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Learning To Solve Non-routine Mathematical Problems*

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ABSTRACT. Recent studies have pointed out that many schoolchildren do not master the skill of solving non-routine mathematical problems. In this article, a trial study designed to encourage seventh and eighth grade students to learn and use problem solving strategies is discussed. The strategies consist of six heuristic strategies known as Simplify the Problem, Guess and Check, Look for a Pattern, Make a Drawing, Make a Systematic List and Work Backward**.** Classroom activities consisted of a short whole-class introduction, group studies and a final whole-class discussion on the given problem. The teacher's role was to encourage and guide the pupils towards engaging in the problem. It is observed that in these classes some of these strategies are effective in learning, others are not.
Key Words: Problem solving, problem solving strategies, non-routine problems, mathematics teaching.

INTRODUCTION

Many research studies and projects have pointed out the importance of learning problem solving in school mathematics courses (Ford, 1994; Higgins, 1997; National Council of Teachers of Mathematics, 1989; Verschaffel, De Corte, Lasure, Van Vaerenbergh, Bogaert, Ratinckx, 1999). One of the major goals of mathematics education is the acquisition of the skill of learning how to solve problems. There are, however, conflicting views about the attainability of these goals (Verschaffel et al., 1999). Despite long years of instruction many research studies show that children are insufficient and not confident in having the aptitudes required for approaching mathematical problems, especially non-routine ones, in a successful way (Asman and Markowitz, 2001; Higgins, 1997).

The reasons for these deficiencies in primary and secondary school children can be attributed to two factors. The first of them is the lack of specific domain knowledge and skills (e.g. concepts, formulas, algorithms, problem solving). The second factor is shortcomings in the heuristic, metacognitive and affective aspects of mathematical competence. When confronted with unfamiliar complex problem situations, children mostly do not spontaneously apply heuristic strategies such as drawing a suitable schema or making a table, etc. The students usually only glance at the problem and try to decide what calculations to perform with the numbers.

Besides this, many pupils have inadequate beliefs and attitudes towards mathematics itself, learning mathematics, and problem solving. These beliefs exert a strong negative influence on pupils' willingness to engage in a mathematical problem. Some examples of such beliefs and attitudes are that there is only one correct way to solve a problem, that a mathematical problem has only one right answer, and that ordinary students can not solve non-routine problems. These insufficiencies in pupils' beliefs are related to the nature of the problems given in the lessons and the classroom culture. Pupils are mostly confronted with routine problems which require only basic operations and calculations. Non-routine problems which reflect the relations between mathematics and reality are rarely presented. Classroom activities can also contribute to unwanted attitudes towards learning outcomes such as the use of strategies for coping with word problems and to beliefs about what mathematics and problem solving is (Verschaffel et al., 1999). Activities such as these do not give opportunities to students for investigation, reasoning or deciding on the solution process and do not improve problem solving skills.

Taking into consideration the problem solving process for this study, a brief summary of this topic is presented below. There are several approaches to explain the problem solving process.

Polya (1957/1997) proposes four stages, which have sub stages, to explain the problem

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solving process; (i) understand the problem (ii) devise a plan (iii) carry out the plan (iv) look back. Garofalo and Lester (1985), have presented the problem solving process as (i) orientation: strategic behaviour to assess and understand a problem (ii) organization: planning of behaviour and choice of actions (iii) execution: regulation of behavior to conform to plans (iv) verification: with two sub-components: evaluation of orientation and organization, evaluation of execution.

Verschaffel et al. (1999) have used five steps in an experimental study. These are (i) build a mental representation of the problem (ii) decide how to solve the problem (iii) execute the necessary calculations (iv) interpret the outcome and formulate an answer (v) evaluate the solution. Taking this behaviour into account during instruction is helpful in order to solve the problem and to improve problem solving ability. From these problem solving process stages, Polya's stages are well-known and taken into account for the present study. Polya's sub stages and the mental activities involved in these stages are as follows:

(1)Understand the problem: This stage requires understanding the problem. The teacher can ask some questions like:

(i) What are the data? What is the condition?

(ii) What is the unknown? State each part with your own sentence etc.

(2) *Devise a plan:* At this stage a connection between the data and the unknown is investigated. If the operations to be made are known then we have a plan. If not, you may be obliged to consider auxiliary or similar problems in order to find a connection. You may try to solve a part of the problem, examine all the data, guess the answer, etc. After these attempts you should eventually obtain a plan of the solution.

In fact, a plan depends on an appropriate strategy. In a solution of a problem sometimes one strategy, sometimes more than one strategy can be used. Main strategies used in problem solving are; (1) make a systematic list (2) guess and check (3) make a drawing (4) look for a pattern (5) write an equation (6) solve a similar or simpler problem (7) work backward (8) construct a table and (9) reasoning.

(3) Carry out the plan: By using the chosen strategy, the problem is solved step by step in this stage. If the solution cannot be found, the strategy is changed.

(4)Look back: At this stage, the solution is checked in terms of the original problem, to see if the answer is reasonable or not and whether there is another way for the solution or not. Related problems which can be solved by this strategy are also considered (Mason,1999).

There are several studies related to learning to solve mathematical problems; some of them are summarized below.

Three of these, Verschaffel et al. (1999), Holton and Anderson (1999) and Pugalee (2001) are more related to the present study. Verschaffel et al. organized a learning environment to examine how modelling and solving mathematical application problems were developed and tested on fifth graders. The research design consisted of seven control and four experimental classes. Pupils were taught a series of heuristics for solving mathematical application problems. Control classes followed regular mathematics classes. A pretest, posttest and retention tests were conducted to test the effect of the experimental learning environment. The results showed that the intervention had a positive effect on different aspects of pupils' modelling and problem solving ability.

Holton and Anderson (1999), studied how problem solving might be taught in a New Zealand context and who might gain most from such teaching. Their aims were first to introduce problem solving to the students via single lessons which were solely on problem solving and second, to use a problem-solving approach to teach curriculum material. They studied with two Form 4 classes of a single-sex (girls) school. The students in their study were all 14 and 15 years old. They chose one low and one high ability class. The results of pretest and posttest showed that although the performance of both classes increased, the class of lower ability students was better than the other class at that level at the end of the school year. Researchers conjecture that the results were achieved in part as a consequence of the time that was spent in the problem solving lessons which allowed them to practise reading and working with verbal problems, and to work on basic material, especially arithmetic and algebra, at their own pace.

Pugalee (2001) investigated whether students' writing about their mathematical problem solving processes showed evidence of a metacognitive framework. Twenty ninth-grade algebra students provided written descriptions of their problem solving processes as they worked on

mathematics problems. A qualitative analysis of the data indicated the presence of a metacognitive framework. Students' written descriptions demonstrated engagement of various metacognitive behaviour during orientation, organization, execution, and verification phases of mathematical problem solving. The findings of this study underscore the importance of implementing writing as an integral part of the mathematics curriculum and emphasize the need for additional research on writing in mathematics.

The present study is related to seventh and eighth grade students' solutions to non-routine problems. During the study of problem solving, Polya's four stages were taken into account. The learning environment used was similar to that of Verschaffel et al. (1999) in that first small group discussion, then whole class discussion took place. They emphasized modelling in problem solving and they studied problem solving strategies as a part of this modelling. The present study focused on strategies and the learning level of strategies at seventh and eighth grades.

Holton and Anderson's (1999) study is similar to the present study since both of them were planned to make problem solving more effective in their countries. However the present study differs from theirs by including entirely non-routine problems and strategies used to solve them.

The present study also investigated students' writings about their problem solving processes as in Pugalee's (2001) study which showed evidence of a metacognitive framework. The present study is different as it gives importance to observations during problem solving processes and makes group discussions before a whole class discussion for the solution.

METHOD

In this study, a trial study was designed in which a learning environment involving non-routine problems for 7^{th} and the 8^{th} grade students was developed, and afterwards implemented and tested. A plan of the experiment was developed in order to teach non-routine mathematical problems. Although this study is similar to the aforementioned studies in some respects, it focuses only on non-routine problems, which makes it different from the others.

Aim of the study

The major goal of the study was to examine whether or not popular problem solving strategies could be learnt by seventh and eighth grade students. If so, what is the learning level? Besides this, the students' activation, motivation, attitudes and interest to these studies are observed.

Taking into account the aim of this study, the research question can be stated. The main research question in this study is "At what level can 7th and the 8th grade students learn problem solving strategies and use these strategies to solve non-routine mathematical problems?"

In the present study it was expected that the experimental group's scores would be significantly higher than the control groups', because in Turkey, there are very few non-routine problems in textbooks. It was also expected that the learning environment would have a positive effect on pupils' beliefs, willingness and attitudes towards mathematical problem solving.

Organization of the learning environment

The learning environment consisted of 17 lessons designed by the researchers. The lessons can be separated into three major parts. (1) An introduction and an explanation of the concept of the problem and the kind of study in instruction (lesson 1). (2) Systematic acquisition of Polya's problem solving process (lesson 2-7). Each lesson in this part was devoted to a strategy. In order to explain how to use a strategy, students worked on a problem. They were informed about the strategy and how to use it. A second similar problem was given to them and the studies went on as mentioned before. (3) In the third part of the lessons, the problems were given to students without a clue for the strategy, and they were asked to solve it by using an appropriate strategy or strategies.

The teacher of the group was one of the researchers and she had worked as a secondary school mathematics teacher for two years. So she had experience as a mathematics teacher. While one of the researchers contributed to the study as a teacher, another researcher observed the students' activities. Before or after the lessons, they contacted each other about the activities from time to time.

Participants

The experimental group consisted of 15 seventh grade students and 13 eighth grade students. The number in the control group from each class was the same. Both of the classes had 18 female and 10 male students in total.

Design of the study

Firstly, as an instruction material, a set of carefully designed non-routine problems is obtained.

The term non-routine means that the problems are not ordinary problems and cannot be solved in general ways like word problems. Non-routine problems may be related to real life or not, but they always represent a model of a real life situation. Secondly, the use of effective instructional techniques was emphasized. Recent studies like those of De Corte (2004), Vershafell at al. (1999) have shown that a powerful teaching-learning environment for problem solving is a social constructivist approach, because this kind of learning environment fosters the development of self-regulation strategies, and creates opportunities to acquire learning and thinking skills. The teaching-learning techniques consisted of (i) a short presentation to the whole class, (ii) group studies on the problem statement, and (iii) a whole class discussion on solutions. The groups consisted of two or three students, and any member of the groups could contact the other groups occasionally if they needed to. While the group work was in progress, the teacher's role was to arrange study groups, present the problem, manage class discussion, and evaluate the solution.

A pretest, posttest and retention tests were used to assess the success of the experimental group and the control group. The experimental group received the 17 lessons in school hours. Each lesson lasted 45 minutes. These lessons were spread over a period of about 3 months. Mathematics courses were given for 4 hours per week, but only one of them was used by the researchers. The other 3 lessons were given by the regular teacher and the students were not given non-routine mathematics problems in those lessons.

During the same period, the control group students continued to follow the regular mathematics curriculum in their classroom. The control group could not pursue it systematically. According to the mathematics teacher of the control group, studies were suitable for the traditional system and the teacher was more active than the children in the lesson. Students were confronted with routine problems which were included in their textbook.

Instruments

The evaluation material in this study consisted of two main components. One of these was problem solving tests, and the other one was observation. Before and after the intervention three problem solving tests (pretest, posttest and retention test) were administered to both groups. Problem solving tests were prepared as a written test by the researchers. Each test consisted of 10 problems, six of which were non-routine, one of which was routine, three of which were application problems. All three tests were very similar in context, and although their statements were different from each other, they required the same strategy.

The answers were examined separately and scored by two researchers. The coefficient of the Pearson Correlation between the scores of the two researchers was computed as 0.83. The sheet on which there was a disagreement between two scores they were examined again. Each total right answer score was 10 points. If an answer was right or incomplete but had a technical error it was given 10 points. Other answers were scored between 0 and 10 points considering the student's precise attempt, using the appropriate strategy and finding part of the answer.

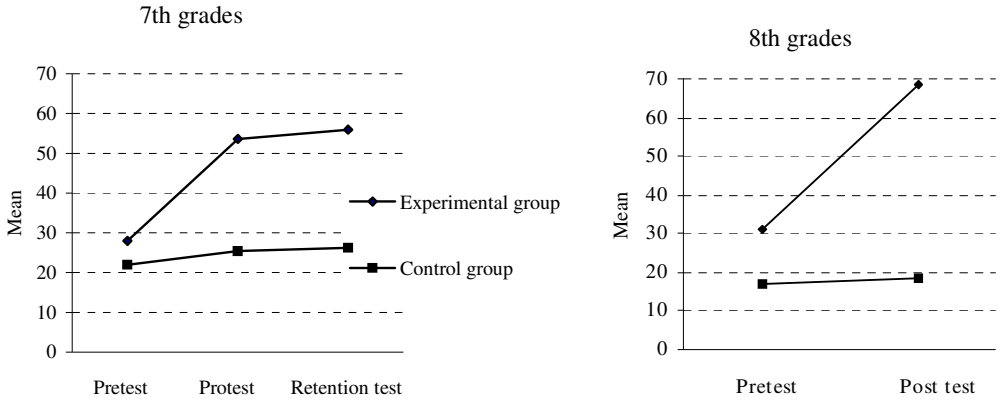
The experimental group students' willingness and attitudes to the study and doing activities were continuously observed throughout the studies. In addition, an attitude test was implemented before and at the end of the intervention.

The experimental group and control group scores, and the experimental group's pretest, posttest and retention test were compared by using t-test for the percentage values.

RESULTS

The results of the analysis are presented in this section. Figure 1 presents the mean scores of the pupils from the experimental and the control groups. The retention test was not administered to the 8th grade students because they had already graduated from their school at that time.

Figure 1. Mean scores of the experimental and the control group in the three versions of the problem solving test.



Although no significant difference was found between the pretest scores of the experimental and the control groups, the former significantly outperformed on the posttest and retention test and this difference was in favour of the experimental group. t values are 0.86, 2.89, 3.86 for the seventh grade, and 2.05, 6.17 for the eighth grade, respectively ($p<0.5$). The Croanbach alpha reliability coefficients of these tests were computed as 0.67, 0.78 and 0.75 for pretest, posttest and retention tests respectively.

In order to find out the improvement level in the experimental group, pretest, posttest and retention test scores were compared by using t test. T value between pretest and posttest is 2.39 for seventh grade and 5.66 for eighth grade. These results revealed that there is a significant difference in favour of posttest, but no significant difference is found between posttest and retention test.

Since the major aim of this study was to encourage learning and using strategies in non-routine problems, the success of the students was examined separately for each strategy taught in the experimental group. In accordance with that aim, each solution in the exam papers was analyzed. When deciding that any strategy was used, that strategy was given one point. In order for a strategy to be pointed 1, it was considered whether it contributed to solution and if it was used properly. Complete and correct solutions were not sought. The Cronbach alpha coefficients for the reliability level of the strategy points were computed as 0.53 and 0.73 for the pretest and posttest respectively. After that, the percentages of use were found dividing the numbers of use into the number of students. The results related to each strategy are shown in Table 1.

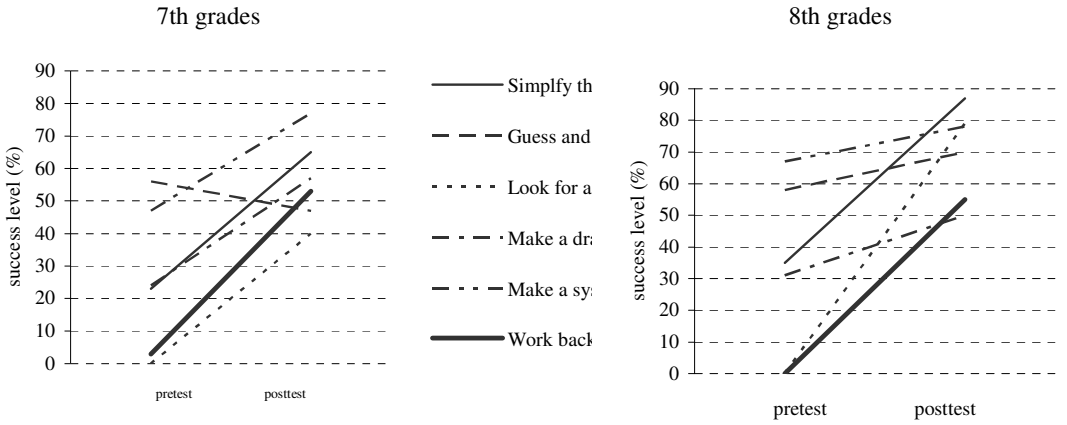
Table 1. *The results of pretest and posttest for each strategy.*

	Seventh Grades			Eighth Grades		
	Pretest(%)	Posttest(%)	t value	Pretest(%)	Posttest(%)	t value
Simplify the problem	23	65	2.56*	35	87	3.45*
Guess and check	56	47	0.06	58	70	0.69
Look for a pattern	0	40	3.16*	0	79	7.51*
Make a drawing	24	57	1.95	31	50	1.08
Make a systematic list	47	77	1.78	67	78	0.68
Work backward	3	53	3.67*	0	55	4.28*

* significant at $p<0.05$ level.

As seen from the table, students had knowledge of some strategies during the pretest although they had not yet been taught in the traditional mathematics syllabus (“make a systematic list”, “guess and check”, “simplify the problem”, and “make a drawing”). Besides these, students did not have any knowledge of the strategies of “work backward” and “look for a pattern”. The results regarding the strategies are shown in Figure 2.

Figure 2. Change in 7th and 8th grade students' use of strategies from pretest to posttest



In both classes, there were significant differences regarding the strategies of "simplify the problem", "look for a pattern" and "work backward" after the experimental study. Although some of them had no significant differences, the success level of all was over 50% except for the "look for a pattern" strategy at seventh grade.

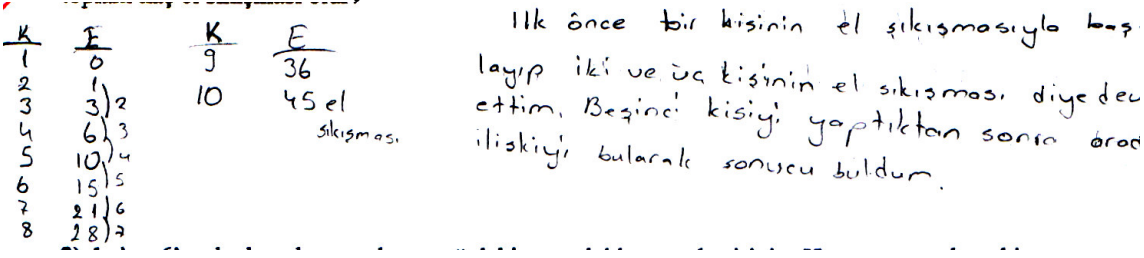
The learning levels of low and high ability students are given in Table 2.

Table 2. *Success level of 7th and 8th grade students in different ability groups*

	Seventh Grades				Eighth Grades			
	Low ability		High ability		Low ability		High ability	
	Pretest (%)	Posttest (%)	Pretest (%)	Posttest (%)	Pretest (%)	Posttest (%)	Pretest (%)	Posttest (%)
Simplify the problem	0	50	50	100	50	88	25	100
Guess and check	30	20	60	80	70	60	50	75
Look for a pattern	0	20	0	100	0	80	0	100
Make a drawing	10	60	54	60	50	52	38	65
Make a systematic list	40	58	54	100	56	70	68	100
Work backward	0	40	10	60	0	44	0	50

*0.05 significant level.

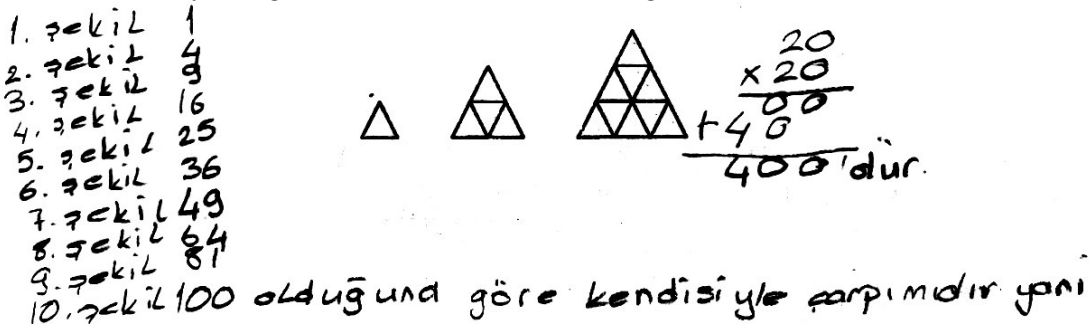
Two examples of the answers in the post test related to the simplify the problem and look for a pattern strategies are as follows. The first problem was "In a meeting of 10 people if everyone shakes hands with each other how many handshakes occur?"



The student writes; "First I started with 1 person, then 2 people,... after the 5th person I looked for the relation between the numbers of handshakes and found the right answer. This answer, besides the use of "simplify the problem", also shows the use of "look for a pattern".

Second example of problems and its answer is given below:

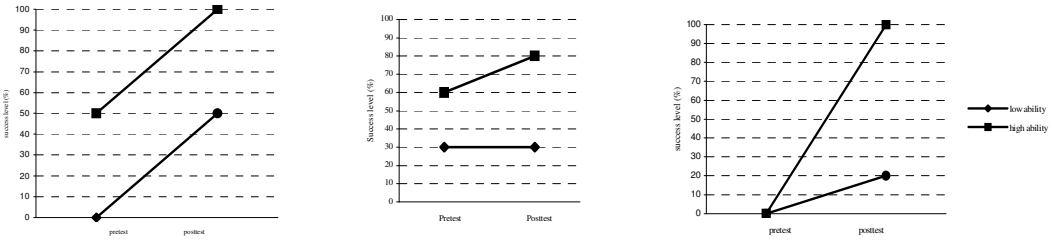
"How many triangles are needed to make 20th figure?"



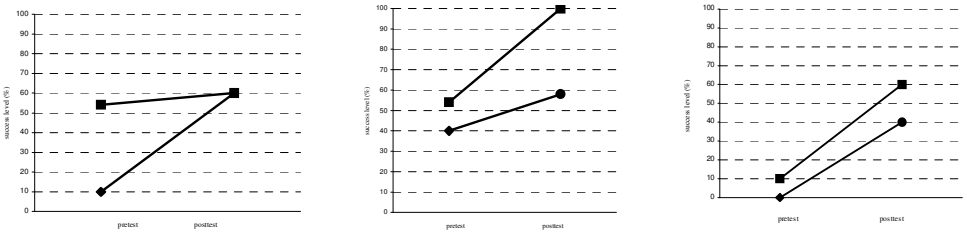
The student writes; "As the 1st figure is 1, the 2nd figure is 4, … and the 10th is 100, the solution is the multiplication of the number with itself."

These values show that some strategies were taught well but some of them were not. The following graphs (Figure 3 and Figure 4) show these results clearly.

Figure 3. A comparison of low ability students with high ability students according to strategy use at 7th grade

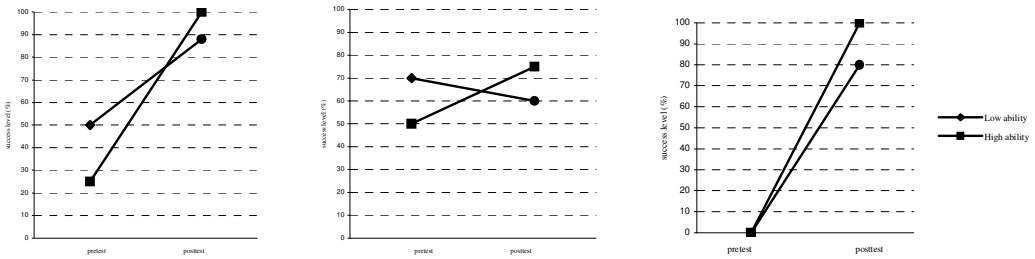


Simplify the problem | Guess and check | Look for a pattern

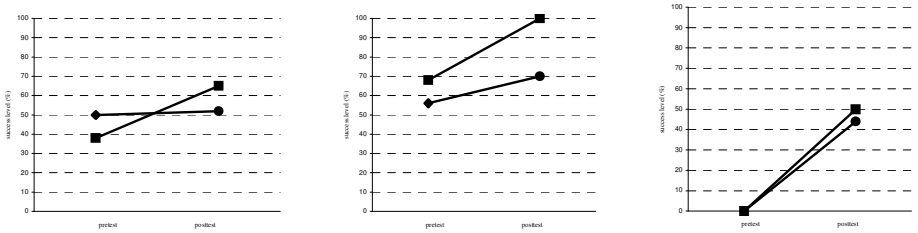


Make a drawing | Make a systematic list | Work backward

Figure 4. A comparison of low ability students with high ability students according to strategy use at 8th grade



Simplify the problem Guess and check Look for a pattern



Make a drawing Make a systematic list Work backward

Figures 3 and 4 indicate that there are similar improvements at both grades. Especially, the "simplify the problem", "look for a pattern" and "make a systematic list" strategies can be taught to both levels of 8th grade and to high ability students at 7th grade.

The results of the 7th grade students can be listed as follows: the differences in the "simplify the problem" and "work backward" strategies are similar in both ability groups, but the learning level of the former was higher than that of the latter. The differences in the "make a systematic list", and "guess and check" strategies were similar and only high ability students showed progress. "Look for a pattern" was learnt perfectly by high ability students, whereas the others could not show any progress. The improvement in the "make a drawing" strategy is interesting. The learning level of both ability groups was approximately 50%.

With the 8th grade students, the strategies of "simplify the problem" and "look for a pattern" can be learnt perfectly by both ability groups. "Make a drawing" and "guess and check" do not show significant improvement. The learning of "make a systematic list" was high in both ability groups. The learning level of "work backward" was approximately 50% for both levels despite the fact that it had been 0% in the pretest.

One of the goals of the present study was to find out whether the students' attitudes towards mathematics were affected by this learning environment or not. To evaluate students' attitudes towards problem solving, a questionnaire consisting of 30 items was applied at the beginning and the end of the study. This questionnaire had been used in Verschaffel (1999)'s study before and involved 35 items, but in our study 5 items were removed and Croanbach's α of this version was 0.77. Pupils had to choose between "strongly agree", "agree", "uncertain", "disagree" and "strongly disagree". Each answer scored from 1 to 5 points and the highest score was given to the belief that was most positive and proper for the instruction's goal. As a result of the scoring, the maximum score which a student could get was 150 (30x5). As shown in Table 3, in spite of positive progress, there is no significant difference between the attitude scores at pretest and posttest.

Table 3. *Attitude scores of experimental group before and after the study*

		Pretest		Posttest		
	n	$\overline{x}$	S	$\overline{x}$	S	t
Experimental group	28	109,04	24,49	116,04	16,37	1,371

*0.05 significant level.

The students were given papers in order to write their opinions related to this experimental study. They wrote positive statements. For example: *"...and I believe that these problems were much more enjoyable than the ones in maths books and I liked them. Now maths is easier and enjoyable"*.

DISCUSSION and CONCLUSIONS

In this article a trial study was designed and presented in order to teach problem solving strategies and to find out at which level 7th and 8th class students have learnt them. This learning environment was implemented and its effects were evaluated with a pretest, posttest and retention test.

According to the results of the written pretest and posttest, the learning environment had a significant positive effect on the acquisition of problem solving strategies. The results of the retention test revealed that the positive effect continued after the experimental lessons. The learning environment also had a positive impact on pupils' enjoyment and attitudes towards the learning of mathematical non-routine problems. The comparison of the results shows that low ability pupils significantly benefited from the learning environment, especially in the use of simplify the problem and work backward strategies. These positive results are similar to that of Follmer (2000), Higgins (1997), Holton and Anderson (1999), Verschaffel et al. (1999).

These positive results can be attributed to a socio-constructivist learning environment and the nature of non-routine mathematical problems. A socio-constructivist learning environment can enrich interaction, sharing of knowledge and experimentation. The nature of non-routine problems is interesting for students because of its imitation of real life situations.

There are some problematic aspects of the research methodology. The number of students was too small to draw reliable and generalizable conclusions regarding the effectiveness of the learning environment. Furthermore the acquisation level of the strategies was gathered from limited numbers of problems.

In addition to this, although it was known that the control group continued the traditional teaching methods, what went on there was not well-known. Besides these shortcomings, some findings were important for designing a mathematics syllabus. It was clear that the learning environment helped students to develop mathematical attitudes. It was also clear that the level of learning the problem solving strategies was different from each other. The learning levels of the "simplify the problem", "make a drawing" and "make a systematic list" strategies were high, whereas the learning levels of the "work backward", "guess and check", and "look for a pattern" strategies were low. In addition, it was shown that the learning level of a strategy depended on the student's age. For example, in spite of the low success level of the "look for a pattern" strategy, its improvement was very rapid for 8th grade students. This result is similar to that of Verschaffel et al. (1999).

The strategy of "make a systematic list" also progressed more rapidly at the 7th grade than at the 8th grade. This argument was beneficial for low ability students. They interacted and cooperated with their group members during a task and behaved effectively in the last lessons.

The students stated that the studies with non-routine problems improved their thinking.

To sum up, it can be stated that the content and objectives of the mathematics syllabus should be changed, taking into consideration non-routine problems, the acquisition of the problem solving process and strategies regarding the age and competence of the children. Additionally, the learning environment should be improved by taking into account the progress of social interaction based on small and whole group discussions. In this study, it has been shown that this kind of learning activity develops the skills of self-regulatory learning, requiring the construction of students knowledge, responsibility for their learning and a positive attitude towards mathematics and mathematics learning.

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Rutin Olmayan Matematiksel Problemlerin Çözümünü Öğrenme

ÖZ: Son çalışmalar ilköğretim öğrencilerinin birçoğunun rutin olmayan matematik problemlerini çözme becerilerinde iyi olmadıklarını göstermektedir. Bu makalede yedinci ve sekizinci sınıf öğrencilerinin rutin olmayan matematiksel problemlerin çözümlerini öğretmek için planlanan deneysel bir çalışma ve bu çalışmanın sonuçları rapor edilmiştir. Bu çalışmadaki stratejiler "Problemi Basitleştirme", "Tahmin ve Kontrol", "Bağıntı Arama", "Şekil Çizme", "Sistematik Liste Yapma" ve "Geriye Doğru Çalışma"dır. Sınıf aktiviteleri, verilen problem üzerinde problemin tüm sınıfa tanıtılması, sonra heterojen grup çalışmaları ve sonunda sınıf tartışmalarından oluşuyordu. Tüm bu aktiviteler boyunca öğretmenin rolü öğrencileri problemlerle meşgul olmaları için cesaretlendirmek ve problem üzerinde çalışmaları için yönlendirmekten ibaretti. Çalışmanın sonucunda bu stratejileri öğretme amacı ile hazırlanan ortamın bazı stratejilerin öğretiminde etkin olduğu, bazılarında ise olmadığı görüldü.
Anahtar kelimeler: problem çözme, problem çözme stratejileri, rutin olmayan problemler, matematik öğretimi

Problem: İlköğretim matematik programları ve değerlendirme standartları ile ilgili son çalışmalar, matematiksel problem çözme gücünü ve muhakeme etme becerilerini geliştirmeye önem vermekte, bu becerileri gerçek hayatta karşılaşılan problemlerin çözümünde kullanabilmeyi öncelikle hedef olarak belirlemektedir (*Verschaffel, De Corte, Lasure, Van Vaerenbergh, Bogaerts & Ratinckx,* 1999). Günümüzde okulların bu amacı gerçekleştirip gerçekleştiremedikleri hususunda ciddi kuşkular vardır. Birçok araştırma, öğrencilerin ilköğretimin ileri sınıflarında bile gerçek hayatta karşılaşılan problemleri çözmenin gerektirdiği matematik yaklaşımları etkili ve başarılı bir biçimde ortaya koyamadıklarını, en azından yetersiz olduklarını göstermiştir. Öğrencilerin problem çözümlerinde büyük ölçüde çözüm sürecine hâkim olma, problemi analiz etme, sonuçları değerlendirme gibi biliş faaliyetleri bakımından eksiklikler gözlenmektedir. Onlar bir problemle karşılaştıklarında daha çok, probleme bir göz atıp; verilen sayılara gerekli işlemleri çabucak uygulayıp sonuca gitme eğilimi göstermektedir.

Amaç: Bu araştırmada yedinci ve sekizinci sınıf öğrencilerinin rutin olmayan matematiksel problemleri çözme stratejilerinden hangilerini öğrenebildiklerini ve bunları hangi düzeyde kullanabildiklerini ortaya koymak amaçlanmıştır.

Yöntem: Çalışmanın ana konusu rutin olmayan problemler olduğu için çalışma öncesinde önce yerli ve yabancı kaynaklardan, ders kitaplarından, internet üzerinden ulaşılan araştırma yazıları ve proje raporlarından rutin olmayan problemler ve bunların çözümünde kullanılan stratejiler taranmıştır. Bu tarama sonucunda öğrencilerin yaş ve olgunluk düzeyi de göz önüne alınarak kaynaklarda sık rastlanan altı temel problem çözme stratejisinin çalışılması karalaştırılmıştır. Bunlar, problemi basitleştirme, tahmin ve kontrol, örüntü arama, şekil çizme, sistematik liste yapma ve geriye doğru çalışma stratejileridir. Stratejilerin her biri için soru dosyaları oluşturulmuş, sonra bu soru dosyalarından seçilen sorular çalışılan öğrencilerin sınıf düzeylerine uygun hale getirilerek çalışmada kullanılmıştır. Çalışmaya yedinci sınıftan 15, sekizinci sınıftan 13 olmak üzere toplam 28 öğrenci katılmıştır. Çalışma grubunu seçmek için gönüllü olarak katılmak isteyen yedinci ve sekizinci sınıf öğrencilerine işlem becerisi ve sıradan problem çözme ağırlıklı 15 soruluk bir başarı testi uygulanmıştır. Bu test sonuçlarına göre her iki sınıftan başarı düzeyi yüksek, orta, düşük olan öğrencilerden 5'er öğrenci seçilmiş ve çalışma bu öğrencilerle yapılmıştır. Çalışma haftada iki gün öğrencilerin normal ders saatlerinin dışındaki 12.30 – 13.30 saatleri arasında 10 hafta süreyle belirlenen günlerde gerçekleştirilmiştir. Eğitim araştırmacı tarafından verilmiştir. Öğretim sırasında öğrenciler iki veya üçer kişilik gruplar halinde çalıştırılmıştır. Gruplar araştırmacı tarafından oluşturulmuş, bu oluşturma sırasında gruplarda farklı düzeyde öğrenci bulunmasına özen gösterilmiştir. Çalışmanın verileri uygulanan testlerden ve çalışma sırasında yapılan gözlemlerden elde edilmiştir. Çalışmanın başında öğrencilerin problem çözmede başvurdukları stratejileri belirlemek

amacıyla 10 soruluk problem çözme testi kullanılmıştır. Bu testte altı tane rutin olmayan problem, bir tane sıradan problem, üç tane gerçek hayat bilgilerinin kullanılmasını gerektiren probleme yer verilmiştir. Çalışmanın sonunda, çalışmanın başındaki testten farklı fakat ona çok benzerlik gösteren bir problem çözme başarı testi uygulanmıştır. Bu testlerde doğru cevaplara 10'ar puan, yanlış cevaplara 0 puan verilmiş, doğru yönteme başvurduğu halde doğru sonuca ulaşamayanlara çözüm girişiminin durumuna göre 0 ile 10 arasında puanlar verilmiştir. Bu çalışmada, deney öncesi ve sonrasında çalışma grubundaki öğrencilerde oluşan farklar incelendiğinden, her bir strateji bazında farkları anlamak için t testinin kullanılması yeterli olmuştur. Verilerin analizinde SPSS Programı kullanılmıştır.

Bulgular ve Sonuç: Ön test sonuçlarına göre yedinci ve sekizinci sınıflarda informal olarak bazı stratejiler kullanılabilmektedir ve kullanma düzeylerinin yüzdelik değerleri bu çalışmada yedinci sınıf öğrencilerinde tahmin ve kontrol (%56), sistematik liste yapma (%47), şekil çizme (%24), problemi basitleştirme (%23) olarak tespit edilmiştir. Bunun yanı sıra geriye doğru çalışma ve örüntü arama stratejisini ise kullanamadıkları gözlenmiştir. Sekizinci sınıf öğrencilerinin stratejileri kullanım yüzdeleri ise tahmin ve kontrol (%58), sistematik liste yapma (%67), şekil çizme (%31) ve problemi basitleştirme (%35) şeklindedir, ancak bağıntı arama ve geriye doğru çalışma stratejilerini kullanamadıkları görülmüştür. Son testten elde edilen verilere göre, eğitim sonrasında stratejilerin oldukça yüksek yüzdelik değerlere ulaşmıştır ve problem çözmede kullanılabildiği gözlenmiştir. Yedinci sınıflarda tahmin ve kontrol (%47), sistematik liste yapma (%77), şekil çizme (%57), problemi basitleştirme (%65), geriye doğru çalışma (%53) ve örüntü arama (%40) olurken sekizinci sınıflarda ise tahmin ve kontrol (%70), sistematik liste yapma (%78), şekil çizme (%50), problemi basitleştirme (%87), geriye doğru çalışma (%55) ve örüntü arama (%79) olarak gözlenmiştir. Bu sonuç problem çözme stratejilerine öğretim programlarında yer verilmesinin öğrencilerin problem çözme becerilerini geliştirmesine katkısının olacağını işaret etmektedir.

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