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# Metacognition Used by Tutors during Peer Tutoring Sessions in Mathematics

# Matematikte Akran Öğretimi Sırasında Öğretici Görevi Üstlenen Öğrencilerde Üstbiliş

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**ABSTRACT.** The aim of this study is to examine metacognitive skills of students who volunteered for teaching mathematical problems to their peers during peer tutoring sessions. In this regard, data were collected through video-recorded peer tutoring sessions of seventh grade students and interviews with them. In data analysis, six metacognitive components are considered: declarative knowledge, procedural knowledge and conditional knowledge under metacognitive knowledge and planning, monitoring and evaluation under metacognitive regulation. According to results it is observed that volunteer tutor students used metacognition during peer tutoring sessions starting from the preparation to the end of the task. The interviews uncovered metacognitive functioning of the tutors through the strategies they used. Besides, it is seen that metacognitive knowledge and regulation the tutors used are interrelated with and overlapping each other.
**Keywords:** Metacognition, Metacognitive Knowledge, Metacognitive Regulation, Peer Tutoring, Teaching Mathematics

**ÖZ.** Bu çalışmanın amacı matematik dersindeki akran öğretimi etkinliğinde öğretici olarak görev alan öğrencilerin üstbilişlerini incelemektir. Bu bağlamda dört yedinci sınıf öğrencisinin akran öğretimi etkinliği video kayıt altına alınarak izlenmiş; aktivite sonunda da performanslarıyla ilgili birebir görüşmeler yapılmıştır. Öğretici öğrencilerle yapılan görüşmelerden elde edilen veriler üst bilişsel beceri ve üstbilişsel bilgi bileşenlerine göre incelenmiştir. Çalışmadan elde edilen sonuçlara göre gönüllü olarak öğretmeyi seçen öğrenciler etkinliklerin planlama aşamasından sonuna kadar üstbilişsel bilgi ve becerilerini ortaya koymuşlardır. Görüşme bulguları öğretici görevi üstelenen öğrencilerin üstbiliş seviyesindeki etkinlik ve stratejilerini göstermiştir. Bunun yanında üstbilişsel bilgi ve becerilerin birbirini destekleyerek uyumlu bir şekilde bilişsel aktivitelerde rol oynadığı görülmüştür.
**Anahtar Sözcükler:** Üstbilişsel Beceri, Üstbilişsel Bilgi, Üstbiliş, Akran Öğretimi, Matematik Eğitimi

**ÖZET**

***Amaç ve Önem:*** Matematik öğretmenlerinin öğrencilere matematiksel bilgiyi ve problem çözmeyi öğretmenin yanında akıl yürütmeyi, bilişsel becerileri ve yüksek seviyede düşünme süreçlerini destekleyici bir öğretim de yapması beklenir. Bu süreçleri bir arada yürütmek kalabalık ve değişken başarı düzeyinde öğrencilerin olduğu sınıflarda daha da zor olmaktadır. Bu çalışmanın birinci yazarı, öğretmenlik yaptığı sınıflarda akran öğretimi yöntemini kullanmaktadır. Akran öğretimi etkinliği sırasında hızlı öğrenen öğrenciler arkadaşlarına öğretmek için sorumluluk almalarının yanında kendi öğrenme süreçleri ve alışkanlıklarını da gözden geçirmektedirler. Bu tür akran destekli öğretim ortamlarının öğrencilerde üstbilişsel mekanizmaları desteklediği alan yazında savunulmaktadır (Whitebread, Bingham, Grau, Pino Pasternak, & Sangster, 2007). Bu sav öğrencilerin aynı zamanda kendi zihinsel süreçleri üzerine düşündüklerini ve bu süreçleri düzenlediklerini düşündürmektedir. Bu bağlamda çalışmada gönüllü olarak öğretici görevi alan öğrencilerin matematik dersinde akran öğretimi sırasında hangi üstbilişsel bilgi ve becerileri gösterdikleri incelenmiştir.

***Yöntem:*** Bu araştırmada matematik dersinde gönüllü olarak akran öğretimine öğretici rolünde katılan öğrencilerin hangi üstbilişsel bilgi ve becerileri gösterdiklerini incelemek için dört hafta süren bir durum çalışması yapılmıştır. Bu bağlamda dört yedinci sınıf öğrencisinin akran öğretimi etkinliği video kayıt altına alınarak izlenmiş, etkinlik sonunda da performanslarıyla ilgili birebir görüşme yapılmıştır. Öğretici öğrencilerle yapılan görüşmelerden elde edilen veriler üst bilişsel beceri ve üstbilşsel bilgi bileşenlere göre incelenmiştir.

***Bulgular:*** Çalışmadan elde edilen sonuçlara göre gönüllü olarak öğretmeyi seçen öğrenciler etkinliklerin planlama aşamasından sonuna kadar üstbilişsel bilgi ve becerilerini ortaya

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koymuşlardır. Görüşme notları öğretici görevi üstelenen öğrencilerin üstbiliş seviyesindeki etkinlik ve stratejilerini göstermiştir. Gönüllü olarak bu görevi üstlenen öğrenciler kendi yeterlilikleri hakkında bilgi sahibi olduklarını birebir görüşmeler sırasında ifade etmişlerdir. Kendi yeterlilikleri hakkında bilgi sahibi olmalarının yanında arkadaşlarının öğrenme özelliklerini düşünmüş olmalarıyla da üstbilişsel bilgiye sahip oldukları görülmüştür. Aynı zamanda etkinlik için hazırlanma aşamasında hangi durumda nasıl bir yöntem kullanacağını düşünerek ve bunun üzerine planlama yaparak üstbilişsel planlama becerilerini de ortaya koymuşlardır. Etkinliğe başlarken kullandıkları yöntem ile ilgili düşünmüş olmaları, bunu nedenleriyle açıklamaları ve etkinlik sırasında öğretimi daha etkili bir hale getirmek için kullandıkları görseller ve sorular üstbilişsel becerileri kullandıklarını desteklemektedir. Etkinlik sırasında arkadaşlarının öğrenmesini düzenlemek ve denetlemek için sordukları sorularla üstbilişsel becerilerden izleme bileşenini aktif olarak kullandıkları görülmüştür. Ayrıca etkinlik sonunda öğretim kalitesini sorgulayıp kendilerini değerlendirmeleri de üstbilişsel becerileri kullandıklarını göstermiştir.

***Tartışma, Sonuç ve Öneriler:*** Bu çalışmada akran öğretimi etkinliğinde öğretici rolü üstlenen öğrenciler etkinlik sırasında arkadaşlarıyla etkileşimleri ve görüşme sırasındaki açıklamalarıyla üstbilişsel beceri ve bilgilere sahip olduklarını göstermişlerdir. Bunun yanında üstbilişsel bilgi ve becerilerin birbirini destekleyerek uyumlu bir şekilde bilişsel aktivitelerde rol oynadığı görülmüştür. Ayrıca, yapılan görüşmelerde bazı öğrencilerin diğerlerine göre daha az üstbilişsel bilgi ve beceriye dayanan açıklamalarda bulunduğu gözlenmiştir. Bu öğrenciler arasında bazı durumlarda üstbilişsel bilgi ve beceri konusunda farklılıklar olabileceğini düşündürmüştür. Çalışmadaki bulgular matematik derslerinde akran öğretimi etkinliğinin öğrenciler arasında üstbilişle desteklenen verimli etkileşimler doğurduğundan, farklı becerileri ortaya çıkardığından ve geride kalan öğrencilere destek sağladığından faydalı olabileceğini göstermektedir. Ayrıca bu çalışma, üstbiliş ve akran öğretimi/genel öğretim konularının bir arada incelendiği alan yazına da katkı sağlamaktadır.

## INTRODUCTION

Supervising the learning of each student in a crowded and mixed-ability classroom is a challenging task for teachers in mathematics lessons. Especially during mathematical problem solving tasks this issue may be further aggravated because of the widely-reported difficulties students have in mathematical problem solving (e.g. Mayer, 1998; Mevarech, Terkieltaub, Vinberger, & Nevet, 2010; Verschaffel, De Corte, Lasure, Vaerenbergh, Bogaerts, & Ratinckx, 1999). Students often need the help of the teacher to understand and solve mathematical problems (Mayer, 1998; Mevarech, 1999). On top of providing such support for the students, a teacher needs to follow the curriculum by ensuring the learning of all students in a limited lesson time. However, in a classroom with about thirty students and one teacher, it is not easy to orchestrate the learning of each student. This is particularly difficult since each student has different cognitive characteristics, abilities, and knowledge levels (Verschaffel et al., 1999).

In a mathematics education context, the responsibility of the teachers is twofold since they do not only teach mathematics content but also aim to develop higher order thinking of their students (Rigelman, 2007). Among such skills, being knowledgeable and aware of their cognitive functions and regulating own thinking processes, i.e. operating as independent learners, are some of the most highly emphasized (Mayer, 1998; Roebers, Cimeli, Röthlisberger, & Neuenschwander, 2012). While aiming to foster mathematical functioning of students, the dual emphasis on content and thinking skills makes teachers' job even more difficult, by necessitating them to support student development on both dimensions.

The first author of this paper, a middle school mathematics teacher, uses peer tutoring approach as a means for tackling some of the difficulties described above. Some students progress faster and the teacher can pick these students so that they can teach their friends in pairs or small groups. This is also expected to encourage students to take responsibility for thinking about and regulating their thoughts. When invited to engage in peer tutoring during her lessons, there are always students who volunteer to teach their classmates. During a regular peer tutoring activity in the classroom of the first author, students who are not successful in solving the problem have extra

time for studying it once more with their peers. While students are peer tutoring, the teacher circulates in the classroom and observes whether the peer groups study comfortably and effectively. An observation from the first author's experiences as a classroom teacher is that, during peer tutoring sessions students who are tutoring their peers take pleasure from this work. Positive effects of peer tutoring on students' attitudes towards subject matter have been previously reported by researchers (Cohen, Kulik, & Kulik, 1982; Tella, 2013). In the class where this particular study is conducted, the first author as classroom teacher also observed that there is interaction between peers through questions and answers rather than just transmission of knowledge during peer tutoring. Through such interactions tutors facilitate the learning of their peers. These interactions are expected to provide opportunities for awareness of and contemplating on the thinking involved (Holton & Clarke, 2006), particularly by the tutors. Yet, beyond such expectations, there needs to be formal investigations in order to learn whether students function in this way during peer tutoring sessions.

The main aim of this study is to investigate tutors' use of metacognitive knowledge and skills during peer tutoring process. This investigation stems from two important teaching considerations: building opportunities for students' exercising of higher order thinking and creating contexts encouraging students' interactions towards learning from peers. A review of literature, presented in the next section, pointed towards students' use of metacognition in a context of peer tutoring in mathematics education.

## Literature Review

### *Metacognition*

At the end of the seventies, Flavell's studies about meta-memory triggered empirical research about metacognition (Desoete & Veenman, 2006). The knowledge about the working of memory, its challenges and strategies is studied under the concept of meta-memory by Flavell and his colleagues (Schneider, 2010). Beside the studies on memory, Flavell (1979) also paid attention to the role of "cognition about cognitive phenomena" conceptualized as metacognition, for various learning tasks (p. 906). Metacognition is defined as the knowledge and regulation of one's cognitive endeavor (Flavell, 1978). It refers to a person's control over her thinking and learning, especially when she has some problems during problem solving and information processing (Desoete & Özsoy, 2009).

Baker and Brown (1984) focused on two components of metacognition: "knowledge of cognition" and "regulation of cognition" (p. 353). According to Schraw (2001) knowledge of cognition refers to the awareness of a person about her own cognition or about cognition in general while regulation of cognition points to a group of activities enabling the individuals to control their learnings. Flavell (1979) explains metacognitive knowledge as awareness of individuals about the three factors *"person, task and strategy*", which are related with each other and affect the cognitive process. *Person* knowledge includes knowledge and beliefs about oneself and also about other individuals as cognitive actors. *Task* knowledge embodies all the knowledge about how one manages the task, which strategies exist during a cognitive enterprise and the characteristics of the cognitive task. Lastly, the *strategy* factor includes the knowledge about why and when to use a particular cognitive action to achieve the goals in a cognitive process.

These three factors, person, task, and strategy, of metacognitive knowledge indicated by Flavell appear in the literature in three subcategories like *declarative knowledge, procedural knowledge,* and *conditional knowledge* respectively. Declarative knowledge indicates knowing "about" things. Procedural knowledge indicates awareness of "how" to do them. Conditional knowledge indicates knowing the "why" and "when" parts of cognition (Schraw & Moshman, 1995).

Regulation is the second component of metacognition comprising of a group of cognitive activities. These activities in self-regulatory mechanism contain controlling the outcome of any cognitive enterprise, planning the following steps, monitoring the current performance regarding its effectiveness, assessing, revising, and evaluating the cognitive process (Baker & Brown, 1980). Metacognitive regulation is considered in three groups of cognitive activities in general: planning, monitoring and evaluating. *Planning* refers to deciding on the appropriate strategy and arranging the resources of individual; *monitoring* or regulating refers to awareness of individual about the course

of events during the performance; *evaluating* refers to assessing the outcomes and the process of a cognitive endeavor (Schraw & Moshman, 1995).

In the last decades, various studies documented the influence of metacognitive processes on student learning and performance (Schneider & Artelt, 2010). Narang and Saini (2013) stated that metacognition has an important role on students' getting better results on a variety of academic tasks. In a mathematics education context, various studies claimed that individuals with higher metacognitive levels perform better on mathematical problem-solving tasks (Artzt & Armour-Thomas, 1992; Pennequin, Sorel, & Mainguy, 2010). Students having the knowledge about their strengths and weaknesses and various strategies to use in new tasks can regulate the process cognitively and master the learning easily (Pintrich, 2002). Swanson (1990) found that the students with high metacognitive skills, regardless of aptitude, relied more on deductive reasoning and evaluation strategies and showed higher performance on mathematical problem solving. It is supposed that "thinking about their thinking" process supports the awareness level of the students about their thinking and this develops the thinking and learning (King, 2002, p.38). Recently, there is also an increased attention on instruction that is supporting students' metacognition in mathematics classrooms to get better results on mathematical problem solving in particular and learning in general (Kramarski, 2004; Mevarech & Fridkin, 2006). All these can be accepted as indicators of the importance of metacognitive processes in mathematics classrooms comprising cognitive processes, problem solving and learning.

### *Peer Tutoring*

According to Vygotsky's concept of the zone of proximal development, social interaction and collaboration with peers facilitate learning, which is stimulating children's mental development (Vygotsky, 1978). One type of peer interaction in learning environment is called peer-assisted learning. Peer-assisted learning (PAL) is a process in which matched pairs having equal status help and support for the learning of other regarding their acquisition of knowledge and skills (Topping & Ehly, 1998). So, the pairs interact with each other in this social learning process with the purpose of teaching or learning. Peer tutoring is just one method of PAL, in which one student takes on a task as tutor/teaching and the other as tutee/learner (Topping & Ehly, 1998). In peer tutoring processes, the tutors demonstrate various skills such as giving feedback, making explanations, scaffolding, and correcting the mistakes together with the tutee managing timing and giving the tutee a chance for attending the process actively (Chi, Siler, Jeong, Yamauchi, & Hausmann, 2001). The tutor is, in general, more advanced, and knowledgeable than the tutee, yet in some peer tutoring conditions this difference in expertise is not very big (Roscoe & Chi, 2007).

Peer tutoring approach is applied by educators for various gains in learning environments. Sharpley and Sharpley (1981) reported the result of a meta-analysis about the effectiveness of peer tutoring by stating that peer tutoring provides some cognitive gains for both tutee and tutors. Among such gains, metacognitive and self-regulatory behaviors were reported to be supported and facilitated in peer-assisted learning environments of children, such as problem solving and learning group activities and peer tutoring activities without adult intervention (Whitebread, Bingham, Grau, Pino Pasternak, & Sangster, 2007). Besides, Holton and Clarke (2006) remarked that peers make reciprocal scaffolding to each other in group work which is a similar activity to metacognitive questioning taking place as the inner dialogue of the learner.

Considering the difficulties students have while solving mathematical problems as emphasized at the beginning of this paper, peer tutoring activities can be used as a method to tackle this issue. Besides, the significant contribution of metacognitive processes to mathematical problem solving and learning is highlighted in research findings. In a mathematics education context, the need of developing metacognitive skills for problem solving and contribution of peer tutoring activities on supporting all students' mathematical functioning directed us to examine these two factors together in the same learning process. Cooperation, questioning and meaningful interaction among students during peer tutoring and self-questioning and inner dialog cited in metacognitive processes demonstrate the similarities in these processes and make it a worthwhile effort to study these together. The aim in this study is to uncover how the tutors use metacognitive skills in peer tutoring activities and what the contributions of metacognitive processes are in peer tutoring while solving

mathematical problems. For this aim, mathematical problem-solving tasks are chosen as a suitable means for observing and investigating the metacognitive processes and peer tutoring activities thoroughly. The specific objective of this study is to examine metacognitive knowledge and skills of the students who are volunteering for teaching their peers during peer tutoring sessions in mathematics classes. Hence, an answer is sought for the following research question:

- Which metacognitive knowledge and skills are used by volunteer students who choose to teach mathematical problems to their peers in peer tutoring sessions (before, during and after the task)?

## METHODOLOGY

### Research Design

A four-week case study was conducted for examining the metacognitive knowledge and skills of four students who volunteered to teach mathematical problems to their peers in peer tutoring sessions. Six peer tutoring sessions were observed without interfering with students' work. After the tutoring sessions, interviews were made with four tutor students.

### Sample

In this study, participants were seventh grade students. Convenience sampling approach, i.e. "working with volunteers or existing groups because of their availability" (Gay, Mills, & Airasian, 2009, p. 134), was adopted by choosing the participants from one of the classrooms of the first author. This sampling approach is deemed suitable since the main aim of the study is exploring a construct and representativeness of the participants is not a major issue. Pair groups were formed consisting of one tutor who is teaching; and one tutee as a learner. An active teaching process, which a tutor can manage by herself was encouraged for efficient data collection. So, volunteer students believing in themselves to teach their peers were purposefully selected as tutors. Among sixty seventh graders of the teacher, there were five students who volunteered to be a tutor and four of them attended the study due to time constraints of one of the volunteers. Six volunteer students participated in the study as tutees. The students did not have any specific training regarding use of metacognitive knowledge and skills.

### Data Collection Process and Materials

Before starting the sessions, mathematical problems (see Appendix A) were prepared and given to tutors so that they can get ready for teaching beforehand. These problems were related to topics in the 7$^{th}$ grade mathematics curriculum. In each session, the tutor taught three problems to her peer. After each session, the set of problems used was reviewed. Due to revisions and additions of problems, the pool of problems enlarged during the study and the set of problems in the sessions varied.

In the study, six peer tutoring sessions were conducted. One peer tutoring session is considered sufficient for observing each tutor while teaching. For each tutor, one problem session was arranged after the school in which the tutor taught the prepared problems to a peer. However, since there were six volunteering tutees, one of four tutors who accepted to do the extra sessions, worked with two more tutees. Each session was observed and video recorded in different days after school. The time interval of teaching in each session ranged from five to ten minutes. After each peer tutoring session, videos of the sessions were watched with the tutors. Stimulated recall interviews including six general questions were conducted. Besides, some particular questions relating to the aim of the study and content of the work in the peer tutoring sessions were addressed while watching the videos. Stimulated recall interview was chosen as the data collection technique since it helps researchers delve into the thinking underlying participants' actions, strategies particularly in situations involving interactions (Dempsey, 2010). So, the combined use of observations while working on the task and interviewing the tutors after the task enabled a thorough investigation of metacognition. Each interview lasted 10 to 20 minutes excluding the time spent for watching the videos. In the interviews, the aim was to bring out students' explanations for their thinking and strategies, their performance and their awareness as related with metacognition. Video recordings of the peer tutoring sessions

constituted secondary data for the study as they were used for stimulated recall interviews with the tutors.

### Data Analysis

After collection of data, transcribed interviews were coded according to the metacognitive components active during tutors' functioning in the peer tutoring sessions. The codes were constructed according to commonly accepted conceptualization of metacognition, as presented in the literature review (Schraw & Moshman, 1995). This is in accordance with using preexisting codes within qualitative data analysis (Yıldırım & Şimşek, 2008). The main components of metacognition used in coding were *metacognitive knowledge and metacognitive regulation*. For each of these, there were three subcategories used in the analysis:

Metacognitive knowledge

- Declarative Knowledge/ Knowledge about the self
- Procedural Knowledge/Knowledge about the task
- Conditional Knowledge/When and why to use the particular strategy
- Metacognitive Regulation
- Planning
- Monitoring/regulation
- Evaluating

Data from the videos were not coded within the analysis, since they did not provide explicit manifestations of tutor's metacognition. They were used in supporting tutors' explanations of their experiences during interviews. Besides they helped us create and present accounts of tutors' thinking.

During the coding of the interview data, some units of analysis were created as well. The teaching task was divided into three parts as Before the Task, During the Task and After the Task. Before the Task part includes the first reactions of tutors to the task and planning/preparation processes upon seeing the problem before implementation with the tutee; During the Task refers to the process through which the tutor teaches the problem and After the Task part includes the evaluations and reflections of the tutors after completing the teaching sessions. These parts enabled the researchers to make the analysis in a systematic way. Each part was analyzed by mainly focusing on particular questions of interviews related with that particular part. Some of the questions and the corresponding responses referred to more than one particular part. Some overarching themes appeared as a result of the analysis, which are presented as subheadings in the results section. Trustworthiness of the study, i.e. data analysis accurately focusing on the phenomenon under investigation (Gay, Mills, & Airasian, 2009), was established by presenting a thorough conceptualization of the phenomenon under study, use of multiple methods to collect data (triangulation), provision of detailed accounts of critical incidents and the context, and having an expert educational researcher other than the authors examine the data analysis. Interview questions were first prepared by three students enrolled in a graduate course in education and then were reviewed and finalized by two expert researchers. Inter rater reliability for the two researchers' coding of the data was above 80% and the differences between the two researchers' codes were discussed and resolved.

## RESULTS

In this study, the aim was to investigate metacognitive knowledge and skills of the tutor students teaching their peers while solving mathematical problems in peer tutoring sessions. Answer for the research question of the study was sought by analyzing the data and presenting the findings according to 3 phases reflected in the question: before, during and after the task.

### Before the Task

*Being a volunteer for teaching the peer*: The first question in the interview was asking the tutors why they became a volunteer for teaching. From the answers, it was seen that they liked teaching in general. Three of them also stated that this was about the self-confidence they have. So, they also believed that they could be successful on this task. Two of them mentioned that they already used

teaching as a strategy while studying at home by acting as if they taught somebody. Because of this, they had self-confidence about teaching. One of them said that she learnt something when she talked to herself. Behaving as if teaching somebody and talking aloud to themselves can be seen similar strategies in learning. Some extracts regarding this issue from the interviews with students are as follows:

> ***Zeynep:*** *Yes, I am always sure that I can do it. So, as I said I have a pretty high level of self-confidence… Maybe I am not very talented on teaching but I thought that I can at the end do this task if I study over it and try a bit…*
> ***Researcher:*** *What kind of trying?*
> ***Zeynep:*** *I mean if I try to teach somebody at home, if I search for effective teaching strategies I think I can do it well, because of this I became a volunteer.*
> ***Yağmur:*** *I always study by teaching myself at home. Because of this characteristic of Zeynep, at the same time I can understand very well what I try to teach myself. Since I have a teaching ability, I became volunteer … This task can require also some self-confidence.*
> ***Sinem:*** *I always talk to myself, while doing this I learn something and I discover some teaching ways…. Maybe I have a great deal of self-confidence. It can be about that.*

Here, these students seem to know about themselves and their capabilities to achieve a task. They refer to their past experiences as well, such as teaching at home. This shows their declarative knowledge as knowledge about oneself. Just one student (Melek) put forward a different reason to become a volunteer. She said she loved to study and studying together with a friend could help her learning as well. So, she appraised this task as a learning environment for herself rather than considering her teaching abilities and capabilities. However, this consideration can still be regarded as this particular student's knowledge and awareness about herself.

*Planning/Preparation:* Students were asked how they planned and prepared for the task beforehand. The main aim was to hear their cognitive strategies planned before the task. All of them stated that they made some preparations for the task. From the answers, it was seen that just one student, Melek, tried to understand the problem well rather than thinking over the teaching method during the preparation. However, the other three made some preparations for discovering effective teaching strategies. For example, two of them expressed that they practiced the process by teaching similar problems to their siblings at home. After this experience, they had a dialogue with their siblings or thought on their own about how their teaching could be improved:

> ***Zeynep:*** *I made some practice about teaching*
> ***Researcher:*** *What kind of practice?*
> ***Zeynep:*** *For example, I have a small board at home, I taught my brother by using it and my toys. I asked some questions like "did you understand Hakan?" What could be better to do? …*
> ***Sinem:*** *Yes, I made a preparation. I taught to my brother and he listened to me. Then he said, "do like this and like that", for example being more explicative.*

The extracts above, indicated tutors' planning towards the cognitive processing necessary for teaching their peers, hence their metacognitive functioning. One tutor (Zeynep) stated that she contemplated how she could explain problems in order to make them more understandable. This was also an indicator of strategic planning. Although she did not mention practicing teaching at home, she had stated previously that she usually used teaching as a strategy to learn something. So, she knew her effective ways for preparing for a task.

The explanation of students about the preparation process showed that the students used metacognitive regulation skills while planning and preparing for the task given to them. While doing this they reviewed what they needed to do for an effective teaching. In this process, they also looked at and appraised the task. Thinking over the process, referring to other people's opinions to have a successful result, making preparation and deciding on some strategies before the task were various indicators of metacognitive regulation skills. These skills were also related with task and strategy knowledge as stated by Flavell (1979). Metacognitive knowledge informed tutors' metacognitive regulation through considerations about the self, the task and strategies. Put differently, metacognitive knowledge and regulation skills worked together through cognitive transactions.

**During the Task**

*Starting Approach:* At the beginning of each teaching session, it was observed that all tutors started by reading the problem. Two of them also explained the problems in their own words. During the interviews, when reasons for their choices of methods were asked, tutors gave the following explanations:

> ***Sinem:*** *I explained it in stages. Firstly, I explained the problem in order to understand it. Then I tried to explain certain things for the solution, about how to solve it. Lastly, it was more about doing the actual solution; we needed to start the solution…. Before starting the solution, I tell firstly what we should do, what we need to find at the end in order not to forget what we should focus on while solving the problem.*
> ***Melek:*** *I firstly read the problem again in order to understand it well, we cannot solve the problem without understanding it. So, I read it piece by piece.*
> ***Zeynep:*** *I narrated the problem by turning it into a story because I wanted my friend to adapt to the problem.*

Here it was seen that the students were aware of why they did something in a particular way during the teaching session. Reading the problem, turning it into a story or summarizing, all served a purpose according to the tutors. From the explanations, it was understood that the choices of tutors were planned actions. They knew why and when to use these strategies. It indicated their conditional knowledge because they knew why a particular strategy was effective for a particular task. They also activated metacognitive regulation skills through planning the strategy they used. Once again, there is evidence for interaction of metacognitive knowledge and regulation within peer tutoring experiences of students.

*Use of Generic Strategies:* The students used various strategies while teaching. One of the most frequently observed strategies was asking questions to tutees. The questions were scattered over the teaching process. While they were explaining the solution, they asked some questions such as "how can we find this?" and "what do you think?" While the questions varied according to its timing during the work on the problem and its content, the reasons for using it as a strategy showed similarities. The following comments were made when tutors were invited to express why they asked questions to the tutees during the teaching process:

> ***Sinem:*** *Because if I only do something…I am already writing, at least he should participate...*
> ***Researcher:*** *Why is his participation important?*
> ***Sinem:*** *What can he (the tutee) understand without participating, by just watching me? I think it is important to participate.*
> ***Melek:*** *...Maybe I could explain him directly but if he does it by himself, he can come up with a new and easy way to find the solution…I already know these, he (the tutee) should learn them, I just provide him to find the answer by asking some questions.*
> ***Yağmur:*** *…So I wanted Emre to think and find the solution himself…You (the teacher researcher) create discussions in the classroom because it becomes more effective and so I want Emre to think and find the answer by himself. Maybe he can find another method while he is thinking…*

From the answers, it can be understood that tutors' strategy of asking questions was often a planned action for varying reasons such as attracting the attention of their peers and trying to create an effective learning environment by providing participation. Their explanations indicated tutors' both procedural and conditional knowledge together with their planning action as a regulation skill. Their awareness of which strategy should be used, when and why, helped them to proceed in a planned way and they asked questions to manage the task by providing the participation of their peers.

There were some other questions frequently asked by tutors during the teaching that could be grouped around a common theme. Rather than attracting their attention, these were mainly about tutors' monitoring skills because they aimed to notice whether their friends understood the solution or whether they had the prerequisite knowledge. Tutors asked various questions regarding their peers' prior knowledge (e.g. "what is a natural number?", "what is the area formula of a rectangle?"),

and questions calling for tutees' explanation of their solution ideas, such as "why do we make a division here?" During the interviews with the tutors, they were asked why they used these questions. They explained the reasons behind the questions they asked during the task. When Zeynep was asked about an incident in her peer tutoring where she stopped the explanation for one of the problems in order to question her tutee, she gave the following explanation:

> ***Zeynep:*** *I asked that question because I wanted to check whether she (the tutee) was on track and whether she was listening to me…I asked because I thought she can understand better this way. If I could not take the answers, I would explain it myself…So, I could notice whether she understood or not.*

Zeynep's comments clearly indicated her efforts to watch the progress of her tutee and considerations about how she would proceed according to the progress. This can be considered as a monitoring process for regulation of the peer tutoring experience. Similarly, in her interview, Yağmur's remarks about the motive for her questioning the knowledge of the tutee about area and perimeter for a question on possible values of area of a rectangle, displayed similarities with Zeynep's:

> ***Yağmur:*** *…Because, the problem said the area is such and such $cm^2$, I want her to know what area is. Maybe she is confused with area and circumference.*
> ***Researcher:*** *So, did you check her prerequisite knowledge?*
> ***Yağmur:*** *Yes, because she could not solve the problem without knowing them.*

The questions which the tutors asked during the task and their explanations about why they asked them, had manifestations of their monitoring skills. Extracts such as those explained above, indicated tutors' awareness of the cognitive progress and also their effort to raise their awareness during the task. They regulated their teaching process when required. According to the answers which they received from the tutees, they steered the teaching by continuing with the current strategy or revising it.

While the students were teaching they also used some representations. For example, in a problem requiring a comparison between two sellers (problem 1), all tutors put the information in the problem on paper. They wrote "first seller" and "second seller" side by side in two columns and listed the given information these columns. They explained their objective for this representation as showing the difference more clearly and enabling the comparison between two sellers. Moreover, they drew some visuals at the beginning of some problems. In a problem about the area of a rectangle (problem 6) Zeynep firstly drew a rectangle on the paper and in a problem about the number of gears in two gearwheels (problem 7) Yağmur depicted the gearwheels while explaining the problem. For the visual shapes which they drew in those problems, they also revealed in the interviews that it was a planned strategy as follows:

> ***Zeynep:*** *Yes, I planned this. Because, I thought that if there is any shape at the beginning, the person can understand well. If I try to tell the lengths of the opposite sides of the rectangle without drawing any rectangle, he might not understand much. So, I tried the method of drawing a shape in the beginning then I made the operations…*
> ***Yağmur:*** *By use of the visuals I actually did what you (the first author) do in the lessons…. Maybe he understands well through these shapes. In the problem, he may not understand well by looking only at the numbers and words.*

The explanations revealed that they thought the visuals were effective for the learner to understand the problems. The tutors' consideration regarding the use of this strategy pointed to their conditional knowledge. They had an idea about what was required in these particular questions for facilitating the learning and teaching environment. Their metacognitive knowledge also affected their preparation and led them to apply the planned strategies. They used some strategies purposefully and aimed to create an effective learning environment by this way.

*About the Tutee:* When the tutors were asked about their teaching methods, they talked about the characteristics of their friends as a tutee. As a reason for their varying scope of explanations on some steps of the problems, they asserted that the explanation was enough for the particular tutee

to understand. They mentioned the perceived achievement level and capacity of their friends according to their experiences as classmates, as in the following excerpt:

> ***Yağmur:*** *Tuğba (the tutee) has understood the similar problem very well in the class… So, I thought she could understand well if I continue through that question.*

In the example of Yağmur, it appeared that the tutor's experience with and observation of classmates during class time influenced her decision of the teaching method. Besides this, the tutors commonly asserted that their explanation was enough because the particular tutee was "a hardworking person and could understand these questions easily". These kinds of reflections appeared also when they were asked what they would do if the tutee was changed. They expressed that they would change the strategy according to who the tutee was:

> ***Zeynep:*** *If there was somebody who was not as good as Ceren at understanding the problem, then I could make more and more explanations about why I did some operations, I would explain all of it.*
> ***Yağmur:*** *I could change my teaching method because every person has a different capacity to learn. For example, Tuğba learns everything easily. But if I tried to teach this topic to Veysel, it would take about one hour rather than ten minutes. (Note: Veysel is a classmate who did not participate in the study.)*

The consideration of Yağmur and Zeynep was about the characteristics of the tutees and their selection of strategies regarding those characteristics. By this way, they decided how much initiative they should take in explaining the solutions to the tutors.

These kinds of considerations indicated metacognitive knowledge about person. However, this corresponds to a specific subcategory, knowledge about others rather than self, according to Flavell's (1979) conceptualization. What the tutors were aware of was the learning characteristics of their friends as tutees. It was seen that while making decisions about the progress of their peer tutoring, they checked their knowledge about the capacity and features of their friends. Thus, it can be claimed that they decided on the strategies and progress by checking their knowledge about their friends. This way of working once again pointed to dynamic and complex metacognitive functioning, at the intersection of metacognitive knowledge and metacognitive regulation. In terms of metacognitive knowledge, they both knew the characteristics of others and why and when to use a given strategy indicating declarative and conditional knowledge respectively. As for metacognitive regulation, some sort of monitoring of process was observed as checking whether the strategies were effective, and made sense for the tutee. The knowledge component informed the regulation process while the regulation served through potential updates for the knowledge base.

**After the Task**

*Taken Decisions:* One of the metacognitive skills under study was tutors' evaluation of various issues like their own performance, the effectiveness of the task and the strategy that they used. After the teaching sessions, the tutors were asked what they would change if they were asked to do the same task again. The aim of this question was to examine the extent to which tutors evaluated the effectiveness of their peer tutoring after working on a task. According to the responses, only one of the tutors (Melek) said she would not change anything; she would do the tutoring in the same way. However, the other tutors pointed to some particular issues they would change in the next teaching session.

> ***Zeynep:*** *I would make more explanations. It would be better if I explained well why I made such operations, especially in the first operation here… Also, I wish I used other strategies on this question, one easier method for example.*
> ***Yağmur:*** *There can be confusion in this part (problem 6), I could ask that problem differently. It could be more understandable. I would be careful about this issue if I remember it when I teach again… At the beginning, I made a mistake here in an operation...I could check the operations again and again (Problem 3) … I could draw some visual representations... I could have taught more effective then (problem 4)*
> ***Sinem:*** *I would ask more questions to my friend by considering his understanding*

Such remarks from the tutors were considered as manifestations of evaluation skills. Through monitoring and evaluations, they were aware of their performance and its effectiveness. However, their considerations focused more on the deficiencies and mistakes in performing the task. During the interviews, the tutors were asked to evaluate their performances by focusing on all characteristics, not only the deficiencies. The interviewing researcher's questions focused on what they learned from the whole task. Responses from two students had similarities regarding their consideration of their preparations for the task. Melek and Zeynep evaluated their performance on various problems as insufficient and they attributed it to their lack of preparation for the task or lack of understanding of the problem as extracted below.

> ***Melek:*** *I learned that if I do not understand well what I teach, then my friends cannot understand too. So, firstly I need to understand what I teach.*
> ***Zeynep:*** *Actually, I learned through these two questions that we should not do a task without being prepared before. Because, I thought I was not as effective as I was in the previous question.*

Certain responses from the tutors also centered on improving the execution of the task through learning a new strategy for solving particular problems or developing some teaching skills. As a metacognitive act, the tutors could assess and face with their "misconceptions" and "knowledge gaps" while they are trying to make precise explanations of the concepts and materials to the tutees during peer tutoring (Roscoe & Chi, 2007). For example, Zeynep asserted that she noticed she was not good at some problems requiring estimation. She explained that she was used to solving problems asking precise answers and these kinds of problems with estimation made her worried:

> ***Zeynep:*** *…here, there was not a definite answer required, so I felt worried... because it was asking for an estimation without a precision. And I am better in the problems requiring precise answers. So, I said that I need to study more over the problems with estimation anymore.*

She thought that she was not effective in teaching the problems with estimation too. As a personal gain, she deduced that she should study more on this kind of problems. Many evaluative comments focused on tutors' developing awareness about their own capabilities and coming up with certain techniques that they tried and used during the task:

> ***Yağmur:*** *While I was teaching, I understood the problem well…When I was teaching Emre, I discovered this strategy.*
> ***Researcher:*** *So, you used it while teaching Tuğba?*
> ***Yağmur:*** *Yes.*
> ***Sinem:*** *It seems like my explanation skill is developed while teaching because while I am teaching somebody I think over my mistakes and I turn back to correct them and try to do it better.*

Such comments indicated that the tutors used both their monitoring skills during the task and also evaluation as metacognitive skills. Often, monitoring and awareness helped tutors to engage in evaluation of the task at a broader level with overarching considerations about improvements for the task and own learning.

## DISCUSSION AND CONCLUSION

Commonalities in peer tutoring and metacognitive functioning such as questioning, strategy selection and planning, led us to the idea that there could be significant co-occurrences of the two. Hence, two distinct research areas in mathematics education, peer tutoring and metacognition, were brought together in this study. The aim was to investigate metacognitive knowledge and skills of the tutor students teaching their peers while solving mathematical problems in peer tutoring sessions. For this objective, the data collected from six peer tutoring sessions and interviews with tutor students were analyzed and the results were presented according to three phases as before, during and after the task.

According to the results, it is observed that the volunteer students displayed metacognitive skills during peer tutoring tasks and evidence was found in the interview notes. During the sessions, the tutors provided some interaction, though limited, through asking some questions in order to monitor and evaluate the teaching process. Use of questions, facilitated towards this aim, served as a

metacognitive process because asking questions as a strategy helped the students to monitor and regulate their cognitive processing and understanding, and to construct new knowledge (King, 2002). Roscoe (2014) found that the interaction between the tutor and the tutee through the questions directed to the tutee supported the tutor's self-monitoring. In the current study, the tutors showed their monitoring skills, regulation capabilities, their awareness of and control on the process by purposeful questions and interacting with their peers. Use of this strategy provided evidence for metacognitive regulation of volunteer tutor students.

An important variable that appeared to be interacting with tutors' metacognitive functioning was self-efficacy, described by Bandura (1982) as the beliefs of the people in their abilities about how well they can perform during a particular progress. Previous research shows that the students with higher self-efficacy level use also effective metacognitive strategies (Coutinho, 2008); and self-efficacy is found as a predictor of both declarative and procedural knowledge (Moores, Chang & Smith, 2006). Three of the four tutors consistently expressed their self-confidence and positive beliefs about their capability to do peer tutoring successfully. This can be considered as their self-efficacy towards this particular task. Such beliefs often interact with students' knowledge and awareness about themselves and monitoring and evaluations from similar past experiences. While self-efficacy might have a determining role in behavioral decisions during such cognitive tasks, student metacognition and the consequent cognitive processes might interact with further self-efficacy considerations, e.g. self-efficacy updates through appraisals and decisions following the outcomes of experiences. Further investigation of the interaction between self-efficacy and metacognition in a context of peer tutoring can prove to be paramount.

All tutors displayed, to some extent, engagement with metacognitive knowledge and regulation skills. In addition to this, they occasionally differed in their metacognitive processing for the particular subjects of investigation. It came to light with the differing perspectives coming from a particular tutor while the others shared common attitudes in some occasions. This can be related with the tutors' different levels of cognitive awareness. If there is a significant level difference on metacognitive knowledge and skills among the individuals, this needs to be documented in more detail through various methods for assessing metacognition. Yet, the particular situation in this study can be an example of how the students can differ according to their levels of metacognitive processing. Besides, investigation of tutors' metacognitive functioning might also inform researchers about potential links between metacognition and tutoring preferences (e.g. whether students beyond certain levels of metacognitive functioning volunteer for tutoring).

Throughout the tasks and comments in the interviews, manifestation of metacognition could often be related with more than one metacognitive component. Although the statements were categorized according to temporal sequence and the sub dimensions of metacognition, metacognitive knowledge and skills are seen as overlapping with each other. For example, metacognitive knowledge level of students affects their metacognitive strategy use. Planning and monitoring of use of such strategies are also interrelated. As Whitebread (1999) asserted "Metacognition itself, of course, is not a unitary process, but contains many aspects and elements, each of which contribute to cognitive functioning and development in different ways, and a number of which may themselves interact with each other" (p. 490). Thus, separation of metacognitive skills and trying to investigate them in isolation is a futile endeavor due to interrelations among different components. What can be more fruitful is to consider these knowledge and skills individually, yet in a context of interactions with others.

From a practice point of view, this investigation can be helpful for mathematics teachers. They can design peer tutoring applications in their classrooms taking the findings from this investigation into consideration. While making decisions for peer tutoring sessions, findings from research studies can inform their decisions on who will be the tutor and what kind of knowledge and skills need to be promoted to support peer tutoring. This study created opportunities to see how interactive teaching activities among students can activate metacognitive characteristics of the students and how the students having cognitive awareness and using metacognitive skills can contribute to the quality of teaching environment in the classroom. It is also contemplated how metacognition has an active role on these kinds of teaching activities of students. Throughout this study, we were pleased to observe, as educators, metacognitive knowledge, and skills of the students

not only for their own problem-solving activities but also for interactive studies with their peers and teaching experience.

Besides implications for teachers, this study sheds light on a fruitful research area: intersection of peer tutoring and metacognition. Studies on metacognition in peer tutoring activities can be linked with a broad range of studies on metacognition for teachers and teaching. This would contribute to understanding how metacognition as a construct influences different stakeholders in education. One way of discovering what the students know about their cognition is proposed by Garner and Alexander (1989) as asking the students to teach a younger one a sound solution for a problem. This links metacognition with peer tutoring and teaching activities naturally. Hence much work needs to be done on metacognition within a context of peer tutoring, also linking it with students' interaction and actual performance of teaching.

Four students may not be enough to draw general conclusions. The small size of sample in this study is one limitation in this sense and this small-scale study should be considered as an effort to make sense of dynamics of a complex phenomenon. Besides, having a teacher researcher as the data collector in touch with the students might have both pros and cons. Studying with your own students might have a risk regarding the trustworthiness of the results, since some students might try to please their teacher. Some researchers claim that in such cases, students may behave differently or utter what you would like to hear while conducting the interviews and making observations (Blakemore, 2012). Yet, more influentially, having a strong relationship with the students and being knowledgeable about their achievement levels, preferences and habits helped the first author of this study to engage in detailed conversations with the participants and draw out rich data. Teachers engaging in research with their students can contribute to unpacking the intricacies of metacognition in classroom settings.

In future studies, it can be meaningful to have larger samples to examine the metacognitive skills and knowledge of tutor students. Working with a larger number of tutors and tutees with various cognitive abilities and knowledge levels can reveal different metacognitive processes during peer tutoring. Further studies could extend the scope of investigation by examining the effect of tutors' metacognitive skills on tutees' learning as well. Besides, researchers can explore different methods for effective implementation of peer tutoring supported by metacognitive skills of tutors in a mathematics classroom.

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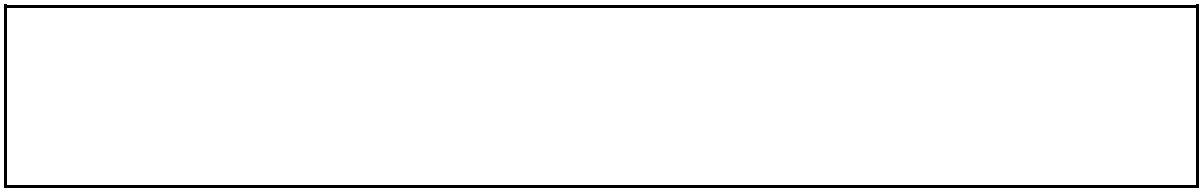
**APPENDIX A: The Problems Used During Peer Tutoring Sessions**

**1.** Mehmet needs to buy 6 lemons from the market. The first seller offers 2 lemons for 75 kuruş and the second seller is selling 4 lemons for 1 lira. Buying from which seller would be a more reasonable move for Mehmet? Why?

**2.** Let's assume you forgot what the result is for 8 x 6 but you know 5 x 6 = 30. How can you find the result for 8 x 6 by using this knowledge?

**3.** You have a garden and you want to produce peaches in 50%, beans in 25%, corn in 15% and carrots in 10% of the garden. If the given shape shows the garden, segment the garden into four parts according to the given information.



**4.** In a classroom, the ratio of number of girls to number of boys is $\frac{4}{7}$. Which of the following cannot be the number of students in the class?

A) 57 B) 55 C) 33 D) 22

**5.** A worker finishes $\frac{2}{7}$th of a duty in 8 hours. How long would it take the worker to finish the whole duty?

**6.** Perimeter of a rectangle is 48 cm. The lengths of short and long sides of the rectangle are both natural numbers.

- What can be the side lengths of the rectangle?

- What are the greatest and smallest possible values for the area of this rectangle? Explain how you reach your answer.

**7.** Two cogwheels connected to each other have 65 teeth in total. When one of the wheels makes 5 turns, the other makes 8 turns. How many teeth does the big cogwheel have?

**8.** A pair of shoes is sold for 220 liras with a 10% profit. If it is sold for 180 liras, what much loss would there be in %?

**9.** When a bike wheel, having a radius of 28 cm, makes 20 turns how far would the bike go? (take π as 3)