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Zero Suffix Method for Solving A Multi-objective Bulk Transportation Problem

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Abstract: In this paper, we approached a Zero Suffix method to study the optimal solution of the Multi-Objective Bulk Transportation Problem (MOBTP) involving trade-off cost against time. This is a very basic alternative approach for evaluating all effective pairs of cost-time transactions. This method is explained by step by step algorithm which is illustrated through a numerical example. The proposed method unifies the existing methods to get all exhaustive solution pairs. A comparative analysis is also present between previous methods and the proposed method.

Keypoints: **Multi-objective bulk transportation problem, Cost- time trade-off, Zero Suffix method**

I. INTRODUCTION:-

Transportation is a special type of network optimization problem for the transport of goods from various origins to different destinations. It is a part of a linear programming problem that occurs in a number of fields. Several strategies for transportation problems have been suggested and different papers on the topic have been published.

The concept of the transportation problem hits back in the year 1941 by Hitchcock. Many of the transportation problems were done in different methods by authors [1] - [4]. However, transportation problems can be categorized as multi-objective transportation problems in real life (MOTP). In the MOTP, the goals may be to minimize the transport time, total transport cost, and the total product degradation during transport, etc. Bhatia et al. have researched minimizing overall costs and travel time[5]. Theodore and Berger subsequently researched similar MOTP[24]. Based on the parametric approach, Aneja and Nair[1] proposed the identification of all non-dominated extremes of the MOTP. Later on, Purusotham et al. [21] developed a method for solving the MOTP. Time minimization BTP was proposed by Bhatia [4] and Foulds and Gibbons [7]. Using branch and bound method , Pareto Optimal solutions of MOBTP is proposed by Prakash et al. [18, 19] and Extremum Difference Method later.
The paper is structured as follows: Mathematical formulation and basic definitions in the first section, Zero Suffix method is summarized in the second section, the comparative study of existing methods and proposed method for solving a MOBTP along with a numerical example of the problem Gupta et al. [10] in the third section and the results are explained in the fourth section. Eventually, the best optimality is presented. The conclusion is being discussed.

II. MATHEMATICAL FORMULATION OF THE PROBLEM:-

In the transportation table, there are m-origins from where goods are to be transported to n-destinations. Let x_{ij} is the number of products transported from ith source to jth destination assuming value 1 or 0 depending upon whether the source fulfills the demand of the destination or not. a_{ij} be the number of products available at the ith source and bj, the demand at the jth destination. The corresponding transportation problem is:
Min.C = $\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}\, x_{ij}$
T = max{t_{ij}: x_{ij} =1, i = 1,2,….,m, j = 1,2,….,n}
Subject to $\sum_{j=1}^{n} b_j\, x_{ij} \le a_i$ for i = 1,2,….,m
$\sum_{i=1}^{m} x_{ij} = 1$, for j = 1,2,….,n
$x_{ij} = 1 \text{ or } 0$ for i = 1,2,….,m and for j = 1,2,….,n.
Where c_{ij} denotes the bulk cost of transportation of units of the product from ithsource to jthdestination. t_{ij}

is the bulk time of transportation of units of the product from ith source to jth destination.

Definition(ith efficient solution/cost- time pair): Let c_1 be the minimum cost of bulk transportation and t_1 be the correspondingtimeofbulktransportation.Let Z_1bethesolutionforthefirstefficientcost-timepair (C_1,T_1). Let$C_2(>C_1)$ be another cost of bulk transportation and $T_2(<T_1)$ be the minimum time of bulk transportation at cost C_2.Then the solution Z_2 for the second efficient cost-time trade- off pair (C_2,T_2) is said to be a second efficient solution if there exists no other solution pair (C,T)s.t.$C_1<C< C_2$ and $T_2<T< T_1$. In a similar manner as above,theithefficient (i>2)solution/cost-time tradeoff pairs may be obtained.

III. ALGORITHM:-

There are two main steps in the proposed algorithm. In step 1, the first efficient cost-time tradeoff pair is obtained while in step 2, the subsequent efficient trade-off pairs are obtained. The steps involved in Zero Suffix method are as follows:

Step1.

1. Deletethecellsfromtheinitialmulti-objectivetransportationtableforwhichrequirementofdestination exceeds the availability of source .
2. Select the least bulk cost for each row and deduct it from all the bulk costs of corresponding row. Apply the same process for each columnalso.
3. Find the suffix values of all the 0‟s in the reduced cost matrix by the following formula

$$S = \frac{\text{Sum of non-zero costs in the ith row and jth column}}{\text{No. of zeroz in the ith row and jth column}}$$

4. Check for the highest suffix value of 0"s. If it is unique, it is required to allocate 1 to the cell in the reduced cost matrixand if it has more than one maximum value, it is possible to allocate the minimum cost to the cell.

Remove the destination from the table whose demand is satisfied and also remove the source whose availability becomes 0 or less than demand of each destination.

5. Repeat the steps (i) to (vi) until all the destinations met their demands.

Step 2:-

To determine the next efficient cost-time trade-off pair, define the new cost matrix of MOBTP as shown below:

$$c_{ij}^1 = \{\infty;\ t_{ij} \geq T_1$$
$$= 0;\ t_{ij} < T_1$$

Repeat step 1 to obtain the subsequent cost-time trade-off pair of MOBTP. In this way, on the repetition of steps 1 and 2, we obtain all the cost-time trade-off pairs of MOBTP.

IV. NUMERICAL EXAMPLE:-

Gupta et al. [10] problem is considered here in which there are three sources S_1, S_2 and S_3 with availabilities 7, 8, 9 units respectively and five destinations D_1, D_2, D_3, D_4 and D_5 with requirements 3, 5, 4, 6, 2 units respectively. In table 1, each pair of entries in each cell denotes the bulk cost$c_{ij}$of transportation and bulk time of transportationt_{ij} respectively. The proposed method is applied on the considered problem to determine the cost-time trade-offpairs.

Table 1;-(Initial Cost and Time Representation of MOBTP)

	D_1	D_2	D_3	D_4	D_5	
S_1	(10, 5)	(9, 6)	(11, 3)	(7, 2)	(8, 3)	7
S_2	(11, 2)	(10, 3)	(13, 5)	(14, 6)	(12, 4)	8
S_3	(8, 4)	(6, 5)	(9, 8)	(10, 3)	(13, 8)	9
	3	5	4	6	2	

Since availability of each source exceeds the requirement of each destination, no cell is deleted from the Table 1. Apply steps (i) and (ii) on the corresponding cost matrix of MOBTP, the reduced cost matrix is shown in

Table 2.

Table 2:- (Reduced table after first two sub-steps of step 1)

	D_1	D_2	D_3	D_4	D_5	
S_1	2	2	1	0 (6.5)	0(6)	7
S_2	0(2.66)	0(1.75)	0(1.5)	4	1	8
S_3	1	0(4.33)	0(4)	4	6	9
	3	5	4	6	2	

Maximum suffix value occurs for x_{14}. Applying steps 1 to 4, we get x_{14} =1, after allocating to this cell the reduced cost matrix table 3 is shown in Table 3.

Table 3:-

	D_1	D_2	D_3	D_5	
S_2	0(0.66)	0(0.25)	0(0.25)	0(1.66)	8
S_3	1	0(2.33)	0(2.33)	5	9
	3	5	4	2	

Applying steps 1 to 4 we get x_{32}=1, we have and the reduced cost matrix is shown in Table 4.

Table 4:-

	D_1	D_3	D_5	
S_2	0(0.33)	0(0)	0((1.66)	8
S_3	1	0(3)	5	4
	3	4	2	

Applying steps 1 to 4 we get x_{33} =1we have and the reduced cost matrix is shown in Table 5.

Table 5:- (Reduced table after 3rd application of step 1)

	D_1	D_5	
S_2	0(1.5)	0(1)	8
	3	2	

Again, applying step 1, we have x_{21} =1 and the reduced cost matrix is shown in Table 6

Table 6:-

	D_5	
S_2	0	5
	2	

Finally, we have x_{25}=1. Thus, requirements of all destinations are satisfied and the solution of the problem is given by $X_1 = \{x_{21}, x_{32}, x_{33}, x_{14}, x_{25}\}$. Therefore, cost of bulk transportation is $C_1 = C(X_1) = 45$ andcorrespondingtime of bulktransportationis $T_1 = T(X_1) = 8$. Thus, the first cost-time trade-off pair is $(C_1, T_1) = (45,8)$.

To obtain the 2nd cost-time trade-off pair, the given BTP is reduced using step2 and table 7 is obtained as shown below.

Table 7:- (Reduced MOBTP after step 2).

	D_1	D_2	D_3	D_4	D_5	
S_1	(10, 5)	(9, 6)	(11, 3)	(7, 2)	(8, 3)	7
S_2	(11, 2)	(10, 3)	(13, 5)	(14, 6)	(12, 4)	8
S_3	(8, 4)	(6, 5)	(9, ∞)	(10, 3)	(13, ∞)	9
	3	5	4	6	2	

Following the proposed algorithm for the reduced MOBTP, the second efficient solution obtained is $X_2 = \{x_{31},$

x_{32}, x_{23}, x_{14}, x_{25}}. The second efficient cost-time trade-off pair is $(C_2, T_2) = (46,5)$. Similarly, the third and fourth efficient solutions are X_3 = {x_{31}, x_{22}, x_{13}, x_{44}, x_{15}} and X_4 = {x_{21}, x_{22}, x_{13}, x_{34}, x_{15}} and having cost-time trade off pairs as $(C_3, T_3) = (47,4)$ and $(C_4, T_4) = (50,3)$ respectively. Thus, the exhaustive cost-time trade-off pairs are $(45,8)$, $(46,5)$, $(47,4)$ and $(50,3)$.

V. COMPARATIVE STUDY AND DISCUSSION:-

The result obtained is compared on the number of cost-time pairs with other methods. It is clear that, in comparison to the procedure proposed in [10], the proposed method provides more cost-term trade-off pairs. But it give same number of cost-time trade-off as compared to the methods in [18, 19]. This increase in the number of cost-time pairs may be seen to lend a greater flexibility to the user; who can make decisions according to his requirement. Therefore it is concluded that the proposed method is very simple and very easy to apply to calculate the exhaustive cost-time trade-off pairs in MOBTP. The comparative analysis is shown in Table8.

Table 8:- (Comparative Study)

Parameters	Method of Gupta et al. [10]	Extremum Difference method	Proposed Method
1st solution Vector and associated cost-time trade-off pair	(45, 8)	(45, 8)	(45, 8)
2nd solution Vector and associated cost-time trade-off pair	(47, 4)	(46,5)	(46,5)
3rd solution Vector and associated cost-time trade-off pair	(50, 3)	(47, 4)	(47, 4)
4th solution Vector and associated cost time trade-off pair	None	(50, 3)	(50, 3)

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