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Misconceptions of Mathematics and Its Relationship to The Learning Pleasure among Elementary School Students in Saudi Arabia

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Abstract. This study aims to identify misconceptions of math concepts and reveal their relationship to math learning pleasure among elementary students. Content of math books was analyzed from the first to the fourth elementary grades, and a test was prepared on the basic concepts of mathematics taught in those grades (Numbers and operatios on numbers, and fractions and operations on fractions). Moreover, a questionnaire of math learning pleasure was designed, and the study tools were administered to (120) elementary school fifth-graders in Wadi Al-Dawasir Governorate. The data was analyzed using SPSS V20. The results indicated that there were some math misconceptions among elementary school students, with regard to estimation, the numerical phrase, the numerator and denominator, and adding two fractions of different denominators. This study attributes many of the misconceptions to the weak Arabic language skills of many students, which is evident from the students' answers. The study findings also revealed that the level of math learning pleasure among the participating students was generally average. The study revealed a weak correlation of 0.2, which is not significant at 0.05, between the math misconceptions and the pleasure of math learning. The study also revealed that there were significant differences at 0.05 between males and females with regard to math misconceptions, and the study concluded with a set of recommendations and suggestions for future study.

Keywords: Misconceptions, Mathematics concepts, Learning pleasure, Elementary level

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INTRODUCTION

Scientific and technological advancements in all facets of life have revolutionized the teaching and learning of mathematics. There have been several efforts and attempts to develop mathematics curricula in the Kingdom of Saudi Arabia and to keep pace with rising needs and requirements of everyday life. However, recent study conducted to measure elementary and middle school students' actual levels of achievement in mathematics reports students' low performance in solving mathematical problems as shown, for example, in the results of the TIMSS International Competition for years 2007, 2011 and 2015 (Alshamry, 2017; Thomson, Wernert, O'Grady, & Rodrigues, 2017). These problems (Li & Li, 2008; Mohyuddin & Khalil, 2012) may result from the student's poor solution strategies or poor memory of algorithms. They may also be a result of lacking in conceptual understanding, which is also called "misconception". These misconceptions Jordan, Glutting and Ramineni (2010) contend, may result from poor numerical sense of both students and their teachers.

Study asserts that some students have many misconceptions of mathematics as they progress from studying mathematical operations to studying algebra (Ay, 2017; Gourgey, 1984; Kiili, Moeller, & Ninaus, 2018). Furthermore, several studies (e.g. Burgos & Godino, 2020; Drews, Dudgeon, Hansen, Lawton, & Surtees, 2005; Moss, 2005) confirm that middle and high school students may generalize their conceptions of whole number properties and operations when learning rational numbers. Further yet, there were many math misconceptions among some pre-service mathematics teachers (Cincinatus & Sheffet, 2016). Lee and Boyadzhiev (2020), on their part, conducted a study for identifying misconceptions about fractions among

22 college students. After analyzing the participants' answers to 41 problems solving questions about fractions, it was found that the most common misconceptions were related to a lack of understanding of the basic definition of fractions. The problem with these misconceptions is that they could ultimately lead to impeding students' performance and future learning of mathematics, and may even significantly affect their attitudes towards mathematics and the pleasure of learning (Ay, 2017).

According to Barton (2018), a valid method of detecting misconceptions is needed to come up with realistic results that can be used in further study. To identify those misconceptions (of numbers and pertinent operations, fractions and pertinent operations) for the purposes of the current study, a test was prepared and a sample of fifth graders was selected. An in-depth analysis and monitoring of error patterns were used in tackling the sampled students' responses to the test. To conclude, early detection of misconceptions in mathematics contributes to identifying the best methods and approaches to teaching mathematics.

LITERATURE REVIEW

Concepts are the basic building blocks of mathematical construction. That is, all mathematical postulates, principles, generalizations and skills depend on concepts, the basis on which learners build their new knowledge. Therefore, more care must be given to building concepts that have real meaning and significance for the students in teaching mathematics across the curriculum. Moreover, concepts need to be connected to learners' background knowledge to form an authentic concepts-base conducive to further and better learning. Clark (2012) points out that mathematical concepts play a major role in simplifying the real world for the purpose pf achieving better communication and understanding. He maintains that concepts are considered a synthesis of overall learning, which helps learners organize their experiences in a way that makes it easier to retrieve and handle (Al-Bayyari, 2012). Furthermore, mathematical concepts are of utmost importance in coming to terms with the revolution of information. In this sense, concept formation and development has become one of the most important outcomes of teaching/learning mathematics at all stages of public and university education. It is one of the basics of mathematics and mathematical knowledge that learners of different educational levels need to develop (Jayanthi, Gersten, Taylor, Smolkowski, & Dimino, 2017). To achieve this purpose, alternative teaching methods are required to ensure sound mathematical concept formation and retention (Iraqi, 2013).

Concept formation and development is a continuous process that varies in terms of difficulty and simplicity from class to class and from one stage to another. Mathematical concepts hold different meanings for individuals depending on their different experiences and maturity levels (Ay, 2017). That is, the same concept may have different meanings for students; namely, it is not static, but rather dynamic and continually developing with maturity and experience acquired over time (Obaid, 2008). Mareschal (2016) argues that students draw a true or false mental image of the concept as a result of their experiences, and it is very important to detect false mental images correct them by all means. According to Clark (2012), to teach mathematical concepts effectively, they should be presented in a historical manner, as originally framed by mathematicians. Personal experience is not the only defining factor in understanding these concepts. Experiences that result from peer interactions are even more significant. They are considered by specialists as one of the applications of Piaget's theory in mathematics instruction. That is, learners learn more from each other than from a teacher; they feel free to share with peers all sorts of opinions and questions that they would hesitate to share with their teacher (Grady, 2018).

Concepts, thus, play a very important role in building sound cognitive structures about the surrounding world. However, mathematical misconceptions have been observed among students from various grade levels, and elementary stage students in particular (Alshamry, 2017; Ay, 2017; Booth, Barbieri, Eyer, & Paré-Blagoev, 2014; Pantziara & Philippou, 2012). These persistent misconceptions make it difficult for students to proceed from studying simple mathematical operations to higher levels of cognition such as algebraic reasoning. They could also hinder their performance and learning in subsequent units of study (Bush & Karp, 2013) and keep them from using what they learned in everyday life (Kiili et al., 2018).

Definition of Misconceptions

There are several synonyms for the term "misconceptions", including erroneous ideas, preconceptions, alternative ideas, misunderstandings, naive beliefs, and prescientific conceptions (Al-Osaimi, 2019).

Study studies conducted by Mohyuddin and Khalil (2012) and Ay (2017) showed that many people cannot distinguish between the two terms "misconception" and "error", although they are quite different. A misconception is part of an error but not the other way around. In other words, an error may occur as a result of a misconception, and a misconception may be one of the sources of committing errors.

There is a difference between a misconception and an alternative concept. A misconception refers to a concept formed in light of a set of beliefs that do not conform to, rather contradict with, a sound scientific viewpoint. On the other hand, an alternative concept is one that is presented by a learner and is unacceptable by scientists but not necessarily a misconception (Zitun, 2008). Both alternative concepts and misconceptions are often resistant to change. Individuals adherently defend them because they are consistent with the conceptual structures and convictions that they hold dear and believe to be right.

In light of definitions reviewed, a misconception is a false mental image formed by a student as a result of going through a confusing or inadequate educational experience. This mental image causes some operational errors in solving mathematical problems. Therefore, not every operational error that the student commits is a result of an alternative conception. It may result from the student's forgetting of a certain step or assigning wrong place values to some numbers. Therefore, accurate diagnostic tools must be used to detect misconceptions; for example, checking for patterns and frequencies in students' misconceptions in various educational situations.

A study by Al-Thaqafi (2015) revealed a set of misconceptions exhibited by some students in mathematics (related to concepts such as: the theory of categories, polynomials, inequalities, and some geometric concepts). Moreover, Booth et al.'s study (2014) indicated that students had many misconceptions as they moved from arithmetic studies to algebraic thinking, and these misconceptions hindered their performance and their learning of mathematics topics (p.10). The studyers also pointed out that mathematical misconceptions inherent in identifying persistent errors would be hardly corrected or modified through traditional models of teaching but would rather require additional interventions so that students could learn correct strategies. Suggestions included interventions that target misconceptions individually, or curricula that basically aim at improving conceptual understanding through courses of study (p. 21). Furthermore, a study by Lucariello, Tine and Ganley (2014) revealed many mathematical misconceptions related to concepts required for learning algebraic variables, especially those related to the nature of numbers and pertinent operations.

A co mprehensive review of the literature was attempted in a study by Bush and Karp (2013) pertaining to prerequisite skills required to study algebra as well as misconceptions associated with middle school students as a tool for gathering ideas for study and practice. Four content areas related to algebra were used (ratio and proportional relationships, number system, expressions and equations, and functions). The most important misconceptions observed in the study included the following (Busha & Karp, 2013, p. 628):

- Ratios and proportional relationships: Misunderstanding the connections among proportional relationships such as part/whole, part/part, whole/part- Confusion with ratios written as quotients.
- The number system: incorrectly or procedurally using fraction algorithms (without understanding them conceptually)- Ordering decimals incorrectly basing the value of a decimal on incorrect reasoning such as the number of digits- Ignoring negative signs- Believing that the commutative and associative properties are true for subtraction and division.
- Expressions and equations: Believing that the equal sign means, "The answer is" rather than expressing a relationship- Incorrectly viewing variables as labels or units.
- Functions: Lacking understanding of and not seeing the connection between multiple representations of functions- Not understanding a linear function represents a rate of change- Not understanding what makes a function proportional or non-proportional- Difficulty understanding the concept of the independent and dependent variables- Difficulty explaining what the slope and y-intercept represents.

Cincinatus and Sheffet's (2016) traced some misconceptions pertaining to Percent value and Percentage among a group of eighth-grade middle school students. A set of problems covering those concepts were presented to the students and some of their solutions reflected their misconceptions of "percentage". That is, they misconceived of "percentage" as synonymous to "percent" and used numbers without paying attention to the concept of "percentage". On the other hand, Wijaya's study (2017) investigated some misconceptions of fractions among Indonesian fourth-grade students and their relationship to the students' opportunities of learning fractions in school. This study consists with Durkin and Rittle-Johnson's (2015) study that revealed some misconceptions about knowledge of decimals (for example, 0.125 is bigger than 0.7 because 0.125 contains 3 numbers and 0.7 contains only one number). This flawed justification, the studyer pointed out, resulted from generalizing the properties of natural numbers (125 is greater than 7). It also consists with a study by Mohyuddin and Khalil (2012) that investigated misconceptions related to 'Numbers, Operations on Numbers, Fractions, Operations on Fractions, Decimals, Measurement and Geometry'.

Kiili et al. (2018) found out that, based on cumulative evidence, many students fail to complete simple numerical tasks even after great efforts on teachers' part in teaching mathematics. This finding may be attributed to misdiagnosis of underlying causes of students' misconceptions. On the other hand, Ni and Zhou (2005) argued that most of the misconceptions related to rational numbers and fractions are attributed to misconceptions about the associated phenomenon of integer bias. That is, students had a misconception that all properties of natural numbers can equally be applied to rational numbers and fractions (For example, 1/3 + 2/5 can be incorrectly solved by adding the numerator and denominator, i.e. 1+2/3+5 = 3/8), This consists with Sidney and Alibali's study conclusions (2015) that stressed the importance of fostering conceptual knowledge before procedural knowledge. Problems with conceptual knowledge, the studyers maintained, would lead to misconceptions or alternative conceptions, whereas procedural knowledge problems are rather related to solution algorithms.

The teacher's role in detecting misconceptions

Teachers play a vital role in identifying their students' mathematical misconceptions. Kleickmann et al. (2015) state that teachers can use several methods to achieve that depending on the nature of the misconception students have. Teachers can hardly succeed in identifying their students' misconceptions unless they are proficient in two basic areas: Content Knowledge (CK) and Pedagogical Content Knowledge (PCK).

Ben-Hur (2006, p. 57) highlights a set of interactions that would occur between a teacher and students to detect and modify mathematical misconceptions. Figure 1 illustrates the effective role that a teacher plays in identifying students' misconceptions by affording them chances to express conceptual reasons and justifications for their solutions to mathematics problems. Moreover, modifying these misconceptions depends on selecting appropriate teaching strategies.

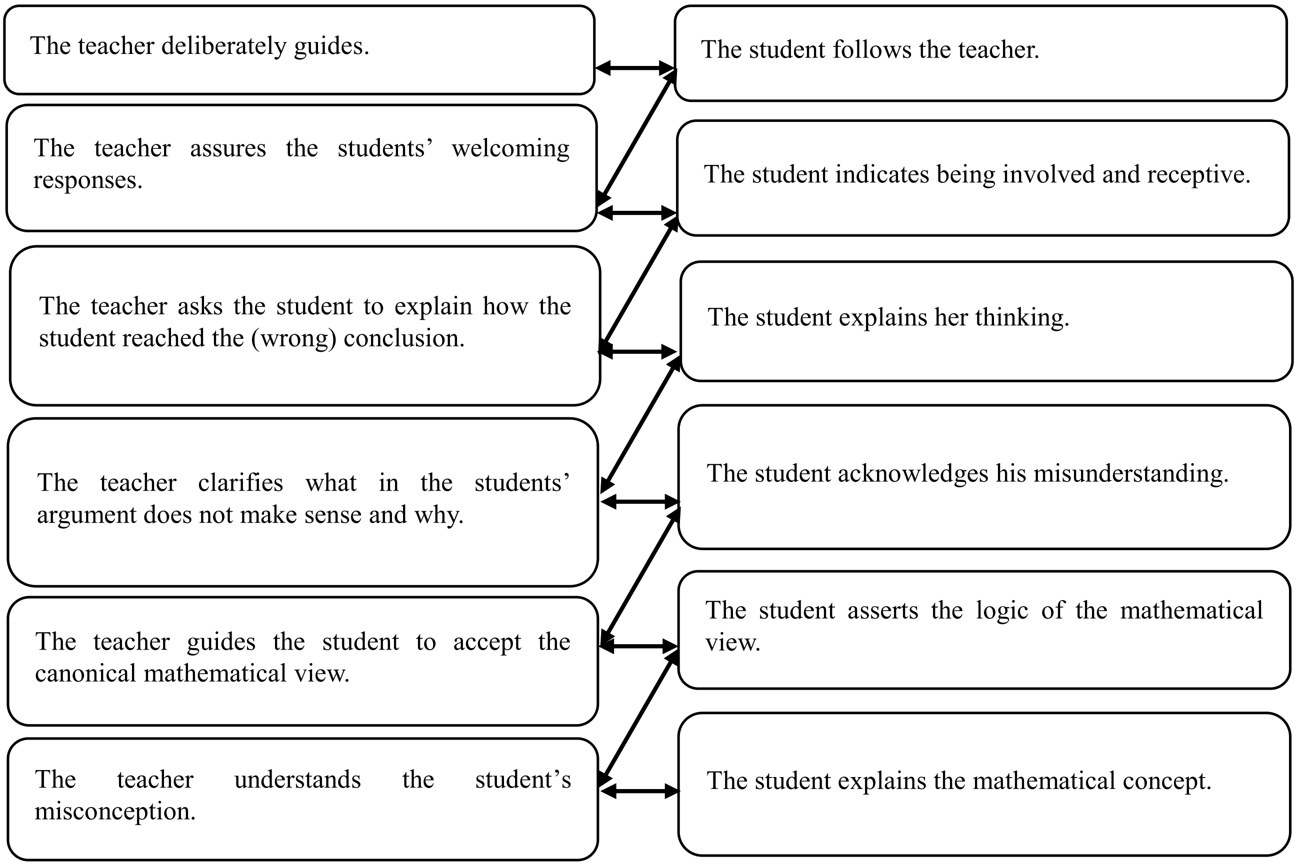

Figure 1: Teacher-student interactions to identify and modify mathematical misconceptions concepts

Characteristics of misconceptions

Reviewing prior study, characteristics of misconceptions can be summarized as follows (Abraham, Williamson, & Westbrook, 1994; Al-Osaimi, 2019; Barton, 2018):

- They make sense from a student's point of view because they are consistent with his/her cognitive conceptions and conceptual structures.
- They do not appear suddenly, but rather grow in the student and gradually get clearer and more solid; then more misconceptions and alternative ideas are built on them.

- They negatively affect students’ concept formation and learning, impede their proper understanding, and even support patterns of misunderstanding, which hinders subsequent learning.
- They grow in a student from many sources, most important of which are his/her perceptions, previous experiences, flawed information and misconceptions imparted by a teacher, flawed analogies and examples unintentionally misused by a teacher, and/or inaccurate ideas and information integrated in syllabus and curriculum plans.
- They are resistant to change and modification, especially with traditional teaching methods, and diagnosing them accurately is an important step for changing and modifying them.

These varied characteristics of misconceptions, in terms of their consistency, stability, and resistance to change, are quite unfavorable for understanding the nature of science and learning new concepts. That is, they affect students' thinking even after finishing school and getting ahead with their lives confronting new situations and problems that need to be solved. The use of non-traditional teaching and learning strategies, especially those related to methods of conceptual change, could possibly help in modifying alternative concepts. These strategies provide learners with opportunities to take on an active role in building their own knowledge and modifying their misconceptions.

In this sense, misconceptions play a role in impeding students' learning and their acquisition of accurate mathematical concepts. Thus, it is very important to uncover and identify these misconceptions at early stages of education, especially at the elementary stage. A variety of methods and techniques are needed in this respect to identify, diagnose, and handle these misconceptions.

Methods of diagnosing misconceptions

Using well-defined tools for detecting misconceptions lends depth and accuracy to studyers. It helps them understand what lies behind operational errors displayed in students’ answers, which in turn makes it easier to identify the necessary methods for modifying these misconceptions effectively.

There are varied methods of detecting students’ misconceptions and misunderstandings about concepts and natural phenomena. Bahadır (2017) and Al-Osaimi (2019) highlighted some methods of diagnosing misconceptions, including free sort tasks, concept maps, free association, concept structuring analysis technique, drawing methods, clinical interviews, classroom discussions, and demonstrate-observe-explain (DOE) strategy.

Surveys conducted by Türkdoğan, Güler, Bülbül, and Danişman (2015) pinpointed common methods that were used to identify mathematical misconceptions. Multiple-choice questions, concept maps, semantic analysis diagrams, predict-observe-interpret were all useful in deeply scrutinizing misconceptions. Barton (2018), on his part, stressed the importance of using open-ended questions instead of closed-ended questions in identifying and understanding operational errors and alternative conceptions in mathematics. However, open-ended questions may take a long time and class time is never sufficient for all students to discuss using open-ended questions. Using multiple-choice questions, thus, was suggested as a good alternative for identifying students' alternative misconceptions in mathematics. In this respect, Barton (2018) highlighted five conditions, also called golden rules, necessary for a multiple-choice alternative; they are summed up as follows: Every item should be clear, test a single skill/concept, students should be able to answer it in less time, each incorrect response refers to one misconception at most, it cannot be answered correctly while still holding a misconception (Barton, 2018, 37).

STUDY PROBLEM

Teaching/learning mathematical concepts is the one of the basics of elementary education. It is the elementary stage where solid foundations of mathematics learning are constructed. Important as this stage is, studyers have been interested in investigating underlying difficulties and problems encountered by students in acquiring the necessary skills. Previous study reports the low level of elementary stage students in Saudi Arabia when it comes to understanding mathematical concepts, which leads to lower levels of achievement in mathematics in general. This low achievement is evident in the results of international tests (Thomson et al. al., 2017), such as Trends in International Mathematics and Science

Study (TIMSS). This may be partially attributed to students' misconceptions of mathematics concepts. Therefore, the current study aims to investigate such misconceptions, and identify their root causes through analyzing students' answers and justifications on the test prepared for that purpose.

STUDY QUESTIONS

1. What are the misconceptions about mathematics concepts that are most common among elementary school students?
2. What is the level of mathematics learning pleasure for elementary school students?
3. What is the correlation between the misconceptions and the level of mathematics learning pleasure among elementary school students?
4. How does Students' misconceptions of mathematics concepts differ based on gender?
5. How does Students' math learning plessure differ based on gender?

STUDY HYPOTHESES

H_o1: There is no significant difference between the male and female students on their misconceptions about math concepts.

H_o2: There is no significant difference between the male and female students on their level of math learning pleasure.

STUDY OBJECTIVES

The current study aimed to investigate misconceptions in mathematics (specifically, numbers and pertinent operations and fractions and pertinent operations) of a sample of elementary-school fifth graders in Wadi Al-Dawasir Governorate in the Kingdom of Saudi Arabia. A potential correlation was also investigated between these misconceptions and the pleasure of learning mathematics.

STUDY SIGNIFICANCE

Theoretical significance: This study provides a checklist of mathematics misconceptions that could be building blocks for future study aimed at modifying these misconceptions early on in students' education.
Practical significance: This study could help elementary stage mathematics teachers avoid committing teaching errors that may lead to these misconceptions among students, it could draw the attention of educators who are in charge of developing mathematics curricula to put these misconceptions into consideration throughout their curriculum planning activities, and early detection of misconceptions in mathematics contributes by all means to identifying the best methods and approaches to teaching mathematics.

TERMINOLOGY

Concept: A mental perception of one or more characteristics of something that is repeated in more than one situation.
Misconceptions of mathematics: A flawed mental image formed by a student as a result of going through a confusing or incomplete educational experience. This mental image is inferred from repeated operational errors in solving mathematical problems.
Pleasure of mathematics learning: A student's internal sense of happiness, tendency and desire to engage in mathematics activities on his/her own.
Elementary school students: They are students who enroll in formal education from the age of 9 till the culmination of the elementary stage at the age of 12.

METHODOLOGY

Study Design: Two methods were adopted in the current study. The descriptive analytic method was used for the purpose of collecting data, describing the problem's dimensions, analyzing the content of mathematics books, and analyzing the results of the mathematical concepts test. On the other hand, the quasi-experimental method was used to investigate the relationship between the two dependent study variables.
Participants: The sample was randomly selected from fifth-grade students in 6 elementary schools in Wadi Al-Dawasir Governorate in Saudi Arabia. The number of participants was 120, including 60 male and 60 female students. Table 1 below shows the sample distribution.

Table 1: Participants' distribution by gender

N	School	Gender	
		Male	Female
1	Prince Sultan Elementary School	20	0
2	King Saud Elementary School	19	0
3	Al Nuweima Elementary School for Boys	21	0
4	Al Nuweima Elementary School for Girls	0	19
5	Nazwa Elementary School for Girls	0	19
6	Al-Khamaseen Elementary School for Girls	0	22
	Total	60	60
		120	

Instruments

Content analysis scale:

Seven units of elementary school mathematics textbooks for fourth and fifth grades were analyzed, and 30 basic mathematical concepts were extracted. To ensure the validity of Content analysis, the tool was submitted in its initial form to a group of specialized reviewers in curriculum and instruction of mathematics, and in psychology and educational evaluation. All suggested modifications and changes were made. To check reliability, the content of the selected units was re-analyzed after two weeks, then the agreement coefficient between the two analyses was calculated, and it was 0.98.

Test in mathematical concepts:

The test aims: To reveal misconceptions in some mathematical concepts was designed. It's consisted of (30) questions, every question consisted of two-step multiple-choice questions. A table of specification of test was designed to assign test items to the topics that have been identified.

Test validity: The initial version of the test was submitted to a number of jurors specialized in mathematics instruction and psychology, and necessary adjustments were made in light of the jurors' suggestions.

Test piloting: The modified version of the test was administered to a sample of fifth-grade elementary school students (N = 45) at the beginning of the first semester of the academic year 2019/2020. Then, the test was re-administered two weeks later for the purpose of adjusting its psychometrics (in terms of reliability, facility and difficulty, and discrimination coefficients, as well as test timing) as follows:

Test reliability: The reliability coefficient was calculated for the test as a whole and for each of its dimensions (numbers and pertinent operations, fractions and pertinent operations) by re-administering the test to a group of elementary stage students in the pilot study. Then, Pearson correlation coefficient was calculated to measure correlation between the test and re-test as shown in Table 2.

Table 2: Pearson's correlation coefficients for the test dimensions

Test dimensions	Correlation coefficient
Numbers and pertinent operations	0.87
Fractions and pertinent operations	0.89
Reliability coefficient for the test as a whole	0.91

Table 2 shows that the mathematical concepts test (with its two dimensions) has an acceptable level of reliability.

Test timing: The time required for the test was estimated by calculating the average time between the lower and upper quartiles, which was 60 minutes.

The final version of the test: The test consisted of 30 questions in its final version. Table 3 shows the relative weights of the achievement test questions.

Table 3: Specification of the test in mathematical concepts:

N	Domain	Topic	Number of concepts	Questions		
				Item	Number	Relative Weigh
1	Numbers & pertinent operations	– Place value – Adding and subtracting	16	1-16	16	53%

		− Estimation & approximation − Algebra & Patterns				
2	Fractions & pertinent operations	− Common fractions & pertinent operations − Decimal fractions & pertinent operations − Comparison of decimals and fractions	14	17-30	14	47%
Total	2	7	30		30	100%

Test scoring: The test was scored by assigning one point for each correct answer and zero for each wrong answer or each unanswered question. Then, the points were added to calculate the total score of the test. Moreover, a scoring rubric was prepared and the recurrences of answer-per-option in each question were calculated in order to determine the level of mathematical misconceptions.

Questionnaire of math learning pleasure:
The objective of the questionnaire was to identify the level of pleasure that the students feel about learning mathematics.
Initial version of the questionnaire: The questionnaire, in its initial version, consisted of 16 phrases.
Validity of the questionnaire: The questionnaire was submitted in its initial version to a number of jurors specialized in curriculum and instruction of mathematics and educational assessment. The jurors were asked to ensure that the questionnaire statements are clear enough and appropriately measure what they are intended to measure. All necessary adjustments were made as advised; namely, some phrases were modified and some were deleted. The final version of the questionnaire consisted of 14 statements.
Piloting of the questionnaire: The modified version of the questionnaire was administered to a sample of students (N = 45) at the beginning of the first semester of the academic year 1441 AH. Then, it was re-administered two weeks later to ensure internal validity and reliability as shown in the following procedures:
Reliability of the questionnaire: Alpha Cronbach coefficients were calculated and the questionnaire proved reliable as shown in Table 4.

Table 4: Cronbach's Alpha α Coefficients for the Math Learning Pleasure Questionnaire

Statement number	Coefficient	Statement number	Coefficient
1	0.530	8	0.603
2	0.521	9	0.668
3	0.490	10	0.757
4	0.588	11	0.793
5	0.608	12	0.750
6	0.618	13	0.842
7	0.350	14	0.841

Table 4 shows that all correlation coefficients are statistically significant at levels 0.01 and 0.05, which indicates that there is internal consistency between the questionnaire statements and the overall score of the questionnaire.
Final version of the questionnaire: The questionnaire, in its final version, consisted of two main sections, the first related to the students' basic data, and the second contained statements that measure the pleasure of learning mathematics. The number of the questionnaire statements was 14, including 7 positive statements and 7 negative statements on a three-point Likert scale. Positive statements were

graded as follows: often (3 points), sometimes (2 points), rarely (1 point), and negative statements were graded as follows: often (1 point), sometimes (2 points), and rarely (3 points).
Statistical Standard: The points of the responses were divided by the number of the following level categories: (low, medium, high) as follows: The range for the scale values = the largest value in the estimate - the smallest value in the estimate (3 - 1 = 2). Since the number of categories needed to interpret the means of the scored responses of the study population are 3 categories, the length of each category = the range of the questionnaire values / the number of interpretation values (2/3 = 0.66). Accordingly, the categories adopted to interpret the mean scores of the participants' responses are shown in Table 5.

Table 5: Interpretation of mean scores on the Math Learning Pleasure Questionnaire

Estimate score	1 – 1.66	1.67 – 2.33	2.34 - 3
Level	Low	Medium	High

Treatment:
The tools were administered to (120) elementary school fifth graders at the beginning of the first semester of the academic year 2019/2020.
In order to identify the misconceptions that caused operational errors, multiple choice questions were used, comprising two parts: the first part includes a concept followed by four options (a, b, c, d), followed by the second part which includes an explanation or a mathematical reason for choosing the correct answer in the first part. There are four explanations/reasons numbered 1, 2, 3 and 4. The student puts a circle around the letter s/he chose in the first part, then puts a circle around the number that s/he thinks represents the interpretation or mathematical reason for what s/he chose in the first part. Consider the following question:
Chose the right answer, 6 / 2 = ….

(A)	(B)	(C)	(D)
8	12	4	3

Because of:

(1)	(2)	(3)	(4)
Adding 2 to 6	Multiply 6 by 2	Subtract 2 from 6	Divide 6 in two groups

As shown in the previous example, every option in the first part, except for the correct option, represents a Procedural error resulting from the student's alternative conception of 'division'. Namely, option A may be the result of the student's misconception of the process of dividing as adding. On the other hand, option B may be the result of the student's misconception of the 'dividing' process as a multiplying process. Furthermore, option C may be the result of the student's misconception of the process of dividing as Subtracting. Thus, option D is the correct option.

RESULTS

Study question 1

What are the misconceptions about mathematics concepts that are most common among elementary school students?
Frequencies and percentages were calculated for mathematical misconceptions and it was found out that these misconceptions were exhibited by the students with a frequency of least 25%, as shown in table 6 below.

Table 6: Descending sorting of frequencies & percentages of mathematical misconceptions

No	Misconceptions	Freq.	Percent.
1	Consider that estimation means giving the same result of adding three different numbers	69	58%
2	Consider that estimation to the nearest ten is the same if the "units" are less than 5	67	56%
3	Conceive that a fraction with a larger denominator is bigger than a fraction with a smaller denominator	64	53%
4	Misconceive of place value	62	52%
5	Conceive that numerators are added together, and denominators are added together in the process of adding two fractions with different denominators	62	52%

6	Consider that there is no difference between a numerical statement and a numerical sentence	61	51%
7	Conceive that a decimal with the most digits is the largest regardless of the place value of the numbers.	57	48%
8	Consider that if the numerator in a common fraction equals 10, it is called a decimal	56	47%
9	Conceive that numerators are added together, and denominators are added together in the process of adding two fractions with the same denominator	54	45%
10	Conceive that estimation is always about rounding to the largest number	52	43%
11	Consider that the whole shaded figure represents the sixth, on the premise that the sixth resembles six	52	43%
12	Conceive that the word "more than" refers to the process of adding variables	51	43%
13	Lack knowledge of the difference between the contrast signs ">, <, =" in number comparisons	49	41%
14	When you subtract a large number from a small number, you miscalculate it zero	45	38%
15	Conceive that the words two-triangles mean two-thirds	45	38%
16	Conceive that numerators are subtracted from numerators and denominators are subtracted from denominators in the process of subtracting two fractions of different denominators	43	36%
17	Consider that rounding to the nearest hundred equals hundred in all cases	40	33%
18	Consider the distributive property of multiplication over addition, namely, including addition and multiplying in the same process.	40	33%
19	Conceive that approximation means having the same result unchanged	39	33%
20	Consider that a numerical statement only includes numbers	37	31%
21	Consider that if a numerator in a common fraction equals 10, it is called a decimal	36	30%
22	Add a number in the tenths place with a number in the hundredths place	36	30%
23	Subtract the first number subtracted from the first number subtracted from, without taking decimal place values into account	36	30%
24	Use the distributive property of multiplication over addition without taking numbers’ place values into account	35	29%
25	When converting a common fraction to a decimal, the decimal point is placed only between the numerator and the denominator	35	29%
26	Rounding to the nearest tenth rather than to the integer one	35	29%
27	Conceive that estimation to the nearest ten equals 10 if ones are greater than or equal to 5	34	28%
28	Conceive that the word "more than" means “equal”	34	28%
29	Consider that the associative property means the commutative property	33	28%
30	Consider that the concept of the neutral element of addition means the integer one	31	26%
31	The smaller number is subtracted from the larger number, regardless of the number subtracted and subtracted from.	30	25%
32	Consider that the distributive property of multiplication over addition is the neutral element for multiplication	30	25%
33	Conceive that a numerator adds with a numerator and a denominator adds with a denominator in the process of subtracting two different fractions	30	25%
34	Conceive that the denominator of one of the equivalent common fractions must equal the numerator of the other	30	25%

It is evident from Table 6 that the most common misconceptions exhibited by students related to two main statements. The first was “estimation means giving the same result of adding three different numbers” with a frequency of 58%. The second was “estimation to the nearest ten is the same if the “units” are less than 5” with a frequency of 56%. On the other hand, the least common misconception was " the denominator of one of the equivalent common fractions must equal the numerator of the other” with a frequency of 25%.

Study question 2

What is the level of mathematics learning pleasure for elementary school students?

Chi-square was calculated for the mathematics learning pleasure questionnaire results as shown in table 7 and table 8 below.

Table 7: Descriptive Statistics

	N	Mean	SD.	Minimum	Maximum
Learning Pleasure Level	120	2.1524	0.49082	1.00	3.00

As shown in Table 7, the participants' level of learning pleasure in mathematics was 2.15. Reviewing the questionnaire level estimates in table 5, it can be concluded that this score (2.15) reflects the average level of mathematics learning pleasure for the students participating in the study, with a standard deviation of 0.4908. Table 8 displays the Chi-Square values for the questionnaire items.

Table 8: Chi-Square test results

Item	Assessment	Observed N	Expected N	Residual	Chi-Square	df	Asymp. Sig.
a1	always	34	40.0	-6.0	1.400	2	0.497
	sometimes	42	40.0	2.0			
	Rarely	44	40.0	4.0			
	Total	120					
a2	always	16	39.7	-23.7	22.202	2	0.000
	sometimes	56	39.7	16.3			
	Rarely	47	39.7	7.3			
	Total	119					
a3	always	29	39.7	-10.7	11.563	2	0.003
	sometimes	33	39.7	-6.7			
	Rarely	57	39.7	17.3			
	Total	119					
a4	always	39	39.7	-.7	6.118	2	0.047
	sometimes	29	39.7	-10.7			
	Rarely	51	39.7	11.3			
	Total	119					
a5	always	22	39.7	-17.7	27.244	2	0.000
	sometimes	31	39.7	-8.7			
	Rarely	66	39.7	26.3			
	Total	119					
a6	always	31	39.7	-8.7	6.924	2	0.031
	sometimes	35	39.7	-4.7			
	Rarely	53	39.7	13.3			
	Total	119					
a7	always	31	39.7	-8.7	4.655	2	0.098
	sometimes	38	39.7	-1.7			
	Rarely	50	39.7	10.3			
	Total	119					
a8	always	42	39.7	2.3	3.042	2	0.218
	sometimes	31	39.7	-8.7			
	Rarely	46	39.7	6.3			
	Total	119					

a9	always	35	39.7	-4.7	2.639	2	0.267
	sometimes	36	39.7	-3.7			
	Rarely	48	39.7	8.3			
	Total	119					
a10	always	58	39.3	18.7	16.542	2	0.000
	sometimes	22	39.3	-17.3			
	Rarely	38	39.3	-1.3			
	Total	118					
a11	always	64	39.7	24.3	22.706	2	0.000
	sometimes	30	39.7	-9.7			
	Rarely	25	39.7	-14.7			
	Total	119					
a12	always	44	39.7	4.3	0.824	2	0.662
	sometimes	36	39.7	-3.7			
	Rarely	39	39.7	-.7			
	Total	119					
a13	always	44	39.7	4.3	0.824	2	0.662
	sometimes	36	39.7	-3.7			
	Rarely	39	39.7	-.7			
	Total	119					
a14	always	53	39.3	13.7	7.746	2	0.021
	sometimes	29	39.3	-10.3			
	Rarely	36	39.3	-3.3			
	Total	118					

Table 8 shows that Chi-Square data results of the questionnaire items varied in terms of statistical value and significance. A detailed statistical analysis of the questionnaire results that showed statistically significant differences between the observed frequencies and the expected frequencies of the estimates (always - sometimes - rarely).

- Regarding item a2, which states: “I feel happy when a math teacher assigns me homework to do”, there was a statistically significant difference at the levels 0.01 and 0.05 between the observed and expected frequencies of the estimates (always - sometimes - rarely). “Always” had the least frequency in this item (16), which indicates a low level of math learning pleasure in this aspect.
- Regarding item a3, which states: “I feel the importance of mathematics in my daily interactions”, there was a statistically significant difference at the levels 0.01 and 0.05 between the observed and expected frequencies of the estimates (always - sometimes - rarely). “Always” had the least frequency in this item as well (29), which indicates a low level of math learning pleasure in this aspect.
- Regarding item a4, which states: “I would like the school broadcasting program to include a section on mathematics”, there was a statistically significant difference at the level 0.05 between the observed and expected frequencies of the estimates (always - sometimes - rarely). “Rarely” had the most frequency in this item (51), which indicates a low level of math learning pleasure in this aspect.
- Regarding item a5, which states: “I make sure to pay attention in math classes, under all circumstances”, there was a statistically significant difference at the levels 0.01 and 0.05 between the observed and expected frequencies of the estimates (always - sometimes - rarely). “Rarely” had the most frequency in this item (66), which indicates a low level of math learning pleasure.
- Regarding item a6, which states: “Mathematics is my most favorite subject”, there was a statistically significant difference at the level 0.05 between the observed and expected frequencies of the estimates (always - sometimes - rarely). “Rarely” had the most frequency in this item (53), which indicates a low level of math learning pleasure in this aspect.

- Regarding item a10, which states: “I feel that mathematics is not that important in my daily interactions”, there was a statistically significant difference at the levels 0.01 and 0.05 between the observed and expected frequencies of the estimates (always - sometimes - rarely). “Always” had the most frequency in this item (58), which is a negative statement that indicates a low level of math learning pleasure.
- Regarding item a11, which states: “I get busy doing something else during math classes”, there was a statistically significant difference at the levels 0.01 and 0.05 between the observed and expected frequencies of the estimates (always - sometimes - rarely). “Always” had the most frequency in this item (64), which is a negative statement that indicates a low level of math learning pleasure.
- Regarding item a14, which states: “I wish the math period could be removed from the schedule”, there was a statistically significant difference at the level 0.05 between the observed and expected frequencies of the estimates (always - sometimes - rarely). “Always” had the most frequency in this item (53), which is a negative statement that indicates a low level of math learning pleasure.
- Regarding the rest of the questionnaire statements, there were no statistically significant differences at the level 0.05 between the observed and expected frequencies of the estimates (always - sometimes - rarely).

Study question 3

What is the correlation between the misconceptions and the level of mathematics learning pleasure among elementary school students?

Pearson’s correlation was calculated between the students' scores on both the Mathematics Concepts Test and the Math Learning Pleasure Questionnaire as shown in table 9.

Table 9: Correlation between mathematical misconceptions and the level of pleasure in mathematics learning

		Level of pleasure in math learning
Numbers & pertinent operations	Pearson Correlation	0.170
	Sig. (2-tailed)	0.064
	N	120
Fractions & pertinent operations	Pearson Correlation	0.133
	Sig. (2-tailed)	0.148
	N	120
Overall concepts Test	Pearson Correlation	0.173
	Sig. (2-tailed)	0.059
	N	120

Table 9 shows that there was a poor, non-statistically significant positive correlation at the level 0.05 between the mean scores of the students in mathematical misconceptions and the level of pleasure in learning mathematics with regard to the two test dimensions separately on the one hand and the overall test on the other hand.

Study question 4

How does Students’ misconceptions of mathematics concepts differ based on gender?

H_o1: There is no significant difference between the male and female students on their misconceptions about math concepts.

MANOVA was used to investigate the effect of the gender variable on the students’ performance level in the mathematics concepts test.

Table 10: Multivariate Test for Gender variable

Independent variable		Value	F	Hypothesis df	Error df	Sig.	Partial Eta Squared
Gender	Wilks' Lambda	0.821	12.72	2	117	0.000	0.179

As shown in table 10, there was a statistically significant difference at the significance level a <0.05 in the two dimensions of the test (numbers and pertinent operations and fractions and pertinent operations) in terms of gender. That is, f value was 12.72, which is statistically significant (p = 0.000). Furthermore, Wilks' Lambda value was 0.821, and the Partial Eta Square value was 0.179. To measure the effect of gender on each of the two dimensions, analysis of variance was used as shown in table 11.

Table 11: Between-Subjects Effects

Independent variable	Dependent Variable	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Gender	Mathematical Misconceptions score	1134.675	1	1134.675	16.199	.000	.121
	numbers and pertinent operations	120.000	1	120.000	4.577	.034	.037
	fractions and pertinent operations	516.675	1	516.675	25.413	.000	.177

It is evident from table 11 that regarding numbers and pertinent operations, f value was 4.577 at the significance level P = 0.037, which means that there were no statistically significant differences at the level a <0.025 between males and females. As for fractions and pertinent operations, f value was 25.413, which is statistically significant at the level P = 0.000. Regarding the overall test, f value was 16.199, which is statistically significant at the level P = 0.000. In other words, the gender variable had an impact on the students' scores on the mathematics concepts test in general and on the fractions-and-operations dimension particularly. As for numbers and operations, there were non-statistical gender-related differences at the level <0.025 in favor of females.

To find out which gender group had a greater impact on the dependent variables, the mean and standard error were calculated as shown in Table 12.

Table 12: Descriptive Statistics

	Gender (Male/Female	Mean	Std. Deviation	N
Mathematics concepts test	Male	16.25	7.082	60
	Female	22.40	9.484	60
	Total	19.32	8.888	120
Numbers and pertinent operations	Male	12.53	4.707	60
	Female	14.53	5.503	60
	Total	13.53	5.197	120
Fractions and pertinent operations	Male	3.72	3.189	60
	Female	7.87	5.522	60
	Total	5.79	4.950	120

Table 12 shows that the mean score of female students in the concepts test in the dimension of fractions and pertinent operations was 7.87, which is higher than the mean score of male students that was 3.72. Moreover, the mean score of female students in the overall test was 22.40, which is higher than the mean of the male students' scores that was 19.32. This result leading to rejection of the null hypothesis (H_o1).

Study question 5

How does Students' math learning plessure differ based on gender?

H_o2: There is no significant difference between the male and female students on their level of math learning pleasure.

T-test was calculated for Independent Samples Test as shown in table 13.

Table 13: t-test results

	Gender (Male/Female	N	Mean	Std. Deviation	Std. Error Mean
Math learning pleasure	Male	60	2.0195	.52049	.06719
	Female	60	2.2853	.42315	.05463

Table 14: Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means						
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
									Lower	Upper
Score of Math learning pleasure	Equal variances assumed	2.654	0.106	3.070	118	0.003	-.26583	.08660	-.43732-	-.09434-
	Equal variances not assumed			3.070	113.280	0.003	-.26583	.08660	-.43740-	-.09427-

It is evident from Table 14 that the t value is 3.070 and the degree of freedom is 118, which is statistically significant at level 0.05. This result indicates a statistically significant difference in terms of gender with regard to the mathematics learning pleasure, in favor of females with a mean of 2.28, which is higher than the mean of males' math learning pleasure (2.02).
This result leading to rejection of the null hypothesis (H_o2).

DISCUSSION

The first study result indicates that many math misconceptions were exhibited by elementary school students, (for example: the dimension of numbers and pertinent operations included misconceptions like misconceiving place value, considering that there is no difference between a numerical statement and a numerical sentence, considering that a numerical statement includes numbers only, misconceiving that approximation is always about rounding to the largest number, miscalculating zero as the result of subtracting a large number from a small number, considering that rounding to the nearest hundred always equals hundred, misconceiving that approximation means having the same result unchanged, conceiving that the word "more than" refers to the process of adding variables, considering that the concept of the neutral element of addition means the integer one. These results are consistent with recent study results (e.g. Ay, 2017; Barton, 2018; Bush & Karp, 2013; Yang & Lin, 2015). Regarding the dimension of fractions and pertinent operations, the most important misconceptions included considering that estimation means giving the same result of adding three different numbers, and this misconception may be attributed to students' interest in obtaining an accurate result of arithmetic operations and their ignorance of the importance of computational estimation in practical life.

It was also found that there was a misconception regarding comparing common fractions; namely considering that a fraction with a greater denominator is larger than a fraction with a smaller denominator. Furthermore, one of the most common misconceptions was considering that numerators are added together and denominators are added together in the process of adding or subtracting two fractions with same and different denominators. Other common misconceptions included: Conceiving that a decimal with the most digits is the largest regardless of the place value of the numbers, considering that if a numerator in a common fraction equals 10, it is called a decimal, considering that "a sixth" means "six parts" (similarity in Arabic pronunciation), considering that the word "triangle" means "two-thirds" (similarity in Arabic pronunciation), this may be due to the weak Arabic language skills of many students. These results are consistent with recent study results (e.g. Ay, 2017; Booth et al., 2014; Burgos & Godino, 2020; Cincinatus & Sheffet, 2016; Kiili et al. al., 2018; Pantziara & Philippou, 2012; Wijaya, 2017).

According to the second and third results, they indicate the poor levels of math learning pleasure exhibited by the participants in general, and particularly those students who had mathematical misconceptions. This result is consistent with a study by Moyer (2001), which indicated that many teachers hardly used appropriate teaching methods, such as the use of manipulatives, that would enhance learning mathematics pleasure and a study by Clement, Narode and Rosnick (1981), which confirmed that misconceptions of basic mathematics concepts were considered a source of mathematical anxiety for students. This result is also consistent with the results of a study by Divjak and Tomić (2011) that revealed poor levels of readiness and pleasure of learning mathematics due to ineffective methods of teaching and presenting mathematics to students and shunning favorable methods such as electronic games.

The fourth result further indicates that male students exhibited more misconceptions of some mathematical terms than females did. This may be attributed to the interesting activities practiced by math teachers in presenting mathematics to the students. The fifth result of the study confirms this and indicates that the female students' level of math learning pleasure exceeded that of male students.

CONCLUSION

It is imperative that the Misconceptions of mathematics concepts be detected early. For this purpose, the present study was designed, which answered five questions and two hypotheses. Misconceptions has been a subject of scrutiny and investigation in prior study, yet little attention has been devoted to misconceptions in relation to math learning pleasure. To date, the role of pleasure in math activities has received little attention. Findings here are rather predictive, implying that learners could hardly engage in cognitively demanding tasks if their misconceptions persist, the matter which supports the affective-cognitive model of teaching and learning in guiding future curricular design. This study provides a context-specific categorization of the most common misconceptions exhibited by elementery stage students, which could possibly contribute in planning appropriate instruction that is beneficial to those learners.

The first result indicated that there are many misconceptions to elementery school students in the Kingdom of Saudi Arabia, related to numbers and operations on them, fractions and operations on them. Looking at these misconceptions, we find that some of them are related to the lack of awareness of the concepts for what they are, some of them are related to the method of teaching followed, and others are related to the generalization of some previous experiences on new concepts, this study attributes many of the misconceptions to the weak Arabic language skills of many students, which is evident from the students' answers. The second result indicated that the level of learning mathematics pleasure was average level for the students participating in the study. The third result indicated that there is a weak relationship between the presence of misconceptions among students and the level of learning mathematics pleasure for them. The fourth result confirms that males are higher in having misconceptions than girls, and the fifth result indicates a lower level of math learning pleasure for males, and this may be attributed to the teaching methods used in girls 'schools, and this is the nature of the educational environment in Wadi Al-Dawasir, where the distribution of Gifts for students on an ongoing basis, and the implementation of many classroom and extracurricular activities in mathematics education.

Study Recommendations

In light of the study results, the following recommendations are proposed:

- The study recommends the necessity of paying attention to students' Arabic language skills, which greatly supports overcoming misconceptions.
- Methods of teaching mathematics should be diversified to avoid chances of misconceptions.

- Accuracy and balance should be considered with regard to using technology in teaching and learning mathematics.
- Workshops and training courses should be held for mathematics teachers to train them on detecting students' misconceptions and attempt solutions to modify them before constructing new mathematical knowledge.
- Mathematics teachers should verify prior knowledge of math concepts before embarking on teaching new concepts.
- Math teachers should try to pronounce math concepts correctly.
- Emphasis on teaching numbers and fractions in more realistic ways

Suggestions For Further Study

Building on the current study, the following study studies are suggested:

- Mathematical misconceptions in other branches of mathematics.
- The relationship between students' learning style and their mathematical misconceptions.

Study Limitations

- *Temporal Limitations:* The study was carried out in the first semester of the academic year 2019/2020.
- *Spatial delimitations*: The experiment was administered to a sample of elementary-school fifth graders in Wadi Al-Dawasir Governorate, Saudi Arabia.
- *Content delimitations:* The content of mathematics books taught to elementary-school fourth graders was analyzed to determine the basic concepts of mathematics related to: numbers and pertinent operations and fractions and pertinent operations

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