Ilkogretim Online - Elementary Education Online, 2021; 20 (1): pp. 397-406
http://ilkogretim-online.org
doi: 10.17051/ilkonline.2021.01.036

Investigating the generalization skills of 9-11th grade students in the context of the "cube breaking" question

Handan Demircioğlu, *Cumhuriyet University, Turkey*, hdemircioglu@cumhuriyet.edu.tr, ORCID 0000-0001-7037-6140
Seda Peker, *Özel Antalya Final Okulları, Antalya* sedapeker88@gmail.com, ORCID 0000-0003-0751-5181

Abstract. The aim of this study is to investigate the generalization skills of 9-11th grade students according to grade and academic success. Case study, one of the qualitative research methods, was used.63 students from the 9th grade, 56 students from the 10th grade, and 61 students from the 11th grade have participated to the study in the 2019-2020 academic year from a private Science high school. The data were collected with "the cube breaking" question. Content analysis was used in the analysis of the data. The findings obtained showed that students from 11th grader were more successful when generalizing but students from 10th grader could not reach generalization. At the same time, when generalization skills are compared according to academic success, it is determined that the thinking processes in the generalization process differ according to academic success.

Keywords: Generalization, generalization strategies, mathematics education, academic success

Received: 22.05.2020 Accepted: 18.08.2020 Published: 01.12.2020

INTRODUCTION

Generalization is expressed as one of the sub-dimensions of mathematical thinking, a high-level cognitive skill (Krutetskii, 1976), a mental process used in the development of concepts and one of the stages of algebraic thinking (Tanışlı & Yavuzsoy-Köse, 2011). Therefore, as Mason (1996) states, it is the essence of mathematics. As a reflection of this, one of the aims of teaching mathematics is to gain students generalization skills. Since mathematical thinking is a thinking process that is not different from daily thinking, gaining generalization skills will contribute to overcoming the problems encountered in daily life. As Burton (1984) stated, since the recognition of a pattern will cause generalization, generalization are frequently made in repetitive situations in daily life.

Polya (1957, p. 108) defined generalization as "*passing from the consideration of one object to the consideration of a set containing that object; or passing from the consideration of a restricted set to that of a more comprehensive set containing the restricted one*" . Yerushalmy (1993) stated that generalizations are the special type of prediction and are realized by reasoning. In general, it can be defined as the expression of an argument in a wider context (Harel & Tall, 1991) extending the field of validity and reaching a general judgment from special cases (Dreyfus, 1991). As can be seen from these definitions, generalization is described as both a process and a product.

The behaviors exhibited during the generalization process, the steps followed and the generalization strategies used have been the subject of many studies. Ellis (2007) introduced a generalization taxonomy that examined the generalization processes of students. In this taxonomy, Ellis expressed his mental activities as generalization actions and individuals' expressions as reflection generalizations and subdivided them. According to Polya (1957), generalization takes place gradually in the form of explaining the observed phenomenon, giving examples about the subject and then examining the special examples. According to Radford (2010), generalizations consist of three stages: factual, contextual and symbolic. The first stage is the factual generalization, where actions are carried out operatively and the generalization made remains in the physical dimension. The second stage is the contextual generalization in which language is used to describe the more abstract and generalizations that the student interprets on the next term based on the figures. The third stage is the symbolic generalization stage in which generalization is expressed by making algebraic notations. Varhol, Drageset and Hansen (2020) studied how 8th grade students cooperate and contribute to generalizing their mathematical generalization problems. In this study, the framework proposed by Radford was used. The findings showed that the collaboration groups formed started with algebraic generalization and then proceeded through factual and contextual generalization. In addition, all groups have shown that they produce solutions at the symbolic generalization level.

Similarly, Garcia-Cruz and Martinón (1998) examined the generalization levels of students in linear patterns in three stages. In the procedural activity or local generalization phase, the student realizes the repetitive or recursive feature of the pattern and can see the common difference with the strategies used. So he/she achieves a local generalization. In the next stage, conceptual understanding, the student applies similar action to a similar problem and develops a strategy as a generalization product. The rule created in the previous problem has now become the stimulus. This stimulus is taken into an appropriate scheme in the mind with the assimilation-regulation processes. Thus, a strategy is created by taking into account all the performance of the student in this process. This strategy is used in a new and similar problem. Thus, conceptual understanding or global generalization is realized. The generalization, which is at the level of scientific thought, in which the relationships and connections between objects, mental analysis is made and which requires a higher thought, is called theoretical generalization. Of course, it is as important as the strategies used in this process as well as understanding the generalization process. It is seen that these are stated as dividing the pattern into pieces, obtaining the next term by counting from the previous term, strategies used in changing number and shape patterns, iterative thinking, clear thinking, visual thinking, numerical thinking, pragmatic thinking (Sucuoğlu, 2015; Lannin, 2005; Becker & Rivera, 2003).

When studies on generalization are examined; it is seen that they are collected under headings such as revealing generalization strategies (Stacey, 1989; Orton & Orton, 1999; Sasman, Linchevski & Olivier, 1999; Chua & Hoyles, 2010; Tanışlı & Yavuzsoy Köse, 2011; Sucuoğlu, 2015), analyzing the generalization process (Lan-Ma, 2007), generalization difficulties (Aslan, 2011), the effect of numerical and formal clues of the generalization process (Rivera & Becker, 2003), the effect of problems on solution strategies related to linear patterns (Samson, 2007). There was no study where variables such as academic achievement, mathematics curriculum and different grade levels were handled together and the effect of these on generalization skills was investigated.

In order for the teaching of mathematics to be effective, it is important for both the student, the teacher, the researchers and the curriculum makers to take the center of the learner and reveal the actions that take place in the process of creating the information. Since one of the objectives of the program is to improve the generalization skill, it becomes more important to reveal the generalization processes of the students and to determine the ways of thinking.

Mathematics teachers' awareness of the mental processes of their students and adjusting the learning environment accordingly will both facilitate the learning of mathematics and guide the student in creating new ways of understanding and thinking. On the other hand, the curriculum guides the students in the behaviors we aim to gain. From this perspective, the participants of this study come from different examination systems. It is important in terms of taking and examining as a variable to question the change in generalization skills. Accordingly, the aim of this study is to examine how the generalization skills of students at different grade levels and different academic achievements coming from different exam systems change.

METHODS

In this study, Generalization skills of 9th, 10th and 11th grade students were examined. A case study was used among qualitative research designs. The process followed in the case study; determining and developing research questions, developing the sub-problems of the research, determining the analysis unit, determining the situation to be studied, selecting the individuals to participate in the research, collecting the data and associating the collected data with the suggestions or sub-problems, analyzing and interpreting the data and reporting the case study.
In this study, firstly, a literature review was carried out. After the literature review, a question pool was created to be used to collect data in the study. Then, the selection of application

questions for the pilot study from this question pool was carried out in line with expert opinions. A pilot study was conducted to determine whether these selected questions measure the desired skill. At the same time, the researcher gained experience through pilot study.

Participants

In this study convenience sampling method was used. 63 students from 9th grades, 56 from 10th grades, 61 from 11th grades attended the study who have been studying at a private Science high school in the 2019-2020 academic year. There are six branches in each of the 9th, 10th and 11th grades in this high school. This study was carried out with students from A, B and C branches. These branches are level groups and A is classified as the best, B is intermediate and C is the lower intermediate. Each participant is coded as class and branch. In other words, coding in the form of 9a-Ö30 refers to the student number 30 in the A branch in the 9th grade. While collecting the data the study firstly applied to B, then C and lastly A branches in 11th grades, so the coding was done in this order. Since it was coded in this way in data analysis, it was given in the study as in this form without making any changes. The participants of the study were summarized in Table 1.

Table 1. *Participants of the study*

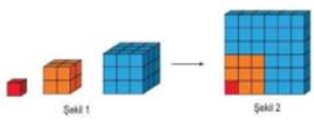
Grade	Branch	Student encodings	f
9	A	Ö1-Ö17	63
	B	Ö18-Ö40	
	C	Ö41-Ö63	
10	A	Ö1-Ö22	56
	B	Ö23-Ö37	
	C	Ö38-Ö56	
11	B	Ö1-Ö20	61
	C	Ö21-Ö41	
	A	Ö42-Ö61	

As can be seen from Table 1, the number of students attending from 9th, 10th and 11th classes is close to each other. The reason for choosing these classes is that their high school entrance exams' are different. The 11th grade students passed to high school with TEOG (Transition from Basic Education to Secondary Education System); 10th graders passed half term TEOG and half term with LGS (Transition to High School System); and 9th grade students passed with LGS exam system. As a result, the students participating in the study are students who are entitled to enroll in Science High School with a different system.

Data Collection Tool and Data Collection

The data were obtained with the cube breaking question given below.

Figure 2 is obtained when the cubes in Figure 1 are combined after they are separated.



a) If the green cube pieces given below with the cubes given in figure 1 are tried to be obtained with the same method, what is the number of cubes in the new shape obtained?

b) If you continue this way until one cube with a side length of 10 br is added, what is the number of cubes of the new shape obtained?

c) If you continue this way until one cube with a side length of n br is added, what is the number of cubes of the new shape obtained?

In the first stage, students were expected to explore the rule in a special case $1^3+2^3+3^3= (1+2+3)^2$ by interpreting the shape given in the visual and starting from this shape. In option a, it is expected to go one step further and do what he/she did with 3 cubes with 4 cubes and discover the rule for the next step 13 + 23 + 33 +43 and give (1 + 2 + 3 + 4) 2 = 102 = 100 answer. It is then expected to forward the rule for 10 cubes, which is a remote case in option b, make an inference, and give the answer to $(1+2+3+...+10)^2= (\frac{10.11}{2})^2 = 55^2 = 3025$. In the last stage, if any number of cubes are taken, that is, student is expected to generalize the rule to any situation and give the answer $1^3+2^3+3^3+4^3+...+ n^3=(1+2+3+...+n)^2$. The data were collected in a single session in each class.

Data analysis

In the analysis of the data, each of the options a, b and c are coded as “true”, “false” and “calculation error”. Table 2 shows the sample analysis table for the 9th grade. The first column of this table contains the code given to the student who participated in the study. Coding 9a means that the student is in the A branch in the 9th grade. Ö1, Ö2,... is the number given to the student in 9A class. A, B and C columns show the options of the question. Thus, thanks to this table, the answers given by each student in the 9th grade in the options a, b and c were compared. The answers of the student to the “a” and “b” options, who made the “c” option true, are also shown. Moreover, it was possible to make comparisons among the branches of the 9th grade, which were formed according to academic success. These tables, prepared for 9th grade, were prepared in the same way in 10th and 11th grades. In this way, comparisons were also made between classes.

Table 2. *Analysis table for 9th grades*

KODLAR	A	B	C	KODLAR	A	B	C
9a-Ö1	✓	✓	✗	9b-Ö33	✓	✗	
9a-Ö2			✗	9b-Ö34			
9a-Ö3	✗	✗		9b-Ö35	✗	✗	✗
9a-Ö4	✓	✗	✗	9b-Ö36			
9a-Ö5	✗	✓	✗	9b-Ö37	✓	✓	✗
9a-Ö6	✓	✓		9b-Ö38			
9a-Ö7	✓	✓		9b-Ö39	✓	✓	✓
9a-Ö8	✓	✗	✓	9b-Ö40		✓	✗
9a-ö9	✓	✓		9c-Ö41			
9a-Ö10	✓	✓	✓	9c-Ö42	✓		
9a-Ö11	✓	✓		9c-Ö43	✓		
9a-Ö12	✓	✓		9c-Ö44			
9a-Ö13	✓	✓		9c-Ö45			
9a-Ö14	✓	✓	✓	9c-Ö46			
9a-Ö15				9c-Ö47	✗	✗	
9a-Ö16	✓	✓	✓	9c-Ö48			
9a-Ö17	✓			9c-Ö49			
9b-Ö18	✗	✓	✗	9c-Ö50	✗	✗	✗
9b-Ö19				9c-Ö51			
9b-Ö20				9c-Ö52			
9b-Ö21	✓			9c-Ö53	✓		✗
9b-Ö22	✓			9c-Ö54			
9b-Ö23				9c-Ö55			
9b-Ö24				9c-Ö56	✗	✗	✗
9b-Ö25				9c-Ö57			
9b-Ö26	✗	✗		9c-Ö58			
9b-Ö27	✓	✗		9c-Ö59			
9b-Ö28				9c-Ö60			
9b-Ö29	✓			9c-Ö61			
9b-Ö30				9c-Ö62			
9b-Ö31	✗	✗		9c-Ö63			
9b-Ö32	✓	✗	✗				

Doğru yapanlar	✓
Yanlış yapanlar	✗
boş bırakanlar	
işlem hatası	✓

In this table, true answers are coded as ✓, false answers as ✗, and answers with calculation errors as ✓. Although there was a calculation error, the answers that showed generalization skills were accepted correct answer, but they were colored yellow to be specific. Wrong answers were handled as answers away from the expected ones. While giving the findings, first of all, total true and false answers were given in a table, and then each class was handled separately according to the students. In this way, both class comparisons were made and comparisons were made among the students according to the options. In the next stage, the answers given in each option of the problem were analyzed by content analysis. The main purpose in content analysis is to reach concepts and relationships that can explain the collected data. The data interpreted in the study are subjected to a deeper process in content analysis, and codes and categories that cannot be detected by a descriptive approach can be discovered as a result of this analysis. For this purpose, the data collected must first be conceptualized, then arranged according to the emerging concepts, and the themes explaining the data must be determined accordingly. In this analysis, the results such as which stages the students used most in the generalization process, which were successful and which failed, and whether they had the ability to generalize were obtained. Some examples of students' solution processes are shown, and then each data is examined one by one according to a certain category system, and codes and themes are created. After getting the expert opinion for the codes and themes, a general evaluation of each solution process was made. In this study, the researcher explained in detail the method, process, data collection and analysis methods of the research, what is done to interpret the findings and reach the results. In the analysis and interpretation of the data, it was objectively approached and the data obtained were directly explained with quotations.

RESULTS

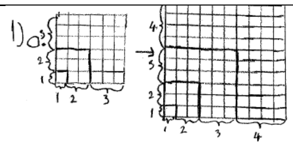
In this section, findings obtained as a result of analyzes made based on the methods and techniques specified are included. In Table 3, the analysis of the question according to the options and the grades are given.

Table 3. *Analysis of the cube breaking question according to options and grades*

	option a			option b			option c		
	True	**False**	**Null**	**True**	**False**	**Null**	**True**	**False**	**Null**
9th grade (63 Ss)	24	9	30	15	12	36	5	11	47
10th grade (56 Ss)	12	10	34	5	10	41	0	10	46
11th grade (61 Ss)	24	11	26	12	15	34	6	16	39

As can be seen from Table 3, for finding the rule for n = 3 and selecting option a for n=4 special case; while 24 students could made from 9th and 11th grades; only 12 students could made from the 10th grades. In option b, 15 students from the 9th grades, 5 students from the 10th grades and 12 students from the 11th grades answered true for the n = 10 special case. As can be seen, the number of students who answered true in option b has almost decreased to half. In option c, 6 students from 5th and 11th grades answered the question true, and no students from the 10th grades responded true. While 5 students from 9th grade who made the c option true are expected to make the a and b options true, only 1 of these students could not make the b option true. However, the student still made the c option true. In other words, there are 4 students who make all three options a, b and c true. In addition, 8 students did not make the c option true even though they did the a and b options true. Although no student in the 10th grade could answer the c option true, it was observed that 5 students did the a and b options true. 6 students from 11th grade answered the c option true. All of these students answered the a and be options true. On the other hand, it was observed that 4 students did not answer c option true even though they made both a and b options. In the cube breaking question, the content analysis of the students' answers in each option was made. The categories obtained from the answers given to option a are given in Table 3.

Table 3. *Findings obtained from the answers given to c option of Cube Breaking question*

<table>
<tr><th>Category</th><th>Student’s answer</th><th>9. grade</th><th>10. grade</th><th>11. grade</th></tr>
<tr><td>Counting unit cubes</td><td>1-) 36 adet küp var. → 1³+2³+3³ = 36
yeşil ⇒ 4³ = 64
36 + 64 = 100 adet küp var.
→ 100 adet küp olur.
(9a-Ö10)</td><td>17</td><td>7</td><td>16</td></tr>
<tr><td>Area of the square</td><td>yeşilden 4
mavi 3
Turuncu 2
kırmızı 1
(1+2+3+4) = 10 birkenarı
10x10 = 100
(11b-Ö3)</td><td>1</td><td>1</td><td>3</td></tr>
<tr><td>Drawing shape</td><td>
(10b-Ö23)</td><td>0</td><td>1</td><td>1</td></tr>
<tr><td>Those only who wrote the result</td><td>100</td><td>6</td><td>3</td><td>4</td></tr>
</table>

As can be seen from Table 3, the answers given were collected under 4 different categories. In this option, students are expected to discover the rule for $1^3+2^3+3^3+4^3$ and give the answer $(1+2+3+4)^2=10^2=100$. Students who answered in the category of “counting unit cubes” reached the result of $1^3+2^3+3^3+4^3=100$ by calculating the number of red, orange and blue cubes given in the question and adding the number of cubes in the green cube. While the number of students responding in this way is 17 in the 9th and 11th grades, it is 7 in the 10th grade. The students in the category of “area of the square” realized that the new shape formed was a square of an edge and reached the result as $(1+2+3+4)^2$ based on the area of the square. While the number of students responding in this way is 1 in 9 and 10th grade, it is 3 in 11th grade. The students who responded in the category of “drawing shape” thought in the same way as the students in the “area of the square” category, but showed that one side of the new square obtained by drawing shape is 1+2+3+4. While the students who answered in this way were 1 in 10th and 11th grades, no students in 9th grade answered in this way. Students who answered in the category of “Those only who wrote the result” wrote only the result as 100. The number of students responding in this way is 6 students in 9th grade, 3 in 10th grade and 4 in 11th grade. The categories obtained from the answers given to option b are given in Table 4.

Table 4 *Findings from the answers given in option b of the cube breakdown question*

	Student’s answer	9. grade	10. grade	11. grade
Counting unit cubes	$1^3+2^3+3^3+4^3+5^3+6^3+7^3+8^3+9^3+10^3$ (10c-Ö56)	3	3	5
Area of the square	$9=(\overbrace{1+2+3+4+5+6+7+8+9+10}^{55})^2=$ (9b-Ö40) $\underbrace{1^3\ 2^3\ 3^3}_{}\ 4^3\ 5^3\ 6^3\ 7^3\ 8^3\ 9^3\ 10^3$ 6x6 kare yapılır 1+2+3+4+5+6+7+8+9+10=55 55x55 kare oluşur (10bÖ23) $1^3+2^3+3^3+4^3+5^3+6^3+7^3+8^3+9^3+10^3$ 1 2 3 4 5 ... 10 tane (1+2+3....+10)=55 $\boxed{55.55}=55^2$ (11b-Ö09) $1^3+2^3+3^3+...+10^3=\left(\frac{10.11}{2}\right)^2=55^2$ (11a-Ö48)	8	2	6
Those only who wrote the result	3025	4	0	1

As seen from Table 4, the answers were collected under 4 different categories. In this option, it is expected to make an inference for 10 special case and give the answer $(1+2+3+...+10)^2=(\frac{10.11}{2})^2=55^2=3025$. Students responding in the category of “counting cubes” made $1^3+2^3+3^3+4^3+5^3+6^3+7^3+8^3+9^3+10^3=1+8+27+...+100=3025$ as in option a. 3 students from the 9th and 10th grades and 5 students from the 11th grades responded in this way. The students who responded in the category of "Area of the square" realized that the length of one side of the square to be formed as in option a in Table 4.5 is 1+2+3+4 and by making use of the area of the square $(1+2+3+4+5+6+7+8+9+10)^2=55^2=3025$. On the other hand, 8 students from 9th grade, 2 students from 10th grade and 6 students from 11th grade responded in this way. 1 student from

9th grade and 4th and 11th grade only wrote the result as 3025. The categories obtained from the answers given to option c are given in Table 5.

As can be seen from Table 5, the answers were collected under two categories. As in options a and b, students responded either by starting from the number of cubes per unit or by starting from the area of the square. A total of 4 students, including 2 students from the 9th and 11th grades, who answered in the category of "counting unit cubes", stated that the number of cubes would be calculated as $1^3+2^3+3^3+4^3+\ldots+ n^3$. A total of 8 students, including 4 students from 9th and 11th grades, answered in the "area of the square" category as $1^3+2^3+3^3+4^3+\ldots n^3 = (1+2+3+4+\ldots n)^2 = \left(\frac{n.(n+1)}{2}\right)^2$.

Table 5. *Findings obtained from the answers given in option c of cube breaking question*

	Student's answer	9. grade	10. grade	11. grade
Counting unit cubes	$1^3+2^3+3^3+4^3---n^3$ = Toplam küp sayısı (9a-Ö8)	2	0	2
Area of the square	$=)\left(\frac{n.(n+1)}{2}\right)^2$ (9a-Ö14) $1^3+2^3+3^3+\ldots+n^3=\left(\frac{n.n+1}{2}\right)^2$ (11a-Ö48)	3	0	4

During the generalization process, the strategies used by 9th, 10th and 11th grades students in options a, b, and c were evaluated together and the findings were summarized in Table 6.

Table 6. *Strategies used when generalizing from proximal to remote*

Category	9. grade			10. grade			11. grade		
	a	b	c	a	b	c	a	b	c
Counting unit cubes	17	3	2	7	3	0	16	5	2
Area of the square	1	8	3	1	2	0	3	6	4
Drawing shape	0	0	0	1	0	0	1	0	0
Those who only wrote result	6	4	0	3	0	0	4	1	0

As can be seen from Table 6, the most preferred strategy for n=4, which is close in 9th, 10th and 11th grades, is the counting unit cubes strategy. However, as generalization is made, the strategy used in all 9th, 10th and 11th grades the number of true answers decreases, while the strategy used increases from the "counting cubes" category to the "area of the square" category.

DISCUSSION and CONCLUSIONS

The students participating in the study are expected to interpret the shape given in the visual before giving any answer to the cube breaking question and find out that it is $1^3+2^3+3^3=(1+2+3)^2$ for the special case n=3. Then they are expected to go one step further for n=4, which is the closest case, and do what they did with 3 cubes with 4 cubes, and discover the rule for $1^3+2^3+3^3 +4^3$ and give the answer $(1+2+3+4)^2=10^2=100$. They are then expected to transfer what they did for n=10, a remote situation, that is, to forward the rule for 10 cubes, make an inference, and give the answer $(1+2+3+\ldots+10)^2= (\frac{10.11}{2})^2 = 55^2 = 3025$ At the last stage, if any number of cubes are taken, they are expected to generalize the rule to any situation and give the answer to $1^3+2^3+3^3+4^3+\ldots+ n^3=(1+2+3+\ldots+n)^2$. The findings of the study showed that while 5 students from the 9th grades and 6th and 11th grades could generalize, no students from the 10th grades could do it. In other words, when generalization skills of different grade levels are compared; it is seen that 11th grades are more successful and 10th grades could not reach generalization. This finding does not coincide with the findings of the studies in which generalization skills were examined according to the grade of education. Indeed, when Akkan (2009) compared the generalization processes of 5-8th grades students, as the grade of education increased, both the solution strategies in the process of generalizing the patterns differed, and the students' strategy culture and their ability to explore distant terms increased. Similarly, Warren (2003) stated that there is not much difference in making generalizations about arithmetic structures among student groups with different learning experiences. Another reason for this result can be shown that the students participating in the study are enrolled in Science High School with different examination systems. Indeed, 9th grades have prepared for LGS exam, 10th grades have prepared for half term TEOG, half term for LGS exam and 11th grades have prepared only for TEOG exam.

When each grade is compared to its academic achievement, that is, when the generalization skills of students in branches A, B and C are compared; it was determined that the thinking processes in the generalization process differ according to academic success. A, B and C branches were formed according to academic success. As a result of the study, it was determined that A branch was more successful than B and C branches and B branch was successful than C branch. This shows that students with high academic skills are also successful in the generalization process. Of course, academic success is considered as a criterion in participant selection in many studies (Sasman, et.all; 1998; Lan- Ma, 2007; Chua & Hoyles, 2010; Samson, 2007; Tanışlı, 2008). The findings obtained from this study showed that Tanışlı (2008) 's success levels were not effective in finding the rule, continuing the pattern to a proximate and finite step and choosing the pattern; on the other hand, it is not in parallel with the finding that patterns are presented (number sequence, function table, shape). However, Dindyal (2007) supports the finding that "students with low mathematics achievement have more difficulty in the generalization step". When generalizations made from proximal to remote are examined; students generalizing for n are expected to do it true for n=4 and n=10. However, only one of these students could not answer correctly for n=10. On the other hand, there are 8 students from the 9th grade, 5 students from the 10th grade and 4 students from the 11th grade, although they give true answer for n=4 and n=10. It was interpreted that these students were able to solve the problem in special cases, but with generalization problems.

When comparisons are made according to the grades and the strategies used in each option; it is seen that the answers given for n=4 are gathered under the categories of "Counting unit cubes ", "Area of the square" and "Those only who write the result". In the counting unit cubes category, the students reached the result of $1^3+2^3+3^3+4^3=100$ by calculating the numbers of the red, orange and blue cubes given in the question and adding the number of cubes in the green cube. Students in the area of the square category realized that the new shape formed was the square of one side and reached the result as $(1+2+3+4)^2$ based on the area of the square. The students who responded in the shape drawing category thought the same way as the students in the area of the square, but showed that one side of the new square obtained by drawing the shape is 1+2+3+4. Only the students who answered in the category of those only who wrote the result gave the result as 100. The findings showed that the majority of students responded by counting unit cubes in 9th, 10th and 11th grades. For n=10, it was discussed that the students did not respond in the shape drawing category, counting the unit cubes, the area of the square and those only who wrote the result. The reason for this may be that it is easy to draw shapes in the proximal case, while they think it is difficult in the remote case. Another important finding is that when the students switch from n=4 to n=10, the students respond more in the area of the square category instead of counting the unit cubes category. This finding is in line with Arcavi's (2003) idea that visual techniques are cognitively more supportive than analytical techniques. At the same time, this finding suggests that Yilmaz (2011) used visualizations in the early stages of the generalization process (that is, in the processes of association and research), and in the final stages they did not need visualization; When the generalization process is considered as a whole, it coincides with the finding that they started the generalization problems visually, and continued with algebraic operations in the later stages. On the other hand, as seen in the findings of many researches in the generalization process, they have adopted two approaches, basically visual and numerical

(Sucuoğlu, 2015; Tanışlı, 2008; Lan Ma, 2007; Orton & Orton, 1999; Sasman et al., 1999; Stacey, 1989). The findings of this study are limited to 9-11th grades students in a Science High School participating in the study. However, the findings show that there are connections between academic achievement and classes. Investigations can be made with more students studying in different high schools and the results can be examined according to the school variable.

REFERENCES

Akkan, Y. (2009). *İlköğretim öğrencilerinin aritmetikten cebire geçiş süreçlerinin incelenmesi*. (Unpublished Doctoral Dissertation). Karadeniz Teknik Universitesi.

Arcavi, A. (2003). The role of visual representations in the learning of mathematics.*Educational Studies in Mathematics, 52*, 215–241.

Aslan, R. (2011). *Örüntü kavramına ilişkin öğrenci güçlüklerini gidermeye yönelik bir ders tasarımı*. (Unpublished Master Dissertation), Dokuz Eylül Universitesi .

Burton, L. (1984). Mathematical thinking: The struggle for meaning. *Journal for Research in Mathematics Education, 15,* 35-49.

Chua, B. L., & Hoyles, C. (2010). Teacher and student choices of generalising strategies: A tale of two views? *Proceedings of the 5th East Asia Regional Conference on Mathematics Education, 2*, 24- 31. Tokyo, Japan: EARCOME.

Dindyal J 2007 *High school students' use of patterns and generalisations* In J. Watson & K.Beswick (Eds), Proceedings of the 30th annual conference of the Mathematics Education Research Group of Australasia 236-245

Dreyfus, T. (1991). Advanced mathematical thinking processes. In D. Tall (Ed.), *Advanced mathematical thinking* (pp. 25-41). Dordrecht, The Netherlands: Kluwer Academic Publishers.

Ellis, A. B. (2007). A taxonomy for categorizing generalizations: Generalizing actions and reflection generalizations. *Journal of the Learning Sciences*, 16(2), 221–262.

Garcia-Cruz, J. A., &Martinon, A. (1998). *Levels of genaralizations in linearpatterns.* Paper presented at the Proceeding of the 22 nd Sonference of the International Group for the Psychology of Mathematics Education, (2), (pp. 329- 336). University of Stellenbosch, South Africa.

Harel, G. & Tall, D. (1991). The general, the abstract, and the generic in advanced mathematics. *For the learning of mathematics*, *11(1)*, 38-42.

Lan- Ma, H. (2007). The potential of patterning activities to generalization. *Proceeding of The 31st Conference of the International Group for the Psychology of Mathematics Education, 3*, 225- 232. Seoul: PME.

Krutetskii, V. A. (1976). *The psychology of mathematical abilities in schoolchildren*. (J. Kilpatrick, & I. Wirszup, Trans.) Chicago: University of Chicago Press.

Lannin, J. K. (2005). Generalization and justification: The challenge of introducing algebraic reasoning through patterning activities. *Mathematical Thinking and learning*, *7(3)*, 231-258.

Mason, J. (1996). Expressing generality and roots of algebra. In N. Bednarz, C. Kieran, & L. Lee (Eds.), Approaches to algebra (pp.65–86). Dordrecht: Kluwer.

Orton, A. and Orton, J. (1999). Pattern and the approach to algebra. In A. Orton (Ed.), *Pattern in the Teaching and Learning of Mathematics* (pp 104-120). London: Cassell.

Polya, G. (1957). How to solve it (2nd ed.). Princeton: Princeton University Press

Radford, L. (2010). *Layers of generality and types of generalization in pattern activities*. In P. Brosnan, D. B. Erchickk, & L. Flevares (Eds.), *32nd annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (Vol. 4, pp. 37–62). Columbus: PME-NA.

Rivera, F. D., & Becker, J. R. (2003). The effects of numerical and figural cues on the induction processes of preservice elemantary teachers. *Proceedings of the 2003 Joint Meeting of PME and PMENA, 4*, 63- 70. Honolulu, Hawaii: Universty of Hawaii. 185

Samson, D. A. (2007). An analysis of the influence of question design on pupils' approaches to number pattern generalisation tasks. Master Thesis, Rhodes University, South Africa.

Sasman, M., Linchevski, L., Olivier, A., & Liebenberg, R. (1998). Probing children's thinking in the process of generalisation. *Proceedings of the Fourth Annual Congress of Association for Mathematics Education of South Africa*, 210- 218. Pietersburg: University of the North.

Stacey, K. (1989). Finding and using patterns in linear generalising problems. *Educational Studies in Mathematics*, *20(2)*, 147-164.

Sucuoğlu, D. (2015). İlköğretim Matematik Öğretmeni Adaylarının Değişen Örüntülere İlişkin Genelleme Stratejileri (Yüksek Lisans Tezi) Gazi Üniversitesi. Eğitim Bilimleri Enstitüsü

Tanışlı, D. (2008). İlköğretim 5*. sınıf öğrencilerinin örüntülere ilişkin anlama ve kavrama biçimlerinin belirlenmesi*. (Yayınlanmamış Doktora Tezi)Anadolu Üniversitesi Eğitim Bilimleri Enstitüsü İlköğretim Ana Bilim Dalı Sınıf Öğretmenliği Doktora Tezi, Eskişehir

Tanışlı, D. & Yavuzsoy Köse, N. (2011). Lineer şekil örüntülerine ilişkin genelleme stratejileri: görsel ve sayısal ipuçlarının etkisi. *Eğitim ve Bilim*, *36* (160), 184- 198.

Varhol, A. Drageset, O.G. & Hansen, M.N. (2020) Discovering key interactions. How student interactions relate to progress in mathematical generalization. Mathematics Education Research Journal https://doi.org/10.1007/s13394-020-00308-z

Yerushalmy, M. (1993). Generalization, induction, and conjecturing: A theoretical perspective. In L. Schwartz, M. Yerushalmy & B. Wilson (Eds.), *The geometric suppose: What is it a case of*? (pp. 57-84). Hillsdale, NJ: Lawrence Erlbaum.

Yılmaz, R. (2011). *Matematiksel Soyutlama ve Genelleme Süreçlerinde Görselleştirme ve Rolü*. (Unpublished Doctoral Dissertation) Gazi University. Ankara