Ilkogretim Online - Elementary Education Online, 2020; Vol 19 (Issue 4): pp.2961-2967
http://ilkogretim-online.org
doi: 10.17051/ilkonline.2020.04.764668

AN APPLICATION OF GENERALIZED DISTRIBUTION SERIES ASSOCIATED WITH ANALYTIC MULTIVALENT FUNCTIONS

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Abstract- The aim of the present paper is to introduce a generalized distribution series. Many interesting results using this series and Dziok-Srivastava linear operator are obtained.

Keywords: Analytic Function, Multivalent Functions, Dziok Srivastava operator, Generalized Distribution..

I. INTRODUCTION :

Let A_p denote the class of function f(z) of the form

$$f(z) = z^p + \sum_{j=p+1}^{\infty} a_j z^j \qquad (p \in N = \{1,2,3,......\}) \qquad (1.1)$$

which are analytic in the open unit disk $U = \{z : z \in C \text{ and } |z| < 1\}$ and satisfy the normalization condition $f(0) = f'(0) - 1 = 0$.

Further let S_p be the subclass of A_p consisting of functions of the form (1.1) which are also multivalent in U and also let T_p be the subclass of S_p consisting of functions of the form

$$f(z) = z^p - \sum_{j=p+1}^{\infty} |a_j| z^j \qquad (1.2)$$

Also suppose that

$$h_p\,[(\alpha_q); (\beta_r); z] = z^p \;{}_qF_r\,(\alpha_1,...,\alpha_q; \beta_1,......\beta_r; z)$$
$$= z^p + \sum_{j=p+1}^{\infty} [B_p^{(\alpha_q),(\beta_r)}(j-p)] z^j \qquad (1.3)$$
$$(q \le r+1;\ \alpha_i \in R;\ \beta_k \in R \setminus Z_0^-;\ Z_0^- = \{0, -1, -2, ...\};\ i = 1,..., q;\ k = 1,...,r;\ z \in U)$$

where ${}_qF_r$ is the generalized hypergeometric function and

$$B_p^{(\alpha_q),(\beta_r)}(j-p) = \frac{\prod_{i=1}^{q} (\alpha_i)_{j-p}}{\prod_{k=1}^{r} (\beta_k)_{j-p}\,(j-p)!} \qquad (1.4)$$

Corresponding to the function $h_p\,[(\alpha_q); (\beta_r); z]$ Dziok and Srivastava [4, p.3, Eq.(3)] introduced a linear operator $H_p\,[(\alpha_q); (\beta_r); z]$ defined by the convolution

$$H_p\,[(\alpha_q); (\beta_r); z] = h_p\,[(\alpha_q); (\beta_r); z] * f(z), \qquad (f \in T_p) \qquad (1.5)$$

Or equivalently by

$$H_p\ [(\alpha_q\ ); (\beta_r\ )]\ f(z) = z^p - \sum_{j=p+1}^{\infty} [B_p^{(\alpha_q\ ),(\beta_r)}(j-p)]\ |a_j| z^j\ , \qquad (z \in U) \qquad (1.6)$$

Here * stands for the convolution of two analytic multivalent functions f and g of the form

$$f(z) = z^p + \sum_{j=p+1}^{\infty} |a_j| z^j \qquad (1.7)$$

and

$$g(z) = z^p + \sum_{j=p+1}^{\infty} |b_j| z^j \qquad (1.8)$$

and is defined by

$$(f * g)(z) = z^p + \sum_{j=p+1}^{\infty} |a_j||b_j| z^j \qquad (1.9)$$

The linear operator $H_p\ [(\alpha_q\ ); (\beta_r\ )]\ f(z)$ includes various other linear operators considered earlier by Hohlov [7], Carlson-Shaffer [2], Goyal and Bhagtani [6], Ruscheweyh [12] etc.

Next by using this operator we introduce the following classes of analytic functions:

Let $T_p(\mu,\delta)$ be the subclass of T_p consisting of functions which satisfy the condition

$$\mathrm{Re}\left\{\frac{z(H_p\ [(\alpha_q\ ); (\beta_r\ )]\ f(z)\ )'}{\mu\ z(H_p\ [(\alpha_q\ ); (\beta_r\ )]\ f(z)\ )' + (1-\mu)\ H_p\ [(\alpha_q\ ); (\beta_r\ )]\ f(z)}\right\} > \delta \qquad (1.10)$$

for some μ $(0 \le \mu < 1)$, δ $(0 \le \delta < 1)$ and for all $z \in U$.

Also, let $C_p(\mu,\delta)$ denote the subclass of T_p consisting of functions which satisfy the condition

$$\mathrm{Re}\left\{\frac{(H_p\ [(\alpha_q\ ); (\beta_r\ )]\ f(z)\ )' + z(H_p\ [(\alpha_q\ ); (\beta_r\ )]\ f(z)\ )''}{(H_p\ [(\alpha_q\ ); (\beta_r\ )]\ f(z)\ )' + \mu\ z\ (H_p\ [(\alpha_q\ ); (\beta_r\ )]\ f(z)\ )''}\right\} > \delta \qquad (1.11)$$

for some μ $(0 \le \mu < 1)$, δ $(0 \le \delta < 1)$ and for all $z \in U$.

From (1.10) and (1.11) it is easy to verify that

$$H_p\ [(\alpha_q\ ); (\beta_r\ )]\ f(z) \in C_p(\mu,\delta) \Leftrightarrow z(H_p\ [(\alpha_q\ ); (\beta_r\ )]\ f(z))' \in T_p(\mu,\delta) \qquad (1.12)$$

It is worthy to note that $T_p\ (0, \delta) \equiv T_p\ (\delta)$, the class of starlike multivalent functions of order δ $(0 \le \delta < 1)$ and $C_p\ (0, \delta) \equiv C_p\ (\delta)$, the class of convex multivalent functions of order δ $(0 \le \delta < 1)$.

For $p = 1$, $q = 1 = \alpha_1, r = 0$ the obtained reduced classes (for univalent functions) $T\ (\mu,\delta)$ and $C\ (\mu,\delta)$ were extensively studied by Altintas and Owa [1] .

Further $T\ (0, \delta) \equiv T^*\ (\delta)$, the class of starlike univalent functions of order δ $(0 \le \delta < 1)$ and $C(0, \delta) \equiv C(\delta)$, the class of convex univalent functions of order δ $(0 \le \delta < 1)$. The classes $T^*\ (\delta)$ and $C\ (\delta)$ were introduced by Silverman [14] and he also obtained coefficient inequalities for these classes.

Let series $S = \sum_{j=0}^{\infty} t_j$ is convergent where $t_j \ge 0$, j = 0, 1, 2,.......

Now , the probability mass function of the generalized distribution is given as

$$p\ (j) = \frac{t_j}{S}\ , \ j = 0,1,2,\ldots\ldots \qquad (1.13)$$

Here p (j) is probability mass function

$p(j) \ge 0$ and $\sum_j p_j = 1$

Now we introduce the series

$$\tau(z) = \sum_{j=0}^{\infty} t_j\ B_p^{(\alpha_q),(\beta_r)}(j)\ z^j \qquad (1.14)$$

where

$$B_p^{(\alpha_q\ ),(\beta_r)}(j) = \frac{(\alpha_1)_j (\alpha_2)_j \ldots\ldots\ldots (\alpha_q)_j}{(\beta_1)_j (\beta_2)_j \ldots\ldots\ldots (\beta_r)_j (j)!} \qquad (1.15)$$

(q ≤ r +1 ; $\alpha_i \in$ R ; $\beta_j \in$ R \ Z_0^-; Z_0^- ={0, -1,- 2,...}; i = 1,..., q ; j = 1,...,r ; z ∈ U)
The series given by (1.14) is convergent for |z| < 1 and for |z| = 1, it is also convergent.
We introduce the generalized distribution series as

$$K_{\tau,p}(z) = z^p + \sum_{j=p+1}^{\infty} \frac{t_{j-p}}{S} z^j \quad (1.16)$$

Further we define

$$T\, K_{\tau,p}(z) = 2\, z^p - K_{\tau,p}(z) = z^p - \sum_{j=p+1}^{\infty} \frac{t_{j-p}}{S} z^j \quad (1.17)$$

Now a convolution operator $T\, K_{\tau,p}(f, z)$ for function f(z) of the form (1.2) as follows

$$T\, K_{\tau,p}(f, z) = T\, K_{\tau,p}(z) * f(z) = z^p - \sum_{j=p+1}^{\infty} |a_j| \frac{t_{j-p}}{S} z^j \quad (1.18)$$

Eminent authors (see [2,3,5,8,9,11,13,14]) have used the hypergeometric functions on different subclasses of analytic functions . In this article, we give the similar conditions for T(s, N, t, z, p) defined by the hypergeometric distribution belong to the $T_p(\mu,\delta)$ and $C_p(\mu,\delta)$.
To characterize our main results, we will require the following theorems:

II. COEFFICIENT INEQUALITIES

Theorem 2.1. A function f(z) of the form (1:2) is in the class $T_p(\mu,\delta)$ ifand only if

$$\sum_{j=p+1}^{\infty} [j - \delta\mu j - \delta + \delta\mu][_p^{(\alpha_q),(\beta_r)}(j-p)]|a_j| \leq [p - \delta\mu p - +\delta\mu] \quad (2.1)$$

The result (2.1) is sharp.

Proof. Suppose that f(z) $\in T_p(\mu,\delta)$. Then we have from (1.10) that

$$\mathrm{Re}\left\{\frac{z(H_p\,[(\alpha_q);\ (\beta_r)]\,f(z))'}{\mu z(H_p\,[(\alpha_q);\ (\beta_r)]\,f(z))' + (1-\mu)\,H_p\,[(\alpha_q);\ (\beta_r)]\,f(z)}\right\} > \delta$$

$$\mathrm{Re}\left\{\frac{p - \sum_{j=p+1}^{\infty} j\,[B_p^{(\alpha_q),(\beta_r)}(j-p)]|a_j||z^{j-p}}{1+\mu(p-1) - \sum_{j=p+1}^{\infty} (\mu j + 1 - \mu) B_p^{(\alpha_q),(\beta_r)}(j-p)|a_j||z^{j-p}}\right\} > \delta \quad (2.2)$$

If we choose z real and let z→ 1^-, we get

$$\mathrm{Re}\left\{\frac{p - \sum_{j=p+1}^{\infty} j\,[B_p^{(\alpha_q),(\beta_r)}(j-p)]|a_j|}{1+\mu(p-1) - \sum_{j=p+1}^{\infty} (\mu j + 1 - \mu)[B_p^{(\alpha_q),(\beta_r)}(j)]|a_j|}\right\} \geq \delta \quad (2.3)$$

which is equivalent to desired result (2.1).

Conversely, suppose that (2.1) holds true. Then, adding

$$-[\,p - \delta\mu p - \delta + \mu\delta]\left[\sum_{j=p+1}^{\infty} (\mu j + 1 - \mu)[B_p^{(\alpha_q),(\beta_r)}(j-p)]|a_j| + \mu(p-1)\right]$$

to both sides of (2.1), we obtain

$$-\mu(p-1)[\,p - \delta\mu p - \delta + \mu\delta] +$$
$$\left[\sum_{j=p+1}^{\infty} j(1 - p\mu + \delta p\mu^2 - \delta\mu^2) - p(1 - \delta\mu - \mu + \delta\mu^2) + (\delta\mu^2 - \delta\mu)\right]\left[B_p^{(\alpha_q),(\beta_r)}(j-p)\right]|a_j| \leq [\,p - \delta\mu p - \delta +$$

$\mu\delta]\left[1+\mu(p-1)-\sum_{j=p+1}^{\infty}(\mu j+1-\mu)[B_p^{(\alpha_q),(\beta_r)}(j-p)]|a_j|\right]$ (2.4)

On the other hand, we see that

$$\left|\frac{z(H_p[(\alpha_q);(\beta_r)]f(z))'}{\mu z(H_p[(\alpha_q);(\beta_r)]f(z))'+(1-\mu)H_p[(\alpha_q);(\beta_r)]f(z)}-[p-\delta\mu p+\delta\mu]\right|$$

$$=\left|\frac{p-\sum_{j=p+1}^{\infty}j[B_p^{(\alpha_q),(\beta_r)}(j-p)]|a_j||z^{j-p}}{1+\mu(p-1)-\sum_{j=p+1}^{\infty}(\mu j+1-\mu)[B_p^{(\alpha_q),(\beta_r)}(j-p)]|a_j||z^{j-p}}-[p-\delta\mu p+\delta\mu]\right|$$

$$\leq\frac{-\mu(p-1)[p-\delta\mu p-\delta+\delta\mu]+\left[\sum_{j=p+1}^{\infty}j(1-p\mu+p\delta\mu^2-\delta\mu^2)-p(1-\mu\delta-\mu+\delta\mu^2)+(\delta\mu^2-\delta\mu)\right][B_p^{(\alpha_q),(\beta_r)}(j-p)]|a_j|}{\left[1+\mu(p-1)-\sum_{j=p+1}^{\infty}(\mu j+1-\mu)[B_p^{(\alpha_q),(\beta_r)}(j-p)]|a_j|\right]}. \quad (2.5)$$

It follows from (2.4) that the last expression in (2.5) is bounded above by

$(p-\delta\mu p-\delta+\delta\mu)$. This implies that $f(z)\in T_p(\mu,\delta)$

Finally, taking the function

$$f(z)=z^p-\frac{(p-\delta\mu p-\delta+\delta\mu)}{(j-\delta\mu j-\delta+\delta\mu)}[B_p^{(\alpha_q),(\beta_r)}(j-p)]z^j \quad (j\geq p+1) \quad (2.6)$$

we can show the result (2.1) is sharp.

Corollary 2.1 If $f(z)\in T_p(\mu,\delta)$, then

$$|a_j|\leq\frac{(p-\delta\mu p-\delta+\delta\mu)}{(j-\delta\mu j-\delta+\delta\mu)}[B_p^{(\alpha_q),(\beta_r)}(j-p)] \qquad (j\geq p+1) \quad (2.7)$$

The equality in (2.7) holds for the function f(z) defined by (2.6).

Corollary 2.2 Taking p=1 , q = $1=\alpha_1, r=0$ the above results get reduced to the results due to Altintas & Owa [1,Theorem 1, pp.43].

Theorem 2.2 A function f(z) defined by (1.2) is in the class $C_p(\mu,\delta)$ if and only if

$$\sum_{j=p+1}^{\infty}j[j-\delta\mu j-\delta+\delta\mu][B_p^{(\alpha_q),(\beta_r)}(j-p)]|a_j|\leq p[p-\delta\mu p-\delta+\delta\mu] \quad (2.8)$$

The result (2.8) is sharp.

Proof. Note that $H_p[(\alpha_q);(\beta_r)]f(z)\in C_p(\mu,\delta)$ if and only if $z(H_p[(\alpha_q);(\beta_r)]f(z))'\in T_p(\mu,\delta)$

Hence replacing a_j by j a_j in Theorem 2.1 , we have the inequality (2.8).

Furthermore , the result (2.8) is sharp for the function

$$f(z)=z^p-\frac{p(p-\delta\mu p-\delta+\delta\mu)}{j(j-\delta\mu j-\delta+\delta\mu)}[B_p^{(\alpha_q),(\beta_r)}(j-p)]z^j \ (j\geq p+1) \quad (2.9)$$

Corollary 2.3 If $f(z)\in C_p(\mu,\delta)$, then

$\left|a_j\right| \leq \frac{p(p-\delta\mu p-\delta+\delta\mu)}{j(j-\delta\mu j-\delta+\delta\mu)}[B_p^{(\alpha_q),(\beta_r)}(j-p)]$ $\quad (j\geq p+1)$ (2.10)

The equality in (2.10) holds for the function f(z) defined by (2.9).

Corollary 2.4 Taking p=1 , q = 1 = α_1, $r = 0$ the above results get reduced to the results due to Altintas & Owa [1, Theorem 2, pp 45].

III. MAIN THEOREMS

Theorem 3.1 Let $T\,K_{\tau,\ p}(z)$ that is of the form (1.17) is in the class $T_p(\mu,\delta)$ iff

$\frac{1}{S}[(1-\delta\mu)\,\tau'(1)+\{p(1-\delta\mu)-\delta(1-\mu)\}\{\tau(1)-\tau(0)\}] \leq [p-\delta\mu p-\delta+\mu\delta]$
(3.1)

Proof. Since we have defined

$T\,K_{\tau,p}(z)= z^p - \sum_{j=p+1}^{\infty}\frac{t_{j-p}}{S}z^j$

according to the Theorem 2.1, we have to show that

$$\sum_{j=p+1}^{\infty}\frac{[j(1-\delta\mu)-\delta(1-\mu)]}{S}\left[B_p^{(\alpha_q),(\beta_r)}(j-p)\right]t_{j-p} \leq p-\delta\mu p-\delta+\delta\mu$$

Now

$$\sum_{j=p+1}^{\infty}\frac{[j(1-\delta\mu)-\delta(1-\mu)]}{S}\left[B_p^{(\alpha_q),(\beta_r)}(j-p)\right]t_{j-p}$$

$=\frac{1}{S}\ [(1-\delta\mu)\sum_{j=p+1}^{\infty}\{(j-p)\}\{B_p^{(\alpha_q),(\beta_r)}(j-p)\}t_{j-p}+p(1-\delta\mu)\sum_{j=p+1}^{\infty}\{B_p^{(\alpha_q),(\beta_r)}(j-p)\}t_{j-p}-\delta(1-\mu)\sum_{j=p+1}^{\infty}\{B_p^{(\alpha_q),(\beta_r)}(j-p)\}t_{j-p}]$

$=\frac{1}{S}[(1-\delta\mu)\sum_{j=1}^{\infty}j\{B_p^{(\alpha_q),(\beta_r)}(j)\}t_j+p(1-\delta\mu)\sum_{j=1}^{\infty}\{B_p^{(\alpha_q),(\beta_r)}(j)\}t_j-\delta(1-\mu)\sum_{j=1}^{\infty}\{B_p^{(\alpha_q),(\beta_r)}(j)\}t_j]$

$=\frac{1}{S}[(1-\delta\mu)\,\tau'(1)+\{p(1-\delta\mu)-\delta(1-\mu)\}\{\tau(1)-\tau(0)\}] \leq [p-\delta\mu p-\delta+\delta\mu]$

Theorem 3.2 Let $T\,K_{\tau,\ p}(z)$ that is of the form (1.17) is in the class $C_p(\mu,\delta)$ iff

$\frac{1}{S}\left[(1-\delta\mu)\,\tau''(1)+(3-2\delta\mu-\delta)\tau'(1)+(1-\delta)\{\tau(1)-\tau(0)\}\right]$
$\leq p[p-\delta\mu p-\delta+\delta\mu]$ (3.2)

Proof: Since

$T\,K_{\tau,\ p}(z)= z^p - \sum_{j=p+1}^{\infty}\frac{t_{j-p}}{S}z^j$

according to the Theorem 2.2, have to prove that

$\sum_{j=p+1}^{\infty}\frac{j[j(1-\delta\mu)-\delta(1-\mu)]}{S}\left[B_p^{(\alpha_q),(\beta_r)}(j-p)\right]t_{j-p} \leq p[p-\delta\mu p-\delta+\delta\mu]$

Now

$$\sum_{j=p+1}^{\infty} \frac{j[j(1-\delta\mu)-\delta(1-\mu)]}{S}\left[B_p^{(\alpha_q),(\beta_r)}(j-p)\right] t_{j-p}$$

$= \quad \frac{1}{S}\Big[(1-\delta\mu)\sum_{j=p+1}^{\infty}(j-1)(j-2)\ \{B_p^{(\alpha_q),(\beta_r)}(j-p)\}\, t_{j-p} + (3-2\delta\mu-\delta)\sum_{j=p+1}^{\infty}(j-1)\{ B_p^{(\alpha_q),(\beta_r)}(j-p)\}\, t_{j-p} + (1-\delta)\sum_{j=p+1}^{\infty}\ \{ B_p^{(\alpha_q),(\beta_r)}(j-p)\}t_{j-p}\ \Big]$

$=\frac{1}{S}\left[(1-\delta\mu)\,\tau''(1)+(3-2\delta\mu-\delta)\tau'(1)+(1-\delta)\{\tau(1)-\tau(0)\}\right]$
$\leq p[p-\delta\mu p-\delta+\delta\mu]$

IV. An Integral Operator

Here we introduce, an integral operator $T\ G_{\tau,p}(z)$ as follows:

$T\ G_{\tau,p}(z) = \mathrm{p}\ \int_0^z \frac{T\ K_{\tau,\ p}(u)}{u}du$ (4.1)

and we get a necessary and sufficient condition for $\mathrm{T\ G_{\tau,p}(z)}$ belonging to the class $C_p(\mu,\delta)$.

Theorem 4.1 If $T\ K_{\tau,p}(z)$ is defined by (1.17) , then $\mathrm{T\ G_{\tau,p}(z)}$ defined by (4.1) is in the class $C_p(\mu,\delta)$ iff (3.1) satisfies.

Proof. Since

$T\ K_{\tau,p}(z) = z^p - \sum_{j=p+1}^{\infty}\frac{t_{j-p}}{S}\, z^j$

$T\ G_{\tau,p}(z) = \mathrm{p}\ \int_0^z \frac{u^p-\sum_{j=p+1}^{\infty}\frac{t_{j-p}}{S}u^j}{u}du$

$= \mathrm{p}[\ \int_0^z u^{p-1}du - \sum_{j=p+1}^{\infty}\int_0^z \frac{t_{j-p}}{S}u^{j-1}\mathrm{du}]$

$= z^p - \sum_{j=p+1}^{\infty}\frac{p}{j\,S}\ t_{j-p}\ \ z^j$

By Theorem (2.2), we have to prove that

$\sum_{j=p+1}^{\infty} j[j(1-\delta\mu)-\delta(1-\mu][B_p^{(\alpha_q),(\beta_r)}(j-p)]\frac{p}{jS}\ t_{j-p} \leq \mathrm{p}[p-\delta\mu p-\delta+\delta\mu]$

or to prove that

$\sum_{j=p+1}^{\infty}[j(1-\delta\mu)-\delta(1-\mu][B_p^{(\alpha_q),(\beta_r)}(j-p)]\frac{1}{S}\ t_{j-p} \leq [p-\delta\mu p-\delta+\delta\mu]$

Now $\sum_{j=p+1}^{\infty}\frac{[j(1-\delta\mu)-\delta(1-\mu)]}{S}\left[B_p^{(\alpha_q),(\beta_r)}(j-p)\right]t_{j-p}$

$= \frac{1}{S}\quad [(1-\delta\mu)\{\sum_{j=p+1}^{\infty}(j-p)\}\quad]\ [\ B_p^{(\alpha_q),(\beta_r)}(j-p)]t_{j-p} + p(1-\delta\mu)\quad \sum_{j=p+1}^{\infty}\quad [\ B_p^{(\alpha_q),(\beta_r)}(j-p)]t_{j-p} - \delta(1-\mu)\sum_{j=p+1}^{\infty}\left[B_p^{(\alpha_q),(\beta_r)}(j-p)\right]t_{j-p}$

$= \frac{1}{S}\,[(1-\delta\mu)\ \sum_{j=1}^{\infty} j\ [\ B_p^{(\alpha_q),(\beta_r)}(j)]t_j + \sum_{j=1}^{\infty}p(1-\delta\mu)\ \ [\ B_p^{(\alpha_q),(\beta_r)}(j)]t_j - \quad \sum_{j=p+1}^{\infty}\delta(1-\mu)\left[B_p^{(\alpha_q),(\beta_r)}(j)\right]t_j$

$= \frac{1}{S}\,[(1-\delta\mu)\tau'(1)+\{p(1-\delta\mu)-\delta(1-\mu)\}\{\tau(1)-\tau(0)\}] \leq [p-\delta\mu p-\delta+\delta\mu]$

Corollary 4.1 Taking p = 1 , q = 1 = α_1, $r = 0$ the results of Theorems 3.1 , 3 .2 and 4.1 get reduced to the results due to Porwal S. [10].

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