *8*(1), 243–248. https://doi.org/10.13189/ujer.2020.080130

Sumintono, B., & Widhiarso, W. (2014). *Aplikasi Model Rasch Untuk Peneliti Ilmu-Ilmu Sosial*. ISBN 978-602-14371-1-7.

Suwito, A., Yuwono, I., Parta, I. N., Irawati, S., & Oktavianingtyas, E. (2016). Solving Geometric Problems by Using Algebraic Representation for Junior High School Level 3 in Van Hiele at Geometric Thinking Level. *International Education Studies*, *9*(10), 27–33. https://doi.org/10.5539/ies.v9n10p27

Syamsuddin, A. (2019). Investigating Student ' s Geometric Thinking in Looking for the Linkages of Quadrilaterals (A Case Study on Students in The Formal Operation Stage). *International Journal of Environmental & Science Education*, *14*(7), 435–444.

Topuz, F., & Birgin, O. (2020). Developing teaching materials supported with Geogebra for circle and disc subject at seventh grade 1. *Ilkogretim Online - Elementary Education Online*, *19*(3), 1–17. https://doi.org/10.17051/ilkonline.2020.735184

Trimurtini, Safitri, T. R., Sari, E., & Nugraheni, N. (2020). The effectivity of contextual teaching and learning (CTL) approach with Geoboard media on mathematics learning for four-grade elementary students. *Journal of Physics: Conference Series*, *1663*. https://doi.org/10.1088/1742-6596/1663/1/012050

Uttal, D. H., Meadow, N. G., Tipton, E., Hand, L. L., Alden, A. R., Warren, C., & Newcombe, N. S. (2013). The Malleability of Spatial Skills : A Meta-Analysis of Training Studies. *Psychological Bulletin*, *139*(2), 352–402. https://doi.org/10.1037/a0028446

Vojkuvkova, I. (2012). The van Hiele Model of Geometric Thinking. *WDS'12 Proceedings*, *1*, 72–75.

Yilmaz, B. (2009). On the development and measurement of spatial ability. *International Electronic Journal of Elementary Education*, *1*(2), 83–96.

Yurt, E., & Tünkler, V. (2016). A Study on the Spatial Abilities of Prospective Social Studies Teachers : A Mixed Method Research *. *Educational Sciences : Theory & Practice*, *16*(3), 965–986. https://doi.org/10.12738/estp.2016.3.0324