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Teaching mathematics through didactic situations

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ABSTRACT- This article aims to show how through the use of didactic situations, plausible reasoning and problem solving, the student can solve non-routine problems about antiderivatives. The problems were applied to students taking integral calculus in the engineering faculty of a public university in Bogota, Colombia. The results obtained show the effectiveness of these schools in the teaching and learning of mathematics. The didactic situations presented in this work are based on the approach of non-routine challenging problems that are attractive to students and that can lead them to make conjectures about the concepts presented or proposed by the teacher.

KEYWORDS: didactic situations, problem solving, plausible reasoning.

I. INTRODUCTION

The theory of didactic situations originated, basically by mathematicians, in the Institutes for Research on Mathematics Education (IREM)created in France after the educational reform of the late 1960s,which imposed the teaching of" Modern Mathematics" Brousseau(1995).

Initially, the objective of the IREMs was to complement the mathematical training of teachers, which was reflected both in the updating of in-service teachers and in the programs and preparation of new teachers in teacher training colleges. Also important was its activity in the production of support materials for the work of teachers in the classroom, i.e. in the creation of mathematics texts, worksheets for students, didactic games and toys, collections of problems and exercises, less on sequences, etc. Brousseau(1986).

Since the fundamental objective of Didactics of Mathematics is to find out how didactic situations work, i.e., which of the characteristics of each situation are determinant for the evolution of students' behavior and, therefore, of their knowledge, this does not mean that it is only interesting to analyze successful didactic situations. Even if a didactic situation fails in its own purpose of teaching something, its analysis can constitute a contribution to Didactics, aslong as it makes it possible to identify the aspects of the situation that were determinant for its failure. Since didactic situations are the object of study of the didactics of mathematics, it has been necessary to develop a methodology for analyzing them. It is common for researchers who have come to educational experimentation with previous training in psychology to design didactic situations that test them in one or several classrooms, and then focust heir interest on the behaviors manifested by the students, with in the experimental situation. Thus, in this work the didactic situation proposed is based on the approach and solution of challenging problems in the concept of antiderivatives.

G.Brousseau(1999)introducetheconceptofdidacticmemorywhentheywonderaboutthe influence on learning of references, at a given moment, to the "mathematical" past of students. They work under the hypothesis that experience plays a decisive role in learning.

II. METHODOLOGY

For this research, which is inscribed in a social science, since it is based on mathematics education, the methodology used is under a qualitative approach and the theoretical bases of action research, since the main objective is to improve the quality of an action, in this case education, through theoretical and practical advances.

Scope of the investigation

This work seeks to generate through the French school of didactic situations and plausible reasoning the construction of a robust meaning of the concept of antiderivatives in students and at the same time to be able to apply it in the solution of problems with emphasis on engineering problems.

Population and Sample

The study population consisted of students from the Universidad Distrital "Francisco Joséde Caldas" in Bogotá, Colombia, and the sample was taken from a second semester differential calculus course in the faculty of engineering. The type of sampling was chosen by convenience, as indicated in Casal(2003).

For this purpose, seven groups of students were selected in which each group consisted of exactly three students and seven problems were proposed to be solved by each group, the problems were solved in class, this subject has an hourly intensity of six hours per week distributed in three days, that is to say, in each class two hours are seen.

During each session the students worked in groups and asked the teacher what they did not understand, the tutor guided them without helping them to solve the problems, leaving the solutions to each of the proposed problems to their own ingenuity.

III. APPLICATIONS AND RESULTS

The main objective of the didactic situations of Brousseau's theory is to allow students to ask themselves questions about what they should learn from their previous knowledge and their own experience. In this way, instead of trying to generate new ideas from scratch, all learning is done starting from an existing base, in our case the concepts of the derivative of a function to arrive at the robust concept of antiderivatives. When Brousseau's theory is applied, the teaching process consists mainly of two parts: the creationofadidacticsituationandtheacquisitionofknowledgethroughdidacticsituations.

The first are artificial scenarios generated by the teacher, who poses to his students problems that could be encountered in real life and that can be approached through logic and reasoning; in our work, problems and situations related to velocity and acceleration of bodies were posed. In this way, students will have to reflect and use their own ideas to try to find a solution to the proposed problems by using the concept of the derivative.

After this process, the didactic situations themselves come into play, in which the theory necessary to correctly solve the problems is provided and the students are helped to generate the appropriate answers to them.

By following this methodology, instead of receiving information passively, students can relate it more easily to their own experience.

When this process is carried out correctly, the three elements of the learning process(teacher, learner and situation)work together to provide the best possible results. Brousseau found that the application of this theory greatly improved the acquisition of knowledge, something that has subsequently been confirmed by a multitude of authors.

It is also a matter of putting into practice the dialectical interaction between the subject and the object, the latter being immersed in a system of relationships with very diverse characteristics. Piaget and Garcia(1982).Ontheonehand,thesubject-objectrelationshipmaybemediatedbytheinterpretationsthatcomefromthesocialcontextinwhichthesubject moves; on the other hand, objects already function in a certain socially established way, in relation to other objects or to other subjects. In the process of interaction, neither the subject nor the object is, there fore, neutral.

It should be noted that this research offers elements for the student to interpret a real situation, the student is not led "as if by a rail" to the solution of the problem ("the situation should lead the student to do what is sought, but at the same time should not lead him(Brousseau,G;1988).if this were to happen, that is, if the student were<led>to the solution of the problem, he would not be making decisions and therefore would not be producing knowledge.

Below are some problems proposed in the class with the solutions given by the students:

1. The first situation consists of a problem where they are presented with the graph of the derivative of a function with some initial conditions, the student must determine the original function based on the concepts of derivative, i.e. not starting from zero, and a series of questions that aims to lead the student to the concept of anti derivative. The following graph is the solution given by a group of three

students.

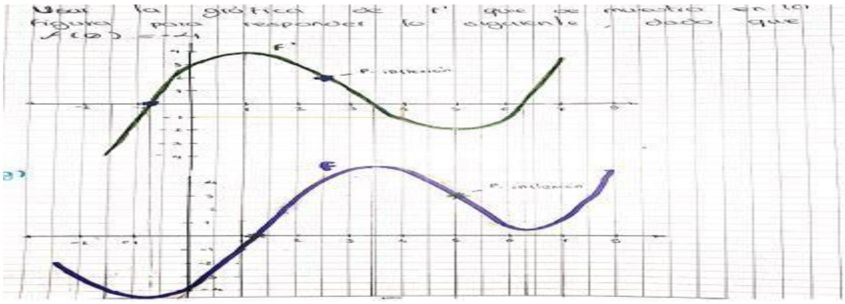


Figure1. problem1

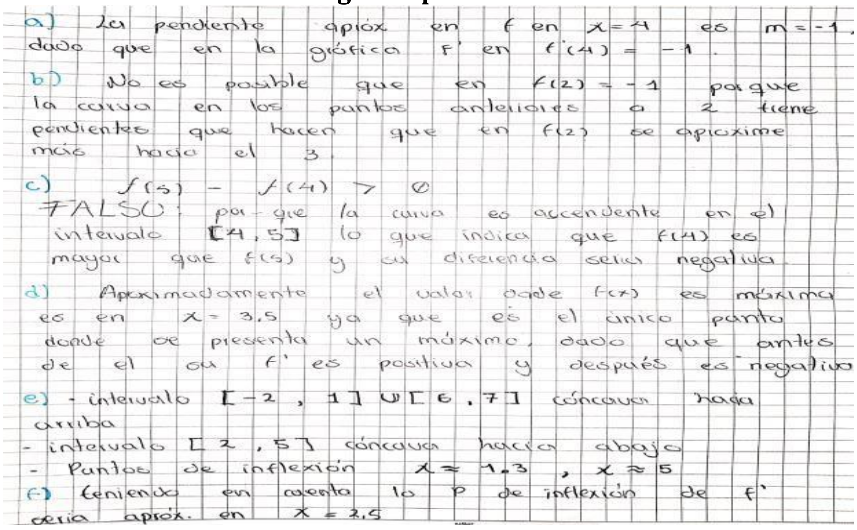

a) La pendiente aprox. en f en $x=4$ es $m=-1$, dado que en la gráfica f' en $f'(4) = -1$.
b) No es posible que en $f(2) = -1$ porque la curva en los puntos anteriores a 2 tiene pendientes que hacen que en $f(2)$ se aproxime más hacia el 3.
c) $f(5) - f(4) > \emptyset$
FALSO porque la curva es ascendente en el intervalo $[4, 5]$ lo que indica que $f(4)$ es mayor que $f(5)$ y su diferencia sería negativa.
d) Aproximadamente el valor donde $f(x)$ es máxima es en $x = 3.5$ ya que es el único punto donde se presenta un máximo, dado que antes de él su f' es positiva y después es negativo
e) - intervalo $[-2, 1] \cup [6, 7]$ cóncava hacia arriba
- intervalo $[2, 5]$ cóncava hacia abajo
- Puntos de inflexión $x = 1.3$, $x \approx 5$
f) teniendo en cuenta los P de inflexión de f' sería aprox. en $x = 2.5$

Figure2. Solution to problem1 presented by groupG1.

2. The following situation poses a motion problem, where students have to apply the concepts of velocity, acceleration and motion of bodies:

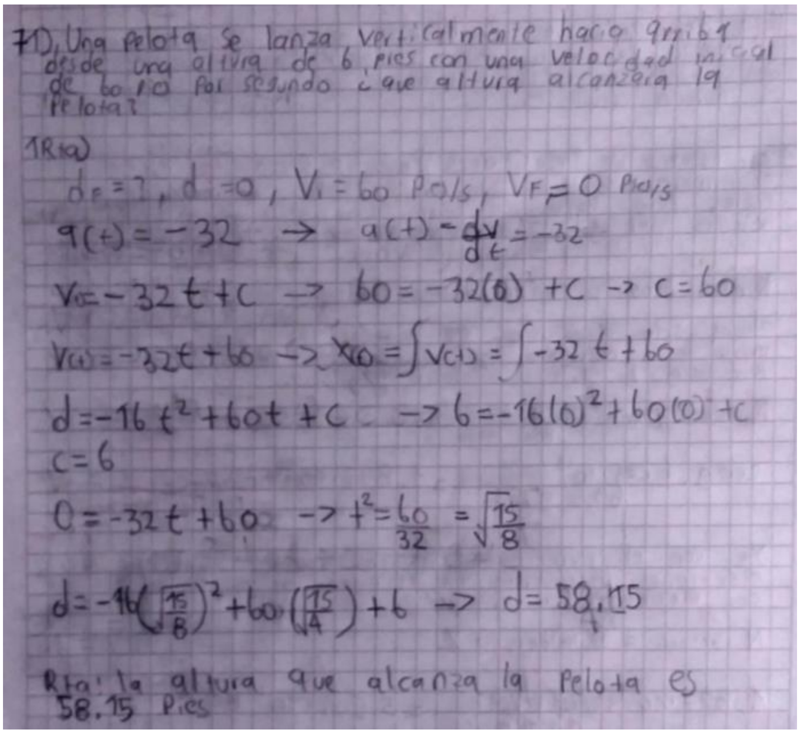

1) Una pelota se lanza verticalmente hacia arriba desde una altura de 6 pies con una velocidad inicial de 60 pies por segundo ¿que altura alcanzaría la pelota?
1Rta)
$d_f = ?,\ d_i = 0,\ V_i = 60$ Pies/s, $V_F = 0$ Pies/s
$a(t) = -32 \rightarrow a(t) = \frac{dv}{dt} = -32$
$V(t) = -32t + C \rightarrow 60 = -32(0) + C \rightarrow C = 60$
$V(t) = -32t + 60 \rightarrow x(t) = \int v(t) = \int -32t + 60$
$d = -16t^2 + 60t + C \rightarrow 6 = -16(0)^2 + 60(0) + C$
$C = 6$
$0 = -32t + 60 \rightarrow t^2 = \frac{60}{32} = \sqrt{\frac{15}{8}}$
$d = -16\left(\sqrt{\frac{15}{8}}\right)^2 + 60\left(\sqrt{\frac{15}{4}}\right) + 6 \rightarrow d = 58.15$
Rta: la altura que alcanza la pelota es 58.15 Pies

Figure3.Solution to problem2 presented by group G2.

It is observed that the student clearly applies the concepts of antiderivative in the motion of bodies where the conditions given by the problem are met.

3. The following situation is intended for the student to demonstrate in a formal way some formulas for motions in physics, using antiderivatives:

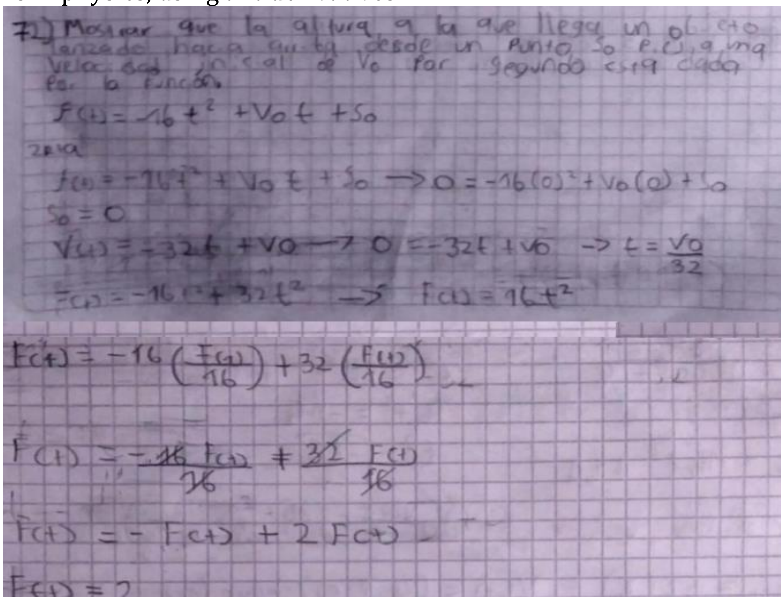


Figure4.Solution toproblem3presentedbygroupG2.

The solution clearly shows that students apply the concepts of derivatives and antiderivatives to find the maximum height of anobject, final velocity and equation of motion.

4. The following didactic situation corresponds to a physics application on free fall, where students must apply concepts of derivatives and antiderivatives in different contexts, some solutions are shown below:

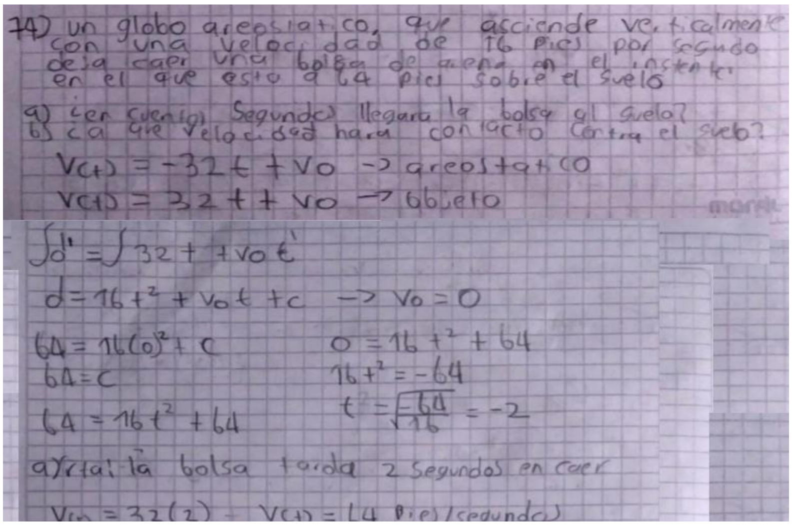
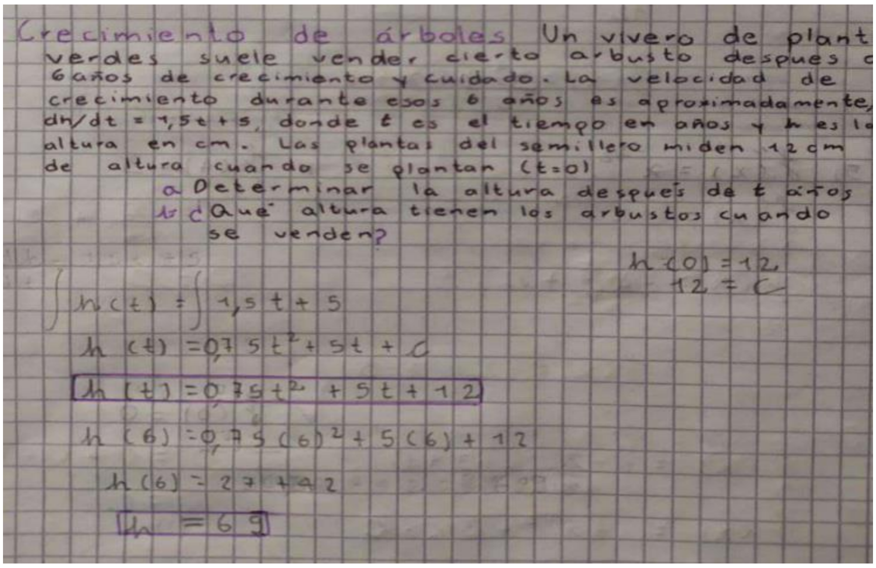


Figure5.Solution toproblem4presentedbygroupG1.

In the solution it can be clearly observed that the students, when applying the concepts of derivative and antiderivative applied to free falling bodies, arrive correctly at the solution.

Inthefollowingproblemthestudentsarepresentedwithadidacticsituationwheretheyhave to find the time and height of an application to the growth of some trees, as an introduction to first order differential equations.



Crecimiento de árboles Un vivero de plant[as] verdes suele vender cierto arbusto después [de] 6 años de crecimiento y cuidado. La velocidad de crecimiento durante esos 6 años es aproximadamente, $dh/dt = 1,5t + 5$, donde t es el tiempo en años y h es la altura en cm. Las plantas del semillero miden 12 cm de altura cuando se plantan $(t=0)$

a Determinar la altura después de t años

b ¿Qué altura tienen los arbustos cuando se venden?

$h(0) = 12$

$12 = C$

$\int h(t) = \int 1,5t + 5$

$h(t) = 0,75t^2 + 5t + C$

$\boxed{h(t) = 0,75t^2 + 5t + 12}$

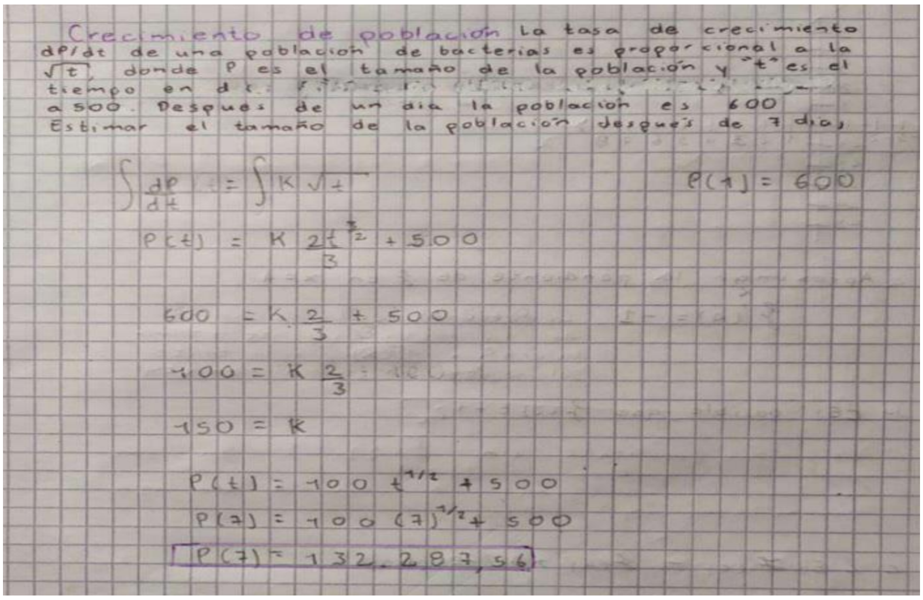
$h(6) = 0,75(6)^2 + 5(6) + 12$

$h(6) = 27 + 42$

$\boxed{h = 69}$

Figure6.Solution toproblem5presentedbygroupG3.

In the last problem, a didactic situation related to population growth was presented, where the starting point was a differential equation, which they solved correctly using the concepts of antiderivatives.



Crecimiento de población La tasa de crecimiento dP/dt de una población de bacterias es proporcional a la $\sqrt{t}$ donde P es el tamaño de la población y "t" es el tiempo en d... a 500. Después de un día la población es 600. Estimar el tamaño de la población después de 7 días.

$\int \frac{dP}{dt} = \int K\sqrt{t}$

$P(1) = 600$

$P(t) = K\frac{2t^{3/2}}{3} + 500$

$600 = K\frac{2}{3} + 500$

$100 = K\frac{2}{3}$

$150 = K$

$P(t) = 100\,t^{3/2} + 500$

$P(7) = 100(7)^{3/2} + 500$

$\boxed{P(7) = 132.287,56}$

Figure7.Solutiontoproblem6presentedbygroupG3.

The following table shows the results of the problems correctly solved by each group of students:

Groups							
	P1	P2	P3	P4	P5	P6	P7
G1	MB	MB	MB	MB	MB	MB	MB
G2	MB	MB	MB	MB	MB	MB	MB
G3	MB	MB	MB	MB	MB	MB	MB
G4	MB	MB	B	MB	MB	MB	MB
G5	MB	MB	MB	MB	MB	M	MB
G6	MB	MB	MB	MB	MB	MB	MB

G7	MB	B	R	MB	MB	B	MB
G8	M	B	MB	M	MB	M B	R
G9	MB	M	MB	B	MB	M B	MB
G10	MB	B	M	MB	B	B	R
G11	MB	MB	MB	B	MB	M B	B
G12	MB	B	MB	B	B	R	MB

Table1.Results

IV. CONCLUSIONS AND RECOMMENDATIONS

Given the solutions presented by the students, it can be seen that by creating well-planned didactic situations it is possible to improve the teaching of mathematics, in our case the concepts of antiderivatives, since the relationship between knowledge and knowledge are fundamental in the relationship between the acquisition of a mathematical concept and the resolution of problems.

Revisions, theoretical reorganizations, decontextualizations, relations between concepts, in short, reflections on entire sections of what has been done, play a fundamental role in the quality of the knowledge that is elaborated.

Finally, it can be deduced from this work that the theory of didactic situations is not ideologically neutral. It takes positions regarding the need to form young people with intellectual autonomy and critical capacity. By placing on the side of the school the responsibility of making students position themselves as theoretical subjects, as producing subjects, it is also made clear that all students have the right to construct and exercise the power that knowledge gives them.

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