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SCHEFFE'S THEORY FOR MATHEMATICAL MODELING

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ABSTRACT- This work is carried out to develop an improved Scheffe's second degree polynomial based model that can be used to optimize the compressive strength of concrete. The compressive strength of fresh and partially set concrete was determined for different ratios which affect the strength of concrete. Control experiments were also carried out and the compressive strength determined. The adequacy of the model developed was tested using student's t-test and Fisher's test. The test results give a good correlation between the model and control results. Compressive strength of all points in the simplex can be derived with the help of this model.

Key words: Scheffe's second degree polynomial, student's t-test and Fisher's test, blend ratio, time lag, compressive strength

I. INTRODUCTION

Compressive strength of concrete is measured by the compression test when it is in hardened condition. It is the ability of concrete to resist loads which tend to compress it. It is done by crushing concrete specimens in compression testing machine.
In order to test the compressive strength of a concrete, cubes 150 mm size were cast, cured for 28 days. The basic constituents of concrete are cement, fine aggregate (sand), coarse aggregate and water. In addition to this the effect of blend ratio, time lag and temperature effect on strength of concrete is considered .The adaptability, strength and durability of cement are of utmost priority over other construction materials Fig. 1.

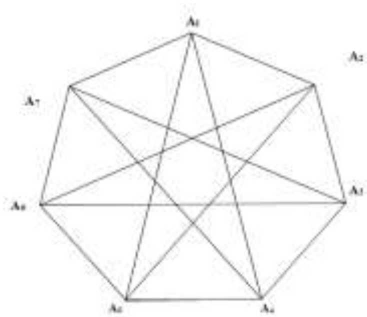

Fig. 1: A four - dimensional factor space holding seven components
Actual components and pseudo components

II. METHODOLOGY

2.1. Purpose of Modeling

The aim of the modeling is to predict the compressive strength from the actual lab strength carried out on the concrete specimens.[1]. Scheffe's mixture model is helpful to compare with a set of estimates of all possible contrasts among the factor level means [2]. Concrete strength affecting factors under consideration are in this research work and the strength due to variation of these parameters can be tested after 28 days curing before they are used. The parameters used are fraction of W/C ratio(A_1) , fraction of cement (A_2), fraction of FA(A_3), fraction of CA(A_4), blend ratio(A_5- ratio of volume of old concrete to fresh concrete), time lag (A_6- time interval between preparation of concrete and its placing) and temperature (A_7).

These models can save the time by making use of experimentations conducted to predict target strengths given the parameters which affect the strength of concrete.

2.1.1. Introduction to factor space in simplex design

Simplex is the representation of shape of the line or planes joining the positions considered for the parameters of the mix. [3] A simplex is nothing but a convex polyhedron with (k + 1) vertices produced by k, intersecting hyper planes in k dimensional space [4]. Scheffe [2] used a regular (q-1) simplex to represent a factor space needed to describe a response surface for mix consisting of several components. If the number of components is denoted by q, then for q = 7, the simplex is a regular heptagon.

2.1.2. Scheffe's factor space

Scheffe [5], introduced polynomial regression to model the response, called ''{q, n} polynomials". These polynomials have to be of low degree (n), otherwise the polynomial contains a large number of coefficients, making interpretation difficult and requiring a large number of design points.

2.1.3. Number of coefficients

P = 7, M = 2

N = (P+M-1)!)/(M!)(P-1)!

N = (7+2-1)!) /[(2!) (7-1) !]

N=28

2.1.4. Factor space for seven components

The first seven pseudo components are located at the vertices of the heptagon simplex.

A_1= [0.45, 1.0, 1.45, 1.75, 0.33, 0.75, 2.8]

A_2= [0.55, 1.0, 1.45, 1.75, 0.33, 0.75, 2.8]

A_3= [0.45, 1.0, 1.45, 1.95, 1.0, 0.75, 3.0]

A_4= [0.55, 1.0, 1.45, 1.95, 1.0, 1.5, 3.0]

A_5= [0.45, 1.0, 1.95, 2.55, 1.5, 2.25, 3.2]

A_6= [0.55, 1.0, 1.95, 2.55, 1.5, 2.25, 3.2]

A_7= [0.60, 1.0, 1.95, 2.55, 1.5, 2.25, 3.2]

A_1= Fraction of W/C ratio

A_2= Fraction of Cement (53 grade)

A_3= Fraction of FA

A_4= Fraction of CA

A_5= Blend Ratio

A_6= Time lag

A_7= Temperature

The corresponding pseudo mix ratios are

B_1 [1,0,0,0,0,0,0], B_2 [0,1,0,0,0,0,0], B_3 [0,0,1,0,0,0,0], B_4 [0,0,0,1,0,0,0],

B_5[0,0,0,0,1,0,0], B_6[0,0,0,0,0,1,0], B_7[0,0,0,0,0,0,1]

A_1		X_{11}	X_{12}	X_{13}	X_{14}	X_{15}	X_{16}	X_{17}		B_1
A_2		X_{21}	X_{22}	X_{23}	X_{24}	X_{25}	X_{26}	X_{27}		B_2
A_3		X_{31}	X_{32}	X_{33}	X_{34}	X_{35}	X_{36}	X_{37}		B_3
A_4	=	X_{41}	X_{42}	X_{43}	X_{44}	X_{45}	X_{46}	X_{47}	*	B_4
A_5		X_{51}	X_{52}	X_{53}	X_{54}	X_{55}	X_{56}	X_{57}		B_5
A_6		X_{61}	X_{62}	X_{63}	X_{64}	X_{65}	X_{66}	X_{67}		B_6
A_7		X_{71}	X_{72}	X_{73}	X_{74}	X_{75}	X_{76}	X_{77}		B_7

'A' represents the actual components while 'B' represents the pseudo components, where 'X' is the constant, a seven by seven matrix. The value of matrix X will be obtained from the first seven mix ratios.

2.1.5. Responses

These are the properties of fresh and hardened remixed concrete. A is a structural depiction of lines joining the atoms of a mixture. The constituent components of concrete mix [6-7] are water, cement, fine and coarse aggregates. In addition to this blend ratio, time lag and temperature are the parameters affecting the strength of concrete. The lattice of this seven -component mixture is a three- dimensional solid equilateral heptagon. Mixture components are subject to the restriction that the summation of all the components must be equal to unity .As a rule; the response surfaces in multi-component systems are very intricate. If a mixture has a total of q components and x_1 be the proportion of the i^{th} component in the mixture such that,

X1 +X2 + X3 + X4 + X5+ X6+ X7=1

For first run

0.45		X_{11}	X_{12}	X_{13}	X_{14}	X_{15}	X_{16}	X_{17}		1
1		X_{21}	X_{22}	X_{23}	X_{24}	X_{25}	X_{26}	X_{27}		0
1.45		X_{31}	X_{32}	X_{33}	X_{34}	X_{35}	X_{36}	X_{37}		0
1.75	=	X_{41}	X_{42}	X_{43}	X_{44}	X_{45}	X_{46}	X_{47}	*	0
0.33		X_{51}	X_{52}	X_{53}	X_{54}	X_{55}	X_{56}	X_{57}		0
0.75		X_{61}	X_{62}	X_{63}	X_{64}	X_{65}	X_{66}	X_{67}		0
2.8		X_{71}	X_{72}	X_{73}	X_{74}	X_{75}	X_{76}	X_{77}		0

X_{11}=0.45	X_{21}=1	X_{31}=1.45	X_{41}=1.75	X_{51}=0.33	X_{61}=0.75	X_{71}=2.8

For second run

0.55		X_{11}	X_{12}	X_{13}	X_{14}	X_{15}	X_{16}	X_{17}		0
1		X_{21}	X_{22}	X_{23}	X_{24}	X_{25}	X_{26}	X_{27}		1
1.45		X_{31}	X_{32}	X_{33}	X_{34}	X_{35}	X_{36}	X_{37}		0
1.75	=	X_{41}	X_{42}	X_{43}	X_{44}	X_{45}	X_{46}	X_{47}	*	0
0.33		X_{51}	X_{52}	X_{53}	X_{54}	X_{55}	X_{56}	X_{57}		0
0.75		X_{61}	X_{62}	X_{63}	X_{64}	X_{65}	X_{66}	X_{67}		0
2.8		X_{71}	X_{72}	X_{73}	X_{74}	X_{75}	X_{76}	X_{77}		0

X_{12}=0.55	X_{22}=1	X_{32}=1.45	X_{42}=1.75	X_{52}=0.33	X_{62}=0.75	X_{72}=2.8

For third run

0.45		X_{11}	X_{12}	X_{13}	X_{14}	X_{15}	X_{16}	X_{17}		0
1		X_{21}	X_{22}	X_{23}	X_{24}	X_{25}	X_{26}	X_{27}		0
1.45		X_{31}	X_{32}	X_{33}	X_{34}	X_{35}	X_{36}	X_{37}		1
1.75	=	X_{41}	X_{42}	X_{43}	X_{44}	X_{45}	X_{46}	X_{47}	*	0
0.33		X_{51}	X_{52}	X_{53}	X_{54}	X_{55}	X_{56}	X_{57}		0

0.75		X_{61}	X_{62}	X_{63}	X_{64}	X_{65}	X_{66}	X_{67}		0
3.0		X_{71}	X_{72}	X_{73}	X_{74}	X_{75}	X_{76}	X_{77}		0

X_{13}=0.45	X_{23}=1	X_{33}=1.45	X_{43}=1.75	X_{53}=0.33	X_{63}=0.75	X_{73}=3.0

For forth run

0.55		X_{11}	X_{12}	X_{13}	X_{14}	X_{15}	X_{16}	X_{17}		0
1		X_{21}	X_{22}	X_{23}	X_{24}	X_{25}	X_{26}	X_{27}		0
1.45		X_{31}	X_{32}	X_{33}	X_{34}	X_{35}	X_{36}	X_{37}		0
1.95	=	X_{41}	X_{42}	X_{43}	X_{44}	X_{45}	X_{46}	X_{47}	*	1
1		X_{51}	X_{52}	X_{53}	X_{54}	X_{55}	X_{56}	X_{57}		0
1.5		X_{61}	X_{62}	X_{63}	X_{64}	X_{65}	X_{66}	X_{67}		0
3.0		X_{71}	X_{72}	X_{73}	X_{74}	X_{75}	X_{76}	X_{77}		0

X_{14}=0.55	X_{24}=1	X_{34}=1.45	X_{44}=1.95	X_{54}=1	X_{64}=1.5	X_{74}=3.0

For fifth run

0.45		X_{11}	X_{12}	X_{13}	X_{14}	X_{15}	X_{16}	X_{17}		0
1		X_{21}	X_{22}	X_{23}	X_{24}	X_{25}	X_{26}	X_{27}		0
1.95		X_{31}	X_{32}	X_{33}	X_{34}	X_{35}	X_{36}	X_{37}		0
2.55	=	X_{41}	X_{42}	X_{43}	X_{44}	X_{45}	X_{46}	X_{47}	*	1
1.5		X_{51}	X_{52}	X_{53}	X_{54}	X_{55}	X_{56}	X_{57}		0
2.25		X_{61}	X_{62}	X_{63}	X_{64}	X_{65}	X_{66}	X_{67}		0
3.2		X_{71}	X_{72}	X_{73}	X_{74}	X_{75}	X_{76}	X_{77}		0

X_{15}=0.45	X_{25}=1	X_{35}=1.95	X_{45}=2.55	X_{55}=1.5	X_{65}=2.25	X_{75}=3.2

For sixth run

0.55		X_{11}	X_{12}	X_{13}	X_{14}	X_{15}	X_{16}	X_{17}		0
1		X_{21}	X_{22}	X_{23}	X_{24}	X_{25}	X_{26}	X_{27}		0
1.95		X_{31}	X_{32}	X_{33}	X_{34}	X_{35}	X_{36}	X_{37}		0
2.55	=	X_{41}	X_{42}	X_{43}	X_{44}	X_{45}	X_{46}	X_{47}	*	1
1.5		X_{51}	X_{52}	X_{53}	X_{54}	X_{55}	X_{56}	X_{57}		0
2.25		X_{61}	X_{62}	X_{63}	X_{64}	X_{65}	X_{66}	X_{67}		0
3.2		X_{71}	X_{72}	X_{73}	X_{74}	X_{75}	X_{76}	X_{77}		0

X_{16}=0.55	X_{26}=1	X_{36}=1.95	X_{46}=2.55	X_{56}=1.5	X_{66}=2.25	X_{76}=3.2

For seventh run

0.6		X_{11}	X_{12}	X_{13}	X_{14}	X_{15}	X_{16}	X_{17}		0
1		X_{21}	X_{22}	X_{23}	X_{24}	X_{25}	X_{26}	X_{27}		0
1.95		X_{31}	X_{32}	X_{33}	X_{34}	X_{35}	X_{36}	X_{37}		0
2.55	=	X_{41}	X_{42}	X_{43}	X_{44}	X_{45}	X_{46}	X_{47}	*	1
1.5		X_{51}	X_{52}	X_{53}	X_{54}	X_{55}	X_{56}	X_{57}		0
2.25		X_{61}	X_{62}	X_{63}	X_{64}	X_{65}	X_{66}	X_{67}		0
3.2		X_{71}	X_{72}	X_{73}	X_{74}	X_{75}	X_{76}	X_{77}		0

X_{17}=0.6	X_{27}=1	X_{37}=1.95	X_{47}=2.55	X_{57}=1.5	X_{67}=2.25	X_{77}=3.2

Putting the values of the constantans we can get [X] matrix as

0.45	0.55	0.45	0.5	0.5	0.55	0.6
1	1	1	1	1	1	1
1.45	1.45	1.45	1.45	2	1.95	1.95
1.75	1.75	1.75	1.95	2.6	2.55	2.55
0.33	0.33	0.33	1	1.5	1.5	1.5
0.75	0.75	0.75	1.5	2.3	2.25	2.25
2.8	2.8	3	3	3.2	3.2	3.2

x_{12}

x_{12}

A_1	0.45	0.55	0.45	0.5	0.5	0.55	0.6		0.5
A_2	1	1	1	1	1	1	1		0.5
A_3	1.45	1.45	1.45	1.45	2	1.95	1.95		0
A_4	1.75	1.75	1.75	1.95	2.6	2.55	2.55	*	0
A_5	0.33	0.33	0.33	1	1.5	1.5	1.5		0
A_6	0.75	0.75	0.75	1.5	2.3	2.25	2.25		0
A_7	2.8	2.8	3	3	3.2	3.2	3.2		0

A_1=0.5	A_2=1	A_3=1.45	A_4=1.75	A_5=2.8	A_6=0.75	A_7=2.8

x_{13}

A_1	0.45	0.55	0.45	0.5	0.45	0.55	0.6		0.5
A_2	1	1	1	1	1	1	1		0
A_3	1.45	1.45	1.45	1.45	1.95	1.95	1.95		0.5
A_4	1.75	1.75	1.75	1.95	2.55	2.55	2.55	*	0
A_5	0.33	0.33	0.33	1	1.5	1.5	1.5		0
A_6	0.75	0.75	0.75	1.5	2.25	2.25	2.25		0
A_7	2.8	2.8	3	3	3.2	3.2	3.2		0

A_1=0.45	A_2=1	A_3=1.45	A_4=1.75	A_5=0.33	A_6=0.75	A_7=2.9

x_{14}

A_1	0.45	0.55	0.45	0.5	0.45	0.55	0.6		0.5
A_2	1	1	1	1	1	1	1		0
A_3	1.45	1.45	1.45	1.45	1.95	1.95	1.95		0
A_4	1.75	1.75	1.75	1.95	2.55	2.55	2.55	*	0.5
A_5	0.33	0.33	0.33	1	1.5	1.5	1.5		0
A_6	0.75	0.75	0.75	1.5	2.25	2.25	2.25		0
A_7	2.8	2.8	3	3	3.2	3.2	3.2		0

A_1=0.475	A_2=1	A_3=1.45	A_4=1.85	A_5=0.665	A_6=1.125	A_7=2.9

x_{15}

A_1	0.45	0.55	0.45	0.5	0.45	0.55	0.6	0.5
A_2	1	1	1	1	1	1	1	0
A_3	1.45	1.45	1.45	1.45	1.95	1.95	1.95	0

A_4	1.75	1.75	1.75	1.95	2.55	2.55	2.55	*	0
A_5	0.33	0.33	0.33	1	1.5	1.5	1.5		0.5
A_6	0.75	0.75	0.75	1.5	2.25	2.25	2.25		0
A_7	2.8	2.8	3	3	3.2	3.2	3.2		0

A_1=0.475	A_2=1	A_3=1.7	A_4=2.15	A_5=0.915	A_6=1.5	A_7=3

x_{16}

A_1	0.45	0.55	0.45	0.5	0.45	0.55	0.6		0.5
A_2	1	1	1	1	1	1	1		0
A_3	1.45	1.45	1.45	1.45	1.95	1.95	1.95		0
A_4	1.75	1.75	1.75	1.95	2.55	2.55	2.55	*	0
A_5	0.33	0.33	0.33	1	1.5	1.5	1.5		0
A_6	0.75	0.75	0.75	1.5	2.25	2.25	2.25		0.5
A_7	2.8	2.8	3	3	3.2	3.2	3.2		0

A_1=0.5	A_2=1	A_3=1.7	A_4=2.15	A_5=0.915	A_6=1.5	A_7=3

x_{17}

A_1	0.45	0.55	0.45	0.5	0.45	0.55	0.6		0.5
A_2	1	1	1	1	1	1	1		0
A_3	1.45	1.45	1.45	1.45	1.95	1.95	1.95		0
A_4	1.75	1.75	1.75	1.95	2.55	2.55	2.55	*	0
A_5	0.33	0.33	0.33	1	1.5	1.5	1.5		0
A_6	0.75	0.75	0.75	1.5	2.25	2.25	2.25		0
A_7	2.8	2.8	3	3	3.2	3.2	3.2		0.5

A_1=0.525	A_2=1	A_3=1.7	A_4=2.15	A_5=0.915	A_6=1.5	A_7=3

x_{23}

A_1	0.45	0.55	0.45	0.5	0.45	0.55	0.6		0
A_2	1	1	1	1	1	1	1		0.5
A_3	1.45	1.45	1.45	1.45	1.95	1.95	1.95		0.5
A_4	1.75	1.75	1.75	1.95	2.55	2.55	2.55	*	0
A_5	0.33	0.33	0.33	1	1.5	1.5	1.5		0
A_6	0.75	0.75	0.75	1.5	2.25	2.25	2.25		0
A_7	2.8	2.8	3	3	3.2	3.2	3.2		0

A_1=0.5	A_2=1	A_3=1.45	A_4=1.75	A_5=0.33	A_6=0.75	A_7=2.9

x_{24}

A_1	0.45	0.55	0.45	0.5	0.45	0.55	0.6	0
A_2	1	1	1	1	1	1	1	0.5
A_3	1.45	1.45	1.45	1.45	1.95	1.95	1.95	0

A_4	1.75	1.75	1.75	1.95	2.55	2.55	2.55	*	0.5
A_5	0.33	0.33	0.33	1	1.5	1.5	1.5		0
A_6	0.75	0.75	0.75	1.5	2.25	2.25	2.25		0
A_7	2.8	2.8	3	3	3.2	3.2	3.2		0

A_1=0.525	A_2=1	A_3=1.45	A_4=1.85	A_5=0.665	A_6=1.125	A_7=2.9

x_{25}

A_1	0.45	0.55	0.45	0.5	0.45	0.55	0.6		0
A_2	1	1	1	1	1	1	1		0.5
A_3	1.45	1.45	1.45	1.45	1.95	1.95	1.95		0
A_4	1.75	1.75	1.75	1.95	2.55	2.55	2.55	*	0
A_5	0.33	0.33	0.33	1	1.5	1.5	1.5		0.5
A_6	0.75	0.75	0.75	1.5	2.25	2.25	2.25		0
A_7	2.8	2.8	3	3	3.2	3.2	3.2		0

A_1=0.5	A_2=1	A_3=1.7	A_4=2.15	A_5=0.915	A_6=1.5	A_7=3

x_{26}

A_1	0.45	0.55	0.45	0.5	0.45	0.55	0.6		0
A_2	1	1	1	1	1	1	1		0.5
A_3	1.45	1.45	1.45	1.45	1.95	1.95	1.95		0
A_4	1.75	1.75	1.75	1.95	2.55	2.55	2.55	*	0
A_5	0.33	0.33	0.33	1	1.5	1.5	1.5		0
A_6	0.75	0.75	0.75	1.5	2.25	2.25	2.25		0.5
A_7	2.8	2.8	3	3	3.2	3.2	3.2		0

A_1=0.55	A_2=1	A_3=1.7	A_4=2.15	A_5=0.915	A_6=1.5	A_7=3

x_{27}

A_1	0.45	0.55	0.45	0.5	0.45	0.55	0.6		0
A_2	1	1	1	1	1	1	1		0.5
A_3	1.45	1.45	1.45	1.45	1.95	1.95	1.95		0
A_4	1.75	1.75	1.75	1.95	2.55	2.55	2.55	*	0
A_5	0.33	0.33	0.33	1	1.5	1.5	1.5		0
A_6	0.75	0.75	0.75	1.5	2.25	2.25	2.25		0
A_7	2.8	2.8	3	3	3.2	3.2	3.2		0.5

A_1=0.575	A_2=1	A_3=1.7	A_4=2.15	A_5=0.915	A_6=1.5	A_7=3

x_{34}

A_1	0.45	0.55	0.45	0.5	0.45	0.55	0.6		0
A_2	1	1	1	1	1	1	1		0
A_3	1.45	1.45	1.45	1.45	1.95	1.95	1.95		0.5
A_4	1.75	1.75	1.75	1.95	2.55	2.55	2.55	*	0.5
A_5	0.33	0.33	0.33	1	1.5	1.5	1.5		0

A_6	0.75	0.75	0.75	1.5	2.25	2.25	2.25		0
A_7	2.8	2.8	3	3	3.2	3.2	3.2		0

A_1=0.475	A_2=1	A_3=1.45	A_4=1.85	A_5=0.665	A_6=1.125	A_7=3

x_{35}

A_1	0.45	0.55	0.45	0.5	0.45	0.55	0.6		0
A_2	1	1	1	1	1	1	1		0
A_3	1.45	1.45	1.45	1.45	1.95	1.95	1.95		0.5
A_4	1.75	1.75	1.75	1.95	2.55	2.55	2.55	*	0
A_5	0.33	0.33	0.33	1	1.5	1.5	1.5		0.5
A_6	0.75	0.75	0.75	1.5	2.25	2.25	2.25		0
A_7	2.8	2.8	3	3	3.2	3.2	3.2		0

A_1=0.45	A_2=1	A_3=1.7	A_4=2.15	A_5=0.915	A_6=1.5	A_7=3.1

x_{36}

A_1	0.45	0.55	0.45	0.5	0.45	0.55	0.6		0
A_2	1	1	1	1	1	1	1		0
A_3	1.45	1.45	1.45	1.45	1.95	1.95	1.95		0.5
A_4	1.75	1.75	1.75	1.95	2.55	2.55	2.55	*	0
A_5	0.33	0.33	0.33	1	1.5	1.5	1.5		0
A_6	0.75	0.75	0.75	1.5	2.25	2.25	2.25		0.5
A_7	2.8	2.8	3	3	3.2	3.2	3.2		0

A_1=0.5	A_2=1	A_3=1.7	A_4=2.15	A_5=0.915	A_6=1.5	A_7=3.1

x_{37}

A_1	0.45	0.55	0.45	0.5	0.45	0.55	0.6		0
A_2	1	1	1	1	1	1	1		0
A_3	1.45	1.45	1.45	1.45	1.95	1.95	1.95		0.5
A_4	1.75	1.75	1.75	1.95	2.55	2.55	2.55	*	0
A_5	0.33	0.33	0.33	1	1.5	1.5	1.5		0
A_6	0.75	0.75	0.75	1.5	2.25	2.25	2.25		0
A_7	2.8	2.8	3	3	3.2	3.2	3.2		0.5

A_1=0.525	A_2=1	A_3=1.7	A_4=2.15	A_5=0.915	A_6=1.5	A_7=3.1

x_{45}

A_1	0.45	0.55	0.45	0.5	0.45	0.55	0.6		0
A_2	1	1	1	1	1	1	1		0
A_3	1.45	1.45	1.45	1.45	1.95	1.95	1.95		0
A_4	1.75	1.75	1.75	1.95	2.55	2.55	2.55	*	0.5
A_5	0.33	0.33	0.33	1	1.5	1.5	1.5		0.5
A_6	0.75	0.75	0.75	1.5	2.25	2.25	2.25		0

A_7	2.8	2.8	3	3	3.2	3.2	3.2		0

A_1=0.475	A_2=1	A_3=1.7	A_4=2.25	A_5=1.25	A_6=1.875	A_7=3.1

x_{46}

A_1	0.45	0.55	0.45	0.5	0.45	0.55	0.6		0
A_2	1	1	1	1	1	1	1		0
A_3	1.45	1.45	1.45	1.45	1.95	1.95	1.95		0
A_4	1.75	1.75	1.75	1.95	2.55	2.55	2.55	*	0.5
A_5	0.33	0.33	0.33	1	1.5	1.5	1.5		0
A_6	0.75	0.75	0.75	1.5	2.25	2.25	2.25		0.5
A_7	2.8	2.8	3	3	3.2	3.2	3.2		0

A_1=0.525	A_2=1	A_3=1.7	A_4=2.25	A_5=1.25	A_6=1.875	A_7=3.1

x_{47}

A_1	0.45	0.55	0.45	0.5	0.45	0.55	0.6		0
A_2	1	1	1	1	1	1	1		0
A_3	1.45	1.45	1.45	1.45	1.95	1.95	1.95		0
A_4	1.75	1.75	1.75	1.95	2.55	2.55	2.55	*	0.5
A_5	0.33	0.33	0.33	1	1.5	1.5	1.5		0
A_6	0.75	0.75	0.75	1.5	2.25	2.25	2.25		0
A_7	2.8	2.8	3	3	3.2	3.2	3.2		0.5

A_1=0.55	A_2=1	A_3=1.7	A_4=2.25	A_5=1.25	A_6=1.875	A_7=3.1

x_{56}

A_1	0.45	0.55	0.45	0.5	0.45	0.55	0.6		0
A_2	1	1	1	1	1	1	1		0
A_3	1.45	1.45	1.45	1.45	1.95	1.95	1.95		0
A_4	1.75	1.75	1.75	1.95	2.55	2.55	2.55	*	0
A_5	0.33	0.33	0.33	1	1.5	1.5	1.5		0.5
A_6	0.75	0.75	0.75	1.5	2.25	2.25	2.25		0.5
A_7	2.8	2.8	3	3	3.2	3.2	3.2		0

A_1=0.5	A_2=1	A_3=1.95	A_4=2.55	A_5=1. 5	A_6=2.25	A_7=3.2

x_{67}

A_1	0.45	0.55	0.45	0.5	0.45	0.55	0.6		0
A_2	1	1	1	1	1	1	1		0
A_3	1.45	1.45	1.45	1.45	1.95	1.95	1.95		0
A_4	1.75	1.75	1.75	1.95	2.55	2.55	2.55	*	0
A_5	0.33	0.33	0.33	1	1.5	1.5	1.5		0
A_6	0.75	0.75	0.75	1.5	2.25	2.25	2.25		0.5
A_7	2.8	2.8	3	3	3.2	3.2	3.2		0.5

A_1=0.575	A_2=1	A_3=1.95	A_4=2.55	A_5=1. 5	A_6=2.25	A_7=3.2

Mixture proportion of control points showing actual and pseudo components
Control Point

x_1

A_1	0.45	0.55	0.45	0.5	0.45	0.55	0.6		0.25
A_2	1	1	1	1	1	1	1		0.25
A_3	1.45	1.45	1.45	1.45	1.95	1.95	1.95		0.25
A_4	1.75	1.75	1.75	1.95	2.55	2.55	2.55	*	0.25
A_5	0.33	0.33	0.33	1	1.5	1.5	1.5		0
A_6	0.75	0.75	0.75	1.5	2.25	2.25	2.25		0
A_7	2.8	2.8	3	3	3.2	3.2	3.2		0

A_1=0.487	A_2=1	A_3=1.45	A_4=1.8	A_5=0.497	A_6=0.937	A_7=2.9

X_2

A_1	0.45	0.55	0.45	0.5	0.45	0.55	0.6		0.25
A_2	1	1	1	1	1	1	1		0.25
A_3	1.45	1.45	1.45	1.45	1.95	1.95	1.95		0.25
A_4	1.75	1.75	1.75	1.95	2.55	2.55	2.55	*	0
A_5	0.33	0.33	0.33	1	1.5	1.5	1.5		0.25
A_6	0.75	0.75	0.75	1.5	2.25	2.25	2.25		0
A_7	2.8	2.8	3	3	3.2	3.2	3.2		0

A_1=0.475	A_2=1	A_3=1.575	A_4=1.95	A_5=0.622	A_6=1.125	A_7=2.95

X_3

A_1	0.45	0.55	0.45	0.5	0.45	0.55	0.6		0.25
A_2	1	1	1	1	1	1	1		0.25
A_3	1.45	1.45	1.45	1.45	1.95	1.95	1.95		0
A_4	1.75	1.75	1.75	1.95	2.55	2.55	2.55	*	0
A_5	0.33	0.33	0.33	1	1.5	1.5	1.5		0.25
A_6	0.75	0.75	0.75	1.5	2.25	2.25	2.25		0.25
A_7	2.8	2.8	3	3	3.2	3.2	3.2		0

A_1=0.5	A_2=1	A_3=1.7	A_4=2.15	A_5=0.915	A_6=1.5	A_7=3

X_4

A_1	0.45	0.55	0.45	0.5	0.45	0.55	0.6		0.25
A_2	1	1	1	1	1	1	1		0
A_3	1.45	1.45	1.45	1.45	1.95	1.95	1.95		0.25
A_4	1.75	1.75	1.75	1.95	2.55	2.55	2.55	*	0.25
A_5	0.33	0.33	0.33	1	1.5	1.5	1.5		0.25
A_6	0.75	0.75	0.75	1.5	2.25	2.25	2.25		0
A_7	2.8	2.8	3	3	3.2	3.2	3.2		0

A_1=0.462	A_2=1	A_3=1.575	A_4=2	A_5=0.79	A_6=1.312	A_7=3

X_5

A_1	0.45	0.55	0.45	0.5	0.45	0.55	0.6		0
A_2	1	1	1	1	1	1	1		0
A_3	1.45	1.45	1.45	1.45	1.95	1.95	1.95		0.25
A_4	1.75	1.75	1.75	1.95	2.55	2.55	2.55	*	0.25
A_5	0.33	0.33	0.33	1	1.5	1.5	1.5		0.25
A_6	0.75	0.75	0.75	1.5	2.25	2.25	2.25		0.25
A_7	2.8	2.8	3	3	3.2	3.2	3.2		0

A_1=0.487	A_2=1	A_3=1.7	A_4=2.2	A_5=1.08	A_6=1.687	A_7=3.1

X_6

A_1	0.45	0.55	0.45	0.5	0.45	0.55	0.6		0
A_2	1	1	1	1	1	1	1		0.25
A_3	1.45	1.45	1.45	1.45	1.95	1.95	1.95		0.25
A_4	1.75	1.75	1.75	1.95	2.55	2.55	2.55	*	0.25
A_5	0.33	0.33	0.33	1	1.5	1.5	1.5		0.25
A_6	0.75	0.75	0.75	1.5	2.25	2.25	2.25		0
A_7	2.8	2.8	3	3	3.2	3.2	3.2		0

A_1=0.487	A_2=1	A_3=1.575	A_4=2	A_5=0.79	A_6=1.312	A_7=3

X_7

A_1	0.45	0.55	0.45	0.5	0.45	0.55	0.6		0
A_2	1	1	1	1	1	1	1		0
A_3	1.45	1.45	1.45	1.45	1.95	1.95	1.95		0
A_4	1.75	1.75	1.75	1.95	2.55	2.55	2.55	*	0.25
A_5	0.33	0.33	0.33	1	1.5	1.5	1.5		0.25
A_6	0.75	0.75	0.75	1.5	2.25	2.25	2.25		0.25
A_7	2.8	2.8	3	3	3.2	3.2	3.2		0.25

A_1=0.525	A_2=1	A_3=1.825	A_4=2.4	A_5=1.375	A_6=2.062	A_7=3.15

Control point X_{12}

A_1	0.45	0.55	0.45	0.5	0.45	0.55	0.6		0.143
A_2	1	1	1	1	1	1	1		0.143
A_3	1.45	1.45	1.45	1.45	1.95	1.95	1.95		0.143
A_4	1.75	1.75	1.75	1.95	2.55	2.55	2.55	*	0.143
A_5	0.33	0.33	0.33	1	1.5	1.5	1.5		0.143
A_6	0.75	0.75	0.75	1.5	2.25	2.25	2.25		0.143
A_7	2.8	2.8	3	3	3.2	3.2	3.2		0.143

A_1=0.507	A_2=1	A_3=1.665	A_4=2.12	A_5=0.928	A_6=1.5	A_7=3.031

Control point X_{13}

A_1	0.45	0.55	0.45	0.5	0.45	0.55	0.6		0.3
A_2	1	1	1	1	1	1	1		0.3
A_3	1.45	1.45	1.45	1.45	1.95	1.95	1.95		0.3
A_4	1.75	1.75	1.75	1.95	2.55	2.55	2.55	*	0.1
A_5	0.33	0.33	0.33	1	1.5	1.5	1.5		0
A_6	0.75	0.75	0.75	1.5	2.25	2.25	2.25		0
A_7	2.8	2.8	3	3	3.2	3.2	3.2		0

A_1=0.485	A_2=1	A_3=1.45	A_4=1.77	A_5=0.397	A_6=0.825	A_7=2.88

Control point X_{14}

A_1	0.45	0.55	0.45	0.5	0.45	0.55	0.6		0.3
A_2	1	1	1	1	1	1	1		0.3
A_3	1.45	1.45	1.45	1.45	1.95	1.95	1.95		0.3
A_4	1.75	1.75	1.75	1.95	2.55	2.55	2.55	*	0
A_5	0.33	0.33	0.33	1	1.5	1.5	1.5		0.1
A_6	0.75	0.75	0.75	1.5	2.25	2.25	2.25		0
A_7	2.8	2.8	3	3	3.2	3.2	3.2		0

A_1=0.48	A_2=1	A_3=1.5	A_4=1.83	A_5=0.447	A_6=0.9	A_7=2.9

Control point X_{15}

A_1	0.45	0.55	0.45	0.5	0.45	0.55	0.6		0.3
A_2	1	1	1	1	1	1	1		0.3
A_3	1.45	1.45	1.45	1.45	1.95	1.95	1.95		0.3
A_4	1.75	1.75	1.75	1.95	2.55	2.55	2.55	*	0
A_5	0.33	0.33	0.33	1	1.5	1.5	1.5		0
A_6	0.75	0.75	0.75	1.5	2.25	2.25	2.25		0.1
A_7	2.8	2.8	3	3	3.2	3.2	3.2		0

A_1=0.49	A_2=1	A_3=1.5	A_4=1.83	A_5=0.447	A_6=0.9	A_7=2.9

Control point X_{16}

A_1	0.45	0.55	0.45	0.5	0.45	0.55	0.6		0.3
A_2	1	1	1	1	1	1	1		0.3
A_3	1.45	1.45	1.45	1.45	1.95	1.95	1.95		0.3
A_4	1.75	1.75	1.75	1.95	2.55	2.55	2.55	*	0
A_5	0.33	0.33	0.33	1	1.5	1.5	1.5		0
A_6	0.75	0.75	0.75	1.5	2.25	2.25	2.25		0
A_7	2.8	2.8	3	3	3.2	3.2	3.2		0.1

A_1=0.495	A_2=1	A_3=1.5	A_4=1.83	A_5=0.447	A_6=0.9	A_7=2.9

Control point X_{17}

A_1	0.45	0.55	0.45	0.5	0.45	0.55	0.6		0.3
A_2	1	1	1	1	1	1	1		0.3
A_3	1.45	1.45	1.45	1.45	1.95	1.95	1.95		0
A_4	1.75	1.75	1.75	1.95	2.55	2.55	2.55	*	0.3

A_5	0.33	0.33	0.33	1	1.5	1.5	1.5		0.1
A_6	0.75	0.75	0.75	1.5	2.25	2.25	2.25		0
A_7	2.8	2.8	3	3	3.2	3.2	3.2		0

A_1=0.495	A_2=1	A_3=1.5	A_4=1.89	A_5=0.648	A_6=1.125	A_7=2.9

Control point X_{23}

A_1	0.45	0.55	0.45	0.5	0.45	0.55	0.6		0.3
A_2	1	1	1	1	1	1	1		0.3
A_3	1.45	1.45	1.45	1.45	1.95	1.95	1.95		0
A_4	1.75	1.75	1.75	1.95	2.55	2.55	2.55	*	0
A_5	0.33	0.33	0.33	1	1.5	1.5	1.5		0.3
A_6	0.75	0.75	0.75	1.5	2.25	2.25	2.25		0.1
A_7	2.8	2.8	3	3	3.2	3.2	3.2		0

A_1=0.495	A_2=1	A_3=1.5	A_4=1.89	A_5=0.648	A_6=1.125	A_7=2.9

Control point X_{24}

A_1	0.45	0.55	0.45	0.5	0.45	0.55	0.6		0.3
A_2	1	1	1	1	1	1	1		0.3
A_3	1.45	1.45	1.45	1.45	1.95	1.95	1.95		0
A_4	1.75	1.75	1.75	1.95	2.55	2.55	2.55	*	0
A_5	0.33	0.33	0.33	1	1.5	1.5	1.5		0
A_6	0.75	0.75	0.75	1.5	2.25	2.25	2.25		0.3
A_7	2.8	2.8	3	3	3.2	3.2	3.2		0.1

A_1=0.525	A_2=1	A_3=1.5	A_4=1.89	A_5=0.648	A_6=1.125	A_7=2.96

Control point X_{25}

A_1	0.45	0.55	0.45	0.5	0.45	0.55	0.6		0.3
A_2	1	1	1	1	1	1	1		0.3
A_3	1.45	1.45	1.45	1.45	1.95	1.95	1.95		0
A_4	1.75	1.75	1.75	1.95	2.55	2.55	2.55	*	0
A_5	0.33	0.33	0.33	1	1.5	1.5	1.5		0
A_6	0.75	0.75	0.75	1.5	2.25	2.25	2.25		0.1
A_7	2.8	2.8	3	3	3.2	3.2	3.2		0.3

A_1=0.535	A_2=1	A_3=1.5	A_4=1.89	A_5=0.648	A_6=1.125	A_7=2.96

Control point X_{26}

A_1	0.45	0.55	0.45	0.5	0.45	0.55	0.6		0.3
A_2	1	1	1	1	1	1	1		0
A_3	1.45	1.45	1.45	1.45	1.95	1.95	1.95		0.3
A_4	1.75	1.75	1.75	1.95	2.55	2.55	2.55	*	0.3
A_5	0.33	0.33	0.33	1	1.5	1.5	1.5		0.1
A_6	0.75	0.75	0.75	1.5	2.25	2.25	2.25		0
A_7	2.8	2.8	3	3	3.2	3.2	3.2		0

A_1=0.465	A_2=1	A_3=1.5	A_4=1.89	A_5=0.648	A_6=1.125	A_7=2.96

Control point X_{27}

A_1	0.45	0.55	0.45	0.5	0.45	0.55	0.6		0.3
A_2	1	1	1	1	1	1	1		0
A_3	1.45	1.45	1.45	1.45	1.95	1.95	1.95		0.3
A_4	1.75	1.75	1.75	1.95	2.55	2.55	2.55	*	0.1
A_5	0.33	0.33	0.33	1	1.5	1.5	1.5		0.3
A_6	0.75	0.75	0.75	1.5	2.25	2.25	2.25		0
A_7	2.8	2.8	3	3	3.2	3.2	3.2		0

A_1=0.455	A_2=1	A_3=1.6	A_4=2.01	A_5=0.748	A_6=1.275	A_7=3

Control point X_{34}

A_1	0.45	0.55	0.45	0.5	0.45	0.55	0.6		0
A_2	1	1	1	1	1	1	1		0
A_3	1.45	1.45	1.45	1.45	1.95	1.95	1.95		0.3
A_4	1.75	1.75	1.75	1.95	2.55	2.55	2.55	*	0.3
A_5	0.33	0.33	0.33	1	1.5	1.5	1.5		0.3
A_6	0.75	0.75	0.75	1.5	2.25	2.25	2.25		0.1
A_7	2.8	2.8	3	3	3.2	3.2	3.2		0

A_1=0.475	A_2=1	A_3=1.65	A_4=2.13	A_5=0.999	A_6=1.575	A_7=3.08

Control point X_{35}

A_1	0.45	0.55	0.45	0.5	0.45	0.55	0.6		0
A_2	1	1	1	1	1	1	1		0.3
A_3	1.45	1.45	1.45	1.45	1.95	1.95	1.95		0.3
A_4	1.75	1.75	1.75	1.95	2.55	2.55	2.55	*	0.3
A_5	0.33	0.33	0.33	1	1.5	1.5	1.5		0
A_6	0.75	0.75	0.75	1.5	2.25	2.25	2.25		0.1
A_7	2.8	2.8	3	3	3.2	3.2	3.2		0

A_1=0.505	A_2=1	A_3=1.5	A_4=1.89	A_5=0.648	A_6=1.125	A_7=2.96

Control point X_{36}

A_1	0.45	0.55	0.45	0.5	0.45	0.55	0.6		0
A_2	1	1	1	1	1	1	1		0.3
A_3	1.45	1.45	1.45	1.45	1.95	1.95	1.95		0.3
A_4	1.75	1.75	1.75	1.95	2.55	2.55	2.55	*	0.3
A_5	0.33	0.33	0.33	1	1.5	1.5	1.5		0
A_6	0.75	0.75	0.75	1.5	2.25	2.25	2.25		0
A_7	2.8	2.8	3	3	3.2	3.2	3.2		0.1

A_1=0.51	A_2=1	A_3=1.5	A_4=1.89	A_5=0.648	A_6=1.125	A_7=2.96

Control point X_{37}

A_1	0.45	0.55	0.45	0.5	0.45	0.55	0.6		0
A_2	1	1	1	1	1	1	1		0
A_3	1.45	1.45	1.45	1.45	1.95	1.95	1.95		0
A_4	1.75	1.75	1.75	1.95	2.55	2.55	2.55	*	0.3

A_5	0.33	0.33	0.33	1	1.5	1.5	1.5		0.3
A_6	0.75	0.75	0.75	1.5	2.25	2.25	2.25		0.1
A_7	2.8	2.8	3	3	3.2	3.2	3.2		0.3

A_1=0.52	A_2=1	A_3=1.8	A_4=2.37	A_5=1.35	A_6=2.025	A_7=3.14

Control point X_{45}

A_1	0.45	0.55	0.45	0.5	0.45	0.55	0.6		0
A_2	1	1	1	1	1	1	1		0
A_3	1.45	1.45	1.45	1.45	1.95	1.95	1.95		0.3
A_4	1.75	1.75	1.75	1.95	2.55	2.55	2.55	*	0.3
A_5	0.33	0.33	0.33	1	1.5	1.5	1.5		0.3
A_6	0.75	0.75	0.75	1.5	2.25	2.25	2.25		0.1
A_7	2.8	2.8	3	3	3.2	3.2	3.2		0

A_1=0.475	A_2=1	A_3=1.65	A_4=2.13	A_5=0.999	A_6=1.575	A_7=3.08

Control point X_{46}

A_1	0.45	0.55	0.45	0.5	0.45	0.55	0.6		0
A_2	1	1	1	1	1	1	1		0
A_3	1.45	1.45	1.45	1.45	1.95	1.95	1.95		0.3
A_4	1.75	1.75	1.75	1.95	2.55	2.55	2.55	*	0.3
A_5	0.33	0.33	0.33	1	1.5	1.5	1.5		0.3
A_6	0.75	0.75	0.75	1.5	2.25	2.25	2.25		0
A_7	2.8	2.8	3	3	3.2	3.2	3.2		0.1

A_1=0.48	A_2=1	A_3=1.65	A_4=2.13	A_5=0.999	A_6=1.575	A_7=3.08

Control point X_{47}

A_1	0.45	0.55	0.45	0.5	0.45	0.55	0.6		0
A_2	1	1	1	1	1	1	1		0
A_3	1.45	1.45	1.45	1.45	1.95	1.95	1.95		0
A_4	1.75	1.75	1.75	1.95	2.55	2.55	2.55	*	0.3
A_5	0.33	0.33	0.33	1	1.5	1.5	1.5		0.3
A_6	0.75	0.75	0.75	1.5	2.25	2.25	2.25		0.3
A_7	2.8	2.8	3	3	3.2	3.2	3.2		0.1

A_1=0.51	A_2=1	A_3=1.8	A_4=2.37	A_5=1.35	A_6=2.025	A_7=3.14

Control point X_{56}

A_1	0.45	0.55	0.45	0.5	0.45	0.55	0.6		0
A_2	1	1	1	1	1	1	1		0
A_3	1.45	1.45	1.45	1.45	1.95	1.95	1.95		0
A_4	1.75	1.75	1.75	1.95	2.55	2.55	2.55	*	0.1
A_5	0.33	0.33	0.33	1	1.5	1.5	1.5		0.3
A_6	0.75	0.75	0.75	1.5	2.25	2.25	2.25		0.3
A_7	2.8	2.8	3	3	3.2	3.2	3.2		0.3

A_1=0.53	A_2=1	A_3=1.9	A_4=2.49	A_5=1.45	A_6=2.175	A_7=3.18

Control point X_{57}

A_1	0.45	0.55	0.45	0.5	0.45	0.55	0.6		0.1
A_2	1	1	1	1	1	1	1		0
A_3	1.45	1.45	1.45	1.45	1.95	1.95	1.95		0
A_4	1.75	1.75	1.75	1.95	2.55	2.55	2.55	*	0
A_5	0.33	0.33	0.33	1	1.5	1.5	1.5		0.3
A_6	0.75	0.75	0.75	1.5	2.25	2.25	2.25		0.3
A_7	2.8	2.8	3	3	3.2	3.2	3.2		0.3

A_1=0.525	A_2=1	A_3=1.9	A_4=2.47	A_5=1.383	A_6=2.1	A_7=3.16

Control point X_{67}

A_1	0.45	0.55	0.45	0.5	0.45	0.55	0.6		0
A_2	1	1	1	1	1	1	1		0
A_3	1.45	1.45	1.45	1.45	1.95	1.95	1.95		0
A_4	1.75	1.75	1.75	1.95	2.55	2.55	2.55	*	0.3
A_5	0.33	0.33	0.33	1	1.5	1.5	1.5		0.1
A_6	0.75	0.75	0.75	1.5	2.25	2.25	2.25		0.3
A_7	2.8	2.8	3	3	3.2	3.2	3.2		0.3

A_1=0.54	A_2=1	A_3=1.8	A_4=2.37	A_5=1.35	A_6=2.025	A_7=3.14

III. MATERIALS AND METHODOLOGY

3.1. Materials

The materials used are the mixture of cement, water, fine and coarse aggregate. The cement used is of 53 grades, conforming to IS 12269:1987[8]. The fine aggregate, whose size ranges from 0.05 to 4.5 mm, was procured from the local river. Crushed granite of 20 mm size downgraded to 4.75 mm obtained from a local stone market was used in the experimental investigation. FA and CA confirming to IS 2386-1963 (Reaffirmed 2002)[9]

3.2. Methodology

The specimen for the compressive strength is concrete cube moulds measuring 150 mm *150mm*150mm. The concrete cubes were cast and cured at 28 days for flexural test. For model formulation eighty four cubes were cast and experimental test was conducted. For the adequacy of the model the eighty four control specimens of cubes were cast and the control test was conducted .For every of the mix ratios, three specimen were cast and cured for 28 days and were crushed after 28 days of curing and the average compressive strength noted. [13,14/10].

IV. RESULTS

4.1. Regression equation for compressive strength

The coefficients of the Scheffe's second degree polynomial were determined as follows;

$$R= 35B_1+34.3 B_2+33.83 B_3+29.9 B_4+32.4 B_5+31.3 B_6+30.88 B_7- B_1B_2-9.66B_1B_3 -3B_1B_4-10.8 B_1B_5 -4.84 B_1B_6+3.44 B_1B_7-3.46 B_2 B_3+3.52B_2B_4+0.2 B_2B_5+0.8 B_2B_6 +0.72B_2B_7+4.3B_3B_4-11.66B_3B_5-10.38 B_3B_6-1.62 B_3B_6-1.62B_3B_7+3.8 B_4B_5-2.36 B_4B_6-1.64 B_4B_7-2.12 B_5B_6+1.64 B_5B_7+1.64 B_6B_7$$

The above is the improved model for the optimization for the compressive strength of a concrete.

4.2. Replication variance

Mean responses, R and the variances of replicates Si^2 are tabulated in Table 6.

Table 1: Matrix Table for Scheffe's (7, 2) Polynomial

S/N	Comp	A_1	A_2	A_3	A_4	A_5	A_6	A_7	Respo-nse	B_1	B_2	B_3	B_4	B_5	B_6	B_7
1	x_1	0.45	1	1.45	1.75	0.33	0.75	2.8	R_1	1	0	0	0	0	0	0
2	x_2	0.55	1	1.45	1.75	0.33	0.75	2.8	R_2	0	1	0	0	0	0	0
3	x_3	0.45	1	1.45	1.75	0.33	0.75	3	R_3	0	0	1	0	0	0	0
4	x_4	0.55	1	1.45	1.95	1	1.5	3	R_4	0	0	0	1	0	0	0
5	x_5	0.45	1	1.95	2.55	1.5	2.25	3.2	R_5	0	0	0	1	0	0	0
6	x_6	0.55	1	1.95	2.55	1.5	2.25	3.2	R_6	0	0	0	1	0	0	0
7	x_7	0.6	1	1.95	2.55	1.5	2.25	3.2	R_7	0	0	0	1	0	0	0
8	x_{12}	0.5	1	1.45	1.75	2.8	0.75	2.8	R_{12}	0.5	0.5	0	0	0	0	0
9	x_{13}	0.45	1	1.45	1.75	0.33	0.75	2.9	R_{13}	0.5	0	0.5	0	0	0	0
10	x_{14}	0.475	1	1.45	1.85	0.665	1.125	2.9	R_{14}	0.5	0	0	0.5	0	0	0
11	x_{15}	0.475	1	1.7	2.15	0.915	1.5	3	R_{15}	0.5	0	0	0	0.5	0	0
12	x_{16}	0.5	1	1.7	2.15	0.915	1.5	3	R_{16}	0.5	0	0	0	0	0.5	0
13	x_{17}	0.525	1	1.7	2.15	0.915	1.5	3	R_{17}	0.5	0	0	0	0	0	0.5
14	x_{23}	0.5	1	1.45	1.75	0.33	0.75	2.9	R_{23}	0	0.5	0.5	0	0	0	0
15	x_{24}	0.525	1	1.45	1.85	0.665	1.125	2.9	R_{24}	0	0.5	0	0.5	0	0	0
16	x_{25}	0.5	1	1.7	2.15	0.915	1.5	3	R_{25}	0	0.5	0	0	0.5	0	0
17	x_{26}	0.55	1	1.7	2.15	0.915	1.5	3	R_{26}	0	0.5	0	0	0	0.5	0
18	x_{27}	0.575	1	1.7	2.15	0.915	1.5	3	R_{27}	0	0.5	0	0	0	0	0.5
19	x_{34}	0.475	1	1.45	1.85	0.665	1.125	3	R_{34}	0	0	0.5	0.5	0	0	0
20	x_{35}	0.45	1	1.7	2.15	0.915	1.5	3.1	R_{35}	0	0	0.5	0	0.5	0	0
21	x_{36}	0.5	1	1.7	2.15	0.915	1.5	3.1	R_{36}	0	0	0.5	0	0	0.5	0
22	x_{37}	0.525	1	1.7	2.15	0.915	1.5	3.1	R_{37}	0	0	0.5	0	0	0	0.5

23	x_{45}	0.475	1	1.7	2.25	1.25	1.875	3.1	R_{45}	0	0	0	0.5	0.5	0	0
24	x_{46}	0.525	1	1.7	2.25	1.25	1.875	3.1	R_{46}	0	0	0	0.5	0	0.5	0
25	x_{47}	0.55	1	1.7	2.25	1.25	1.875	3.1	R_{47}	0	0	0	0.5	0	0	0.5
26	x_{56}	0.5	1	1.95	2.55	1. 5	2.25	3.2	R_{56}	0	0	0	0	0.5	0.5	0
27	x_{57}	0.525	1	1.95	2.55	1. 5	2.25	3.2	R_{57}	0	0	0	0	0.5	0	0.5
28	x_{67}	0.575	1	1.95	2.55	1. 5	2.25	3.2	R_{67}	0	0	0	0	0	0.5	0.5

Table 2: Mixture Proportion of Control Points Showing Actual and Pseudo Components

S/N	Comp	A_1	A_2	A_3	A_4	A_5	A_6	A_7	Respo-nse	B_1	B_2	B_3	B_4	B_5	B_6	B_7
1	x_1	0.45	1	1.45	1.75	0.33	0.75	2.8	C_1	0.25	0.25	0.25	0.25	0	0	0
2	x_2	0.55	1	1.45	1.75	0.33	0.75	2.8	C_2	0.25	0.25	0.25	0	0.25	0	0
3	x_3	0.45	1	1.45	1.75	0.33	0.75	3	C_3	0.25	0.25	0	0	0.25	0.25	0
4	x_4	0.55	1	1.45	1.95	1	1.5	3	C_4	0.25	0	0.25	0.25	0.25	0	0
5	x_5	0.45	1	1.95	2.55	1.5	2.25	3.2	C_5	0	0	0.25	0.25	0.25	0.25	0
6	x_6	0.55	1	1.95	2.55	1.5	2.25	3.2	C_6	0	0.25	0.25	0.25	0.25	0	0
7	x_7	0.6	1	1.95	2.55	1.5	2.25	3.2	C_7	0	0	0	0.25	0.25	0.25	0.25
8	x_{12}	0.5	1	1.45	1.75	2.8	0.75	2.8	C_{12}	0.143	0.143	0.143	0.143	0.143	0.143	0.143
9	x_{13}	0.45	1	1.45	1.75	0.33	0.75	2.9	C_{13}	0.3	0.3	0.3	0.1	0	0	0
10	x_{14}	0.475	1	1.45	1.85	0.665	1.125	2.9	C_{14}	0.3	0.3	0.3	0	0.1	0	0
11	x_{15}	0.475	1	1.7	2.15	0.915	1.5	3	C_{15}	0.3	0.3	0.3	0	0	0.1	0
12	x_{16}	0.5	1	1.7	2.15	0.915	1.5	3	C_{16}	0.3	0.3	0.3	0	0	0	0.1
13	x_{17}	0.525	1	1.7	2.15	0.915	1.5	3	C_{17}	0.3	0.3	0	0.3	0.1	0	0
14	x_{23}	0.5	1	1.45	1.75	0.33	0.75	2.9	C_{23}	0.3	0.3	0	0	0.3	0.1	0
15	x_{24}	0.525	1	1.45	1.85	0.665	1.125	2.9	C_{24}	0.3	0.3	0	0	0	0.3	0.1
16	x_{25}	0.5	1	1.7	2.15	0.915	1.5	3	C_{25}	0.3	0.3	0	0	0	0.1	0.3
17	x_{26}	0.55	1	1.7	2.1	0.91	1.5	3	C_{26}	0.3	0	0.3	0.3	0.1	0	0

					5	5										
18	x_{27}	0.575	1	1.7	2.15	0.915	1.5	3	C_{27}	0.3	0	0.3	0.1	0.3	0	0
19	x_{34}	0.475	1	1.45	1.85	0.665	1.125	3	C_{34}	0	0	0.3	0.3	0.3	0.1	0
20	x_{35}	0.45	1	1.7	2.15	0.915	1.5	3.1	C_{35}	0	0.3	0.3	0.3	0	0.1	0
21	x_{36}	0.5	1	1.7	2.15	0.915	1.5	3.1	C_{36}	0	0.3	0.3	0.3	0	0	0.1
22	x_{37}	0.525	1	1.7	2.15	0.915	1.5	3.1	C_{37}	0	0	0	0.3	0.3	0.1	0.3
23	x_{45}	0.475	1	1.7	2.25	1.25	1.875	3.1	C_{45}	0	0	0.3	0.3	0.3	0.1	0
24	x_{46}	0.525	1	1.7	2.25	1.25	1.875	3.1	C_{46}	0	0	0.3	0.3	0.3	0	0.1
25	x_{47}	0.55	1	1.7	2.25	1.25	1.875	3.1	C_{47}	0	0	0	0.3	0.3	0.3	0.1
26	x_{56}	0.5	1	1.95	2.55	1. 5	2.25	3.2	C_{56}	0	0	0	0.1	0.3	0.3	0.3
27	x_{57}	0.525	1	1.95	2.55	1. 5	2.25	3.2	C_{57}	0.1	0	0	0	0.3	0.3	0.3
28	x_{67}	0.575	1	1.95	2.55	1. 5	2.25	3.2	C_{67}	0	0	0	0.3	0.1	0.3	0.3

Table 3: Results of 28 Day Comp Strength (Response R_i)

S/N	Response Symbol	Average Comp Strength (MPa)	S/N	Response Symbol	Average Comp Strength (MPa)	S/N.	Response Symbol	Average Comp Strength (MPa)	S/N	Response Symbol	Average Comp Strength (MPa)
1	R_1	35	8	R_{12}	34.4	15	R_{24}	32.98	22	R_{37}	31.95
2	R_2	34.3	9	R_{13}	32	16	R_{25}	33.4	23	R_{45}	32.1
3	R_3	33.83	10	R_{14}	31.7	17	R_{26}	33	24	R_{46}	30.01
4	R_4	29.9	11	R_{15}	31	18	R_{27}	32.77	25	R_{47}	29.98
5	R_5	32.4	12	R_{16}	31.94	19	R_{34}	32.94	26	R_{56}	31.32
6	R_6	31.3	13	R_{17}	33.8	20	R_{35}	30.2	27	R_{57}	32.05
7	R_7	30.88	14	R_{23}	33.2	21	R_{36}	29.97	28	R_{67}	31.5

Table 4: Results of 28 Day Comp Strength (For Control Points

S/N	Response Symbol	Average Comp Strength (MPa)	S/N	Response Symbol	Average Comp Strength (MPa)	S/N.	Response Symbol	Average Comp Strength (MPa)	S/N	Response Symbol	Average Comp Strength (MPa)
1	C_1	32.92	8	C_{12}	32.85	15	C_{24}	31.18	22	C_{37}	33.67
2	C_2	33.43	9	C_{13}	33.18	16	C_{25}	31.32	23	C_{45}	32.35
3	C_3	33.57	10	C_{14}	31.33	17	C_{26}	30.99	24	C_{46}	32.05
4	C_4	32.01	11	C_{15}	30.81	18	C_{27}	31.85	25	C_{47}	30.27

5	C_5	31.48	12	C_{16}	31.70	19	C_{34}	31.43	26	C_{56}	31.83
6	C_6	32.37	13	C_{17}	32.16	20	C_{35}	32.84	27	C_{57}	32.95
7	C_7	30.04	14	C_{23}	31.84	21	C_{36}	31.31	28	C_{67}	30.18

Table 5: Coefficients of the Scheffe's Second Degree Polynomial for Compressive Strength

Sr. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Symbol	α_1	α_2	α_3	α_4	α_5	α_6	α_7	α_{12}	α_{13}	α_{14}	α_{15}	α_{16}	α_{17}	A_{23}
Comp Strength	35	34.3	33.83	29.9	32.4	31.3	30.88	-1	- 9.66	-3	- 10.8	- 4.84	3.44	- 3.46
Sr. No.	15	16	17	18	19	20	21	22	23	24	25	26	27	28
Symbol	A_{24}	α_{25}	α_{26}	α_{27}	α_{34}	α_{35}	α_{36}	α_{37}	α_{45}	α_{46}	α_{47}	α_{56}	α_{57}	α_{67}
Comp Strength	3.52	0.2	0.8	0.72	4.3	- 11.66	- 10.38	- 1.62	3.8	- 2.36	- 1.64	- 2.12	1.64	1.64

V. VALIDATION AND TEST OF ADEQUACY OF THE MODEL

The improved model was analyzed statistically using student's t-test and ANOVA method; the adequacy of the model was tested against the experimental results of the control points. The predicted values (R-predicted)) for the test control points were obtained by substituting the corresponding values of B_1, B_2, B_3, B_4 B_5, B_6 and B_7 into the improved model .These Values were compared with the experimental results (R-observed).
The test for adequacy of the model was done using student's t-test and ANOVA at 95% confidence level on the flexural strength at the control points that is, C_1, C_2, C_3, C_4, C_5, C_6 , C_7, C_{12}, C_{13}, C_{14}, C_{15}, C_{16}, C_{17},C_{23}, C_{24}, C_{25}, C_{26}, C_{27},C_{34}, C_{35}, C_{36}, C_{37} ,C_{45}, C_{46}, C_{47}, C_{56}, C_{57} and C_{67}. In this test, two hypotheses were set as follows:

5.1. Null hypothesis

There is no significant difference between the laboratory tests and model predicted strength results.

5.2. Alternative hypothesis

There is a significant difference between the laboratory test and model predicted strength results.

5.3. Student's t-Test

We do a two-tail test (inequality) and if $t_{stat} > t_{critical}$ two tail, we reject the null hypothesis. From the result presented in Tables 8 and 9, $t_{stat} = 0.0727$ and $t_{critical}$ two tail = 2.051, we accept the null hypothesis.

Table 6: Experimental Test Result and Replication Variance

Sr.No.	Response symbol	Replicate	Response R_i(N/mm^2)	$\sum R_i$	R	$\sum R_i^2$	R_i-R	$\sum(R_i$-R)^2	S_i^2
1	R_1	1	34.98	105.00	35.00	3675.02	-0.02	0.02	0.01
2	R_1	2	35.10				0.10		0.00
3	R_1	3	34.92				-0.08		0.00
4	R_2	1	34.10	102.90	34.30	3529.99	-0.20	0.52	0.26
5	R_2	2	34.88				0.58		0.00
6	R_2	3	33.92				-0.38		0.00
7	R_3	1	33.70	101.49	33.83	3433.43	-0.13	0.03	0.01
8	R_3	2	33.90				0.07		0.00
9	R_3	3	33.89				0.06		0.00
10	R_4	1	30.12	89.70	29.90	2682.58	0.22	0.55	0.28
11	R_4	2	30.28				0.38		0.00
12	R_4	3	29.30				-0.60		0.00
13	R_5	1	32.55	97.20	32.40	3149.47	0.15	0.19	0.09
14	R_5	2	32.60				0.20		0.00
15	R_5	3	32.05				-0.35		0.00
16	R_6	1	31.20	93.90	31.30	2939.17	-0.10	0.10	0.05

17	R_6	2	31.55				0.25		0.00
18	R_6	3	31.15				-0.15		0.00
19	R_7	1	30.92	92.64	30.88	2860.79	0.04	0.07	0.03
20	R_7	2	31.04				0.16		0.00
21	R_7	3	30.68				-0.20		0.00
22	R_{12}	1	34.57	103.20	34.40	3550.14	0.17	0.06	0.03
23	R_{12}	2	34.23				-0.17		0.00
24	R_{12}	3	34.40				0.00		0.00
25	R_{13}	1	32.12	96.00	32.00	3072.03	0.12	0.03	0.01
26	R_{13}	2	31.88				-0.12		0.00
27	R_{13}	3	32.00				0.00		0.00
28	R_{14}	1	31.55	95.10	31.70	3015.32	-0.15	0.65	0.32
29	R_{14}	2	31.22				-0.48		0.00
30	R_{14}	3	32.33				0.63		0.00
31	R_{15}	1	30.87	93.00	31.00	2883.04	-0.13	0.04	0.02
32	R_{15}	2	31.15				0.15		0.00
33	R_{15}	3	30.98				-0.02		0.00
34	R_{16}	1	31.75	95.82	31.94	3060.55	-0.19	0.06	0.03
35	R_{16}	2	32.00				0.06		0.00
36	R_{16}	3	32.07				0.13		0.00
37	R_{17}	1	33.75	101.40	33.80	3427.33	-0.05	0.01	0.00
38	R_{17}	2	33.85				0.05		0.00
39	R_{17}	3	33.80				0.00		0.00
40	R_{23}	1	33.00	99.60	33.20	3307.48	-0.20	0.76	0.38
41	R_{23}	2	33.89				0.69		0.00
42	R_{23}	3	32.71				-0.49		0.00
43	R_{24}	1	32.66	98.94	32.98	3263.44	-0.32	0.46	0.23
44	R_{24}	2	32.79				-0.32		0.00
45	R_{24}	3	33.49				0.51		0.00
46	R_{25}	1	33.33	100.20	33.40	3346.70	-0.07	0.02	0.01
47	R_{25}	2	33.52				0.12		0.00
48	R_{25}	3	33.35				-0.05		0.00
49	R_{26}	1	33.12	99.00	33.00	3267.60	0.12	0.60	0.30
50	R_{26}	2	33.48				0.48		0.00
51	R_{26}	3	32.40				-0.60		0.00
52	R_{27}	1	32.70	98.31	32.77	3221.63	-0.07	0.01	0.00
53	R_{27}	2	32.82				0.05		0.00
54	R_{27}	3	32.79				0.02		0.00
55	R_{34}	1	32.90	98.82	32.94	3255.14	-0.04	0.01	0.01
56	R_{34}	2	32.89				-0.05		0.00
57	R_{34}	3	33.03				0.09		0.00
58	R_{35}	1	30.18	90.60	30.20	2736.14	-0.02	0.02	0.01
59	R_{35}	2	30.30				0.10		0.00
60	R_{35}	3	30.12				-0.08		0.00
61	R_{36}	1	30.13	89.91	29.97	2694.67	0.16	0.06	0.03
62	R_{36}	2	30.00				0.03		0.00
63	R_{36}	3	29.78				-0.19		0.00
64	R_{37}	1	31.87	95.85	31.95	3062.44	-0.08	0.03	0.02
65	R_{37}	2	32.10				0.15		0.00
66	R_{37}	3	31.88				-0.07		0.00
67	R_{45}	1	32.14	96.30	32.10	3091.26	0.04	0.03	0.01

68	R_{45}	2	32.19				0.09		0.00
69	R_{45}	3	31.97				-0.13		0.00
70	R_{46}	1	30.21	90.03	30.01	2701.87	0.20	0.07	0.03
71	R_{46}	2	29.98				-0.03		0.00
72	R_{46}	3	29.84				-0.17		0.00
73	R_{47}	1	29.76	89.94	29.98	2696.55	-0.22	0.15	0.08
74	R_{47}	2	29.89				-0.09		0.00
75	R_{47}	3	30.29				0.31		0.00
76	R_{56}	1	31.30	93.96	31.32	2942.85	-0.02	0.02	0.01
77	R_{56}	2	31.23				-0.09		0.00
78	R_{56}	3	31.43				0.11		0.00
79	R_{57}	1	32.12	96.15	32.05	3081.67	0.07	0.06	0.03
80	R_{57}	2	32.18				0.13		0.00
81	R_{57}	3	31.85				-0.20		0.00
82	R_{67}	1	31.45	94.50	31.50	2976.75	-0.05	0.00	0.00
83	R_{67}	2	31.52				0.02		0.00
84	R_{67}	3	31.53				0.03		0.00
								$\sum S_R^2$	2.308
								S_R	1.519
Sr.No.	Response symbol	Replicate	Response R_i(N/mm^2)	$\sum R_i$	R	$\sum R_i^2$	R_i-R	$\sum(R_i$-R$)^2$	S_i^2
1	C_1	1	33.10	98.78	32.93	3252.62	0.17	0.20	0.10
2	C_1	2	33.12				0.19		0.00
3	C_1	3	32.56				-0.37		0.00
4	C_2	1	33.45	100.32	33.44	3354.45	0.01	0.00	0.00
5	C_2	2	33.41				-0.03		0.00
6	C_2	3	33.46				0.02		0.00
7	C_3	1	33.62	100.73	33.58	3382.35	0.04	0.01	0.00
8	C_3	2	33.61				0.03		0.00
9	C_3	3	33.50				-0.08		0.00
10	C_4	1	32.10	96.04	32.01	3074.52	0.09	0.13	0.06
11	C_4	2	32.21				0.20		0.00
12	C_4	3	31.73				-0.29		0.00
13	C_5	1	31.52	94.46	31.49	2974.39	0.03	0.00	0.00
14	C_5	2	31.49				0.00		0.00
15	C_5	3	31.45				-0.04		0.00
16	C_6	1	32.39	97.13	32.38	3144.91	0.01	0.00	0.00
17	C_6	2	32.39				0.01		0.00
18	C_6	3	32.35				-0.03		0.00
19	C_7	1	30.18	90.12	30.04	2707.50	0.14	0.07	0.04
20	C_7	2	30.12				0.08		0.00
21	C_7	3	29.82				-0.22		0.00
22	C_{12}	1	33.00	98.57	32.86	3238.73	0.14	0.03	0.02
23	C_{12}	2	32.82				-0.04		0.00
24	C_{12}	3	32.75				-0.11		0.00
25	C_{13}	1	33.21	99.56	33.19	3303.78	0.02	0.01	0.00
26	C_{13}	2	33.22				0.03		0.00
27	C_{13}	3	33.13				-0.06		0.00
28	C_{14}	1	31.41	94.01	31.34	2945.70	0.07	0.01	0.00
29	C_{14}	2	31.32				-0.02		0.00

30	C_{14}	3	31.28				-0.06		0.00
31	C_{15}	1	30.78	92.44	30.81	2848.51	-0.03	0.01	0.01
32	C_{15}	2	30.91				0.10		0.00
33	C_{15}	3	30.75				-0.06		0.00
34	C_{16}	1	31.68	95.10	31.70	3014.73	-0.02	0.00	0.00
35	C_{16}	2	31.72				0.02		0.00
36	C_{16}	3	31.70				0.00		0.00
37	C_{17}	1	32.20	96.50	32.17	3104.27	0.03	0.00	0.00
38	C_{17}	2	32.15				-0.02		0.00
39	C_{17}	3	32.15				-0.01		0.00
40	C_{23}	1	31.81	95.53	31.84	3042.05	-0.03	0.01	0.00
41	C_{23}	2	31.92				0.08		0.00
42	C_{23}	3	31.80				-0.04		0.00
43	C_{24}	1	31.22	93.57	31.19	2918.17	0.03	0.02	0.01
44	C_{24}	2	31.28				0.03		0.00
45	C_{24}	3	31.07				-0.12		0.00
46	C_{25}	1	31.29	93.96	31.32	2942.91	-0.03	0.00	0.00
47	C_{25}	2	31.30				-0.02		0.00
48	C_{25}	3	31.37				0.05		0.00
49	C_{26}	1	31.03	93.00	31.00	2882.82	0.03	0.04	0.02
50	C_{26}	2	31.12				0.12		0.00
51	C_{26}	3	30.85				-0.15		0.00
52	C_{27}	1	31.95	95.58	31.86	3045.09	0.09	0.07	0.03
53	C_{27}	2	31.98				0.12		0.00
54	C_{27}	3	31.65				-0.21		0.00
55	C_{34}	1	31.51	94.30	31.43	2964.45	0.08	0.01	0.00
56	C_{34}	2	31.40				-0.03		0.00
57	C_{34}	3	31.39				-0.04		0.00
58	C_{35}	1	32.89	98.55	32.85	3237.07	0.04	0.02	0.01
59	C_{35}	2	32.91				0.06		0.00
60	C_{35}	3	32.75				-0.10		0.00
61	C_{36}	1	31.35	93.94	31.31	2941.34	0.04	0.02	0.01
62	C_{36}	2	31.38				0.07		0.00
63	C_{36}	3	31.21				-0.11		0.00
64	C_{37}	1	33.79	101.02	33.67	3401.52	0.12	0.03	0.01
65	C_{37}	2	33.68				0.01		0.00
66	C_{37}	3	33.55				-0.13		0.00
67	C_{45}	1	32.47	97.07	32.36	3141.06	0.11	0.02	0.01
68	C_{45}	2	32.34				-0.02		0.00
69	C_{45}	3	32.26				-0.09		0.00
70	C_{46}	1	32.19	96.17	32.06	3083.17	0.13	0.03	0.01
71	C_{46}	2	32.00				-0.06		0.00
72	C_{46}	3	31.98				-0.07		0.00
73	C_{47}	1	32.31	90.81	30.27	2774.41	2.04	25.44	12.72
74	C_{47}	2	32.35				2.08		0.00
75	C_{47}	3	26.15				-4.12		0.00
76	C_{56}	1	31.89	95.49	31.83	3039.53	0.06	0.01	0.00
77	C_{56}	2	31.80				-0.03		0.00
78	C_{56}	3	31.80				-0.03		0.00
79	C_{57}	1	32.96	98.86	32.95	3258.06	0.01	0.00	0.00
80	C_{57}	2	32.91				-0.04		0.00

81	C_{57}	3	32.99				0.04		0.00
82	C_{67}	1	30.21	90.54	30.18	2732.62	0.03	0.00	0.00
83	C_{67}	2	30.20				0.02		0.00
84	C_{67}	3	30.13				-0.05		0.00
								$\sum S_R^2$	13.09
								S_R	3.61

Table 7: Experimental Test Result and Scheffe's Model Test Results

Sr.No.	Experimental Test Results	Scheffe's Model Test Results	Sr.No.	Experimental Test Results	Scheffe's Model Test Results
R_1	35.00	32.93	C_1	32.93	25.75
R_2	34.30	33.44	C_2	33.44	29.13
R_3	33.83	33.58	C_3	33.58	27.82
R_4	29.90	32.01	C_4	32.01	33.20
R_5	32.40	31.49	C_5	31.49	33.32
R_6	31.30	32.38	C_6	32.38	25.67
R_7	30.88	30.04	C_7	30.04	31.63
R_{12}	34.40	32.86	C_{12}	32.86	24.57
R_{13}	32.00	33.19	C_{13}	33.19	23.18
R_{14}	31.70	31.34	C_{14}	31.34	31.85
R_{15}	31.00	30.81	C_{15}	30.81	31.80
R_{16}	31.94	31.70	C_{16}	31.70	33.39
R_{17}	33.80	32.17	C_{17}	32.17	27.07
R_{23}	33.20	31.84	C_{23}	31.84	26.82
R_{24}	32.98	31.19	C_{24}	31.19	33.29
R_{25}	33.40	31.32	C_{25}	31.32	27.57
R_{26}	33.00	31.00	C_{26}	31.00	28.78
R_{27}	32.77	31.86	C_{27}	31.86	34.86
R_{34}	32.94	31.43	C_{34}	31.43	26.22
R_{35}	30.20	32.85	C_{35}	32.85	24.52
R_{36}	29.97	31.31	C_{36}	31.31	27.41
R_{37}	31.95	33.67	C_{37}	33.67	35.35
R_{45}	32.10	32.36	C_{45}	32.36	26.53
R_{46}	30.01	32.06	C_{46}	32.06	27.53
R_{47}	29.98	30.27	C_{47}	30.27	31.57
R_{56}	31.32	31.83	C_{56}	31.83	34.91
R_{57}	32.05	32.95	C_{57}	32.95	24.46
R_{67}	31.50	30.18	C_{67}	30.18	31.73

Table 8:t Test for Compressive Strength

S/N	Symbol	Lab	Model	Lab -Model	(Lab -Model $)^2$
1	C_1	32.93	25.75	7.18	51.55
2	C_2	33.44	29.13	4.31	18.58
3	C_3	33.58	27.82	5.76	33.18
4	C_4	32.01	33.2	-1.19	1.42

5	C_5	31.49	33.32	-1.83	3.35
6	C_6	32.38	25.67	6.71	45.02
7	C_7	30.04	31.63	-1.59	2.53
8	C_{12}	32.86	24.57	8.29	68.72
9	C_{13}	33.19	23.18	10.01	100.20
10	C_{14}	31.34	31.85	-0.51	0.26
11	C_{15}	30.81	31.8	-0.99	0.98
12	C_{16}	31.7	33.39	-1.69	2.86
13	C_{17}	32.17	27.07	5.1	26.01
14	C_{23}	31.84	26.82	5.02	25.20
15	C_{24}	31.19	33.29	-2.1	4.41
16	C_{25}	31.32	27.57	3.75	14.06
17	C_{26}	31	28.78	2.22	4.93
18	C_{27}	31.86	34.86	-3	9.00
19	C_{34}	31.43	26.22	5.21	27.14
20	C_{35}	32.85	24.52	8.33	69.39
21	C_{36}	31.31	27.41	3.9	15.21
22	C_{37}	33.67	35.35	-1.68	2.82
23	C_{45}	32.36	26.53	5.83	33.99
24	C_{46}	32.06	27.53	4.53	20.52
25	C_{47}	30.27	31.57	-1.3	1.69
26	C_{56}	31.83	34.91	-3.08	9.49
27	C_{57}	32.95	24.46	8.49	72.08
28	C_{67}	30.18	31.73	-1.55	2.40
				74.13	666.98

$t_{stat} = \Sigma(\text{lab-model})/\{[n\ \Sigma(\text{lab-model})^2 - \Sigma(\text{lab-model})^2]/(n-1)\}^{0.5}$

$=0.0727$, $t_{critical}=2.051$ $t_{stat} < t_{critical}$ Therefore, we accept the null hypothesis.

5.4. Analysis of variance

If $F < F_{crit}$, the null hypothesis of the analysis of variance can be accepted. From the result presented in Table 10, $F = 0.382$ and $F_{crit} = 4.019$ so $F < F_{crit}$ .Therefore, we do accept null hypothesis. The difference between the experiment result and the model result was not significant. Hence, the model is satisfactory for use in predicting the probable compressive strength when the mix ratio is known and vice-versa.

VI. CONCLUSION

Scheffe's second degree polynomial was used to develop a model for predicting the compressive strength of concrete. This model could predict the compressive strength of the concrete cubes if the mix ratios are known and vice versa. The model gave highest flexural strength of 35 MPa corresponding to mix ratio of 0.45:1.0:1.45:1.75:0.33:0.75:2.8 for water, cement, fine aggregate, coarse aggregate, blend ratio, time lag and temperature respectively. The minimum strength was found to be 29.9MPa which corresponds to mix ratio of 0.55:1.0:1.45:1.95:1.0:1.5:3.0 for water, cement, fine aggregate, coarse aggregate, blend ratio, time lag and temperature respectively

The strengths predicted by the models are in good agreement with the corresponding experimentally observed results. Any desired compressive strength of concrete, given any mix proportions is easily estimated by the model.

Table 9: ANOVA for Compressive Strength

S/N	Comp Strength Symbol	Yobs	Ypred	$Yobs^2$	$Ypred^2$
1	C_1	35	32.93	1225.00	1084.14
2	C_2	34.3	33.44	1176.49	1118.15
3	C_3	33.83	33.58	1144.47	1127.45
4	C_4	29.9	32.01	894.01	1024.80
5	C_5	32.4	31.49	1049.76	991.46
6	C_6	31.3	32.38	979.69	1048.30
7	C_7	30.88	30.04	953.57	902.48
8	C_{12}	34.4	32.86	1183.36	1079.56
9	C_{13}	32	33.19	1024.00	1101.26
10	C_{14}	31.7	31.34	1004.89	981.90
11	C_{15}	31	30.81	961.00	949.50
12	C_{16}	31.94	31.70	1020.16	1004.91
13	C_{17}	33.8	32.17	1142.44	1034.75
14	C_{23}	33.2	31.84	1102.24	1014.01
15	C_{24}	32.98	31.19	1087.68	972.72
16	C_{25}	33.4	31.32	1115.56	980.97
17	C_{26}	33	31.00	1089.00	960.93
18	C_{27}	32.77	31.86	1073.87	1015.01
19	C_{34}	32.94	31.43	1085.04	988.15
20	C_{35}	30.2	32.85	912.04	1079.02
21	C_{36}	29.97	31.31	898.20	980.44
22	C_{37}	31.95	33.67	1020.80	1133.83
23	C_{45}	32.1	32.36	1030.41	1047.01
24	C_{46}	30.01	32.06	900.60	1027.72
25	C_{47}	29.98	30.27	898.80	916.32
26	C_{56}	31.32	31.83	980.94	1013.17
27	C_{57}	32.05	32.95	1027.20	1086.02
28	C_{67}	31.5	30.18	992.25	910.87
	Total	**899.82**	**894.05**	**28973.49**	**28574.84**

N=total score =56 K=2 Dfb=K-1=1 Dfw=N-K=56-2=54

$\Sigma(lab^2/28)+\Sigma(model^2/28)-\Sigma(lab-model)^2/56=$

SSb= $(899.82)^2/28 + (894.05)^2/28 - (899.82+894.05)^2/56=0.59$

SSw $\Sigma lab^2+\Sigma model^2-(\Sigma lab)^2/28+(\Sigma model)^2/28=28973.49+28574.84-(899.82)^2/28)-(894.05)^2/28=83.97$

MSb= 0.59/1=0.59

MSw=SSw/54 =83.97/54

MSw= 1.554962

F=MSb/MSw **=0.380645**

Fcritical (F-Distribution from table =4.019541

Fcalculated < Fcritical , Accept the model

Table 10 :Anova: Single Factor

SUMMARY

Groups	*Count*	*Sum*	*Average*	*Variance*
Yobs	28	899.82	32.13643	2.092276
Ypred	28	894.0505108	31.93038	1.017648

ANOVA

Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
Between Groups	0.594411	1	0.594411	0.382267	0.538992	4.019541
Within Groups	83.96794	54	1.554962			
Total	84.56236	55				

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