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Solving Mathematical Problems Involving Fraction: Understanding The Error Patterns Exhibited By Junior High School Students

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ABSTRACT

Research on instructional strategies that support student learning in mathematical problem-solving is needed as it is an important part of teaching mathematics. This study aims to analyze the Error Pattern Exhibited by Junior High School Students in Solving Mathematical Problems Involving Fraction in one of the biggest schools in the Division of Zamboanga City. This study was voluntarily participated by 333 Grade 7 students. The results revealed This study indicates that students bring with them conceptions and preconceptions to the present Mathematics classroom that was not corrected previously. The teacher's pedagogical skills or competence also contribute to the errors committed by the students. The latter did not receive quality instruction, which could have prevented the errors and their patterns. Moreover, errors were also due to students' lapses in concentration, hasty reasoning, and lack of long-term learning. They appear to understand the concept of fractions at the end of the unit, but could not retain it. The students also had insufficient exposure to word problems. The aforementioned types of error exhibited by the different participants and respondents as analyzed from their scripts adhere to the findings of the studies of Wijaya (2014).

Keywords: error pattern, fraction, mathematical problems, Junior High school, analysis

INTRODUCTION

In recent decades, a lot of progress has been made in research on teaching students how to solve mathematical problems. However, we still need to learn more about how teachers can help their students complete this difficult activity (Lester and Cai, 2016). Results from the Program for Worldwide Understudy Evaluation (PISA) demonstrate the way that just 53% of understudies from the taking an interest nation could tackle issues requiring more than direct derivation and utilizing portrayals from various data sources (OECD, 2019).

In the Philippines, the same thing has happened. Low academic achievement among graduates is one of the most pressing issues facing our educational system today (Lapus, 2006). It was noticed that graduates of elementary and secondary schools did not perform up to standard in mathematics and science. This observation is supported by a recent paper by Paglinawan (2014), which demonstrates a decline in the quality of elementary and secondary education in the Philippines. The National Career Assessment Examination (NCAE) and the National Assessment Test (NAT) have produced lower-than-anticipated results among elementary and high school students. The subject of mathematics typically receives the lowest scores on these tests. During my years as an educator, I discovered that fractions continue to be one of the most challenging mathematical concepts to comprehend and teach. Perchiasamy et al.'s study, 2014) showed that a lot of people don't know how to use basic fractions, especially the ones that involve adding, subtracting, dividing, and multiplying numbers. However, Yea-Ling's (2005) study found that, particularly for students of low ability, solving problems involving fractions was more challenging than solving problems involving decimals or whole numbers. However, it appears that this holds true for almost all students, especially those in the elementary grades. Fragments are "exceedingly difficult for children to master," according to the National Assessment of Educational Progress (NAEP, 2001, p. 5). According to Groff (1996), this may be the reason why students from lower grade levels are unable to recall previous experiences with fractions.

A crucial skill for most occupations is the ability to comprehend and compute fractions. In addition, it is a necessary foundational skill for algebra (National Mathematics Advisory Panel, 2008), a significant application of other areas of mathematics, and a contribution to one's ability to count (Booth and Newton, 2012; Brown and Quinn, 2010; Cinnappan, 2002; and Wu, 2005). Furthermore, Gabriel et al. (2013) learned from teachers that understanding fractions seem essential because they can cause anxiety in math and reduce opportunities for further engagement in science and math.

The National Conference on Teaching Mathematics (NCTM) asserts that teachers also experience frustration when attempting to teach fractions effectively if their students struggle to comprehend the concept. The same frustration is shared by mathematics instructors of the Department of Maritime Education (DME) of ZCSPC, who have to deal with students with weak foundations in math, especially in fractions. This is despite mathematical ability being an essential tool for maritime work. Maritime students, therefore, need to possess a significant degree of mathematical competency and a strong foundation in mathematics that should have already been acquired in the basic education level and not upon entrance in higher education.

It is in this premise that the researcher would like to understand deeper the error pattern Exhibited by Junior High School Students as may be one of the factors in the future curriculum enhancement in the tertiary level.

RELATED LITERATURE AND STUDIES

Solving Mathematical Problems

Natural and man-made situations can both be numerically and spatially quantified using mathematics. It is used to solve problems and has contributed to technological, social, and economic advancements (Dendane, 2009, as cited by Gurat 2018). Phonapichat et al. (2014) expressed that the fundamental motivation behind showing science is to empower understudies to tackle issues in day-to-day existence. The ability to solve mathematical problems is not only a goal of learning mathematics but also something that is very important in daily life and in the workplace (Pinter, 2012); Being able to solve problems can be beneficial (NCTM, 2000). Therefore, learning ought to be developed in a way that teaches students to recognize and address their challenges (Balm, 2009).

In order to solve problems, a lot of math skills were needed. However, many students lack the fundamental math skills they require (Mohd Nizam & Rosaznisham 2004; Berch & Mazzocco 2007). As a result, a lot of students were having trouble with math, especially with solving math problems (Tay Lay Heong 2005; Tarzimah 2005; Mohd Johan 2002; Zalina 2005; Lim See Kiat 1995). According to Miranda (2006), children may have difficulty thinking and learning when they are unable to pay attention, describe the orientation of shape and space, make visual and auditory perceptions, memorize simple things, and comprehend language. As a result, students may encounter difficulties during various stages of the problem-solving process.

Mathematical problem-solving success is contingent on a variety of factors and abilities. The fact that many skills are required for a learner to be an effective problem solver is one of the main obstacles to learning how to solve problems. Additionally, the teaching of problem-solving skills is one of the most challenging subjects to teach (Dendane, 2009). Critical thinking has a unique significance in the investigation of math (Wilson, Fernandez, and Hadaway, 2011). The primary objective of teaching mathematical problem-solving is for students to acquire a general ability to solve real-world problems and to apply mathematics to real-world situations. It can also be used as a teaching strategy to get students to understand concepts better.

There are two aspects to problem-solving: i) whether the problems are presented linguistically (with words) or non-linguistically (with pictures or problems based on them); and ii) the illumination of the problem structure, which includes the information, goal, and action plan (Zhining et al. 1995). Ibrahim (1997) stated that there are two main steps in solving problems: i) Converting the issue into mathematical phrases; and ii) the calculation of the operation that is required to complete the mathematical sentences. Compared to the other steps in the problem-solving process, the first step revealed students' difficulties more clearly. According to Polya (1981), problem-solving is a process that begins when students are presented with a challenge and continue until the issue is resolved. In addition, error analysis revealed that students lacked knowledge of arithmetic and procedure due to a lack of conceptual understanding (Latha, 2007). According to Mohd Johan (2002), many students lacked the ability to plan and carry out the strategies for solving problems and could not give the problems any meaning. However, few studies have focused on the difficulties of math problem-solving caused by a lack of mathematical skills. Better strategies for overcoming obstacles could be developed if the issues with the required mathematical skills are recognized. In addition, students may have difficulty learning mathematics if the teaching and learning methods used did not meet the students' intellectual needs.

According to Bahar and Maker (2015), scientists refer to the idea of solving problems as a high-level thinking process that involves intellectual ability and major cognitive processes. A problem solver can use Polya's (1973) strategy or steps to solve a problem, which states that we must first comprehend the issue at hand; We must comprehend the request. Second, in order to come up with solutions and plan for them, we need to see how things are connected and how the unknown is connected to data. Thirdly, we implement the strategy. Fourth, we look back at the solutions that were found, go over them once more, and we talk about them.

Techniques for Dealing with Mathematical Problems

In order to improve students' ability to solve math problems, a number of studies were carried out. Hoon, Kee, and Singh (2013) investigated how students responded to and were able to use the heuristics approach to solve mathematical problems. In proving this, Reiss and Renkl (2002) suggested using worked-out heuristic examples. They suggested incorporating this frequently into the mathematics classroom to teach students how to extract relevant information from problems. Novotná (2014) aimed to improve the students' culture of problem-solving through the use of strategies like the analogy, guess-check revise, problem reformulation, solution drawing, systematic experimentation, way back, and use of graphs of functions. Koichu, Berman, and Moore (2004) also aimed to promote heuristic literacy in a regular mathematics classroom.

According to Gurat's (2018) findings, student teachers in the Problem-Solving subject employ cognitive, metacognitive, and other problem-solving strategies. The student teachers employ rehearsal, elaboration, and organization as cognitive strategies. Self-regulation and critical thinking are examples of metacognitive strategies. The cognitive and metacognitive strategies overlap with other strategies. Prediction/orientation, planning, monitoring, and evaluation are all examples of these. Additionally, the findings suggest that the strategies have a significant impact on the student teachers' academic performance.

In addition, a study conducted in 2010 by Tambychik and Mohd Meerah revealed that respondents lacked numerous mathematics skills, including information, visual-spatial, and number-fact skills. The most crucial skill was information literacy. Problem-solving in mathematics is hampered by a lack of these mathematical skills and cognitive abilities. It is anticipated that this comprehension of how the deficits influenced problem-solving will provide useful guidelines for the preparation of diagnostic instruments and learning modules to develop mathematics skills.

Difficulty with Learning Fractions

The fundamental differences between whole numbers and fractions have traditionally been cited as the source of difficulty with fractions. Whole-number bias, or students' overgeneralization of whole number knowledge to fractions, can result from this (DeWolf & Vosniadou, 2015; Ni & Zhou, 2005). Because of the inherent differences between whole numbers and fractions, students assimilate whole number concepts into understanding fractions, resulting in misconceptions about fractions. One numeral is used to represent whole numbers, while two numerals and a fraction bar are used to represent fractions. Students frequently mistakenly interpret a fraction as a single number when they view its numerators and denominators as distinct whole numbers due to whole-number bias. Common mistakes like adding the numerators and denominators of two fractions are often caused by this.
According to Booth & Newton (2012), mastery of fractions is essential for mastering more complex mathematical concepts like algebra. However, many students struggle to attain competency with fractions, and the difficulties associated with learning fractions have been extensively documented (e.g., NMAP, 2008; 2008; Nunes & Bryant;). According to the 2017 NAEP, only 32% of fourth graders correctly identified which fractions were greater than, less than, or equal to a benchmark fraction, 1/2. Only 25% of fourth graders correctly identified a fraction that was closest to 1/2 on the 2009 NAEP. The performance on the two NAEP fraction-related items demonstrates that difficulty with fractions is not new; it lasts for a long time.

Students who struggle to learn mathematics may find it particularly challenging to master fractions. According to the findings of Namkung, Fuchs, and Koziol (2018), students who had difficulty learning mathematics in fourth grade and had whole-number competence below the 10th percentile were 32 times more likely to struggle with fractions than students who had intact whole-number knowledge. Students with intact whole number knowledge were five times more likely than students with less severe mathematics learning difficulties (between the 10th and 25th percentile) to struggle with fractions.

RESEARCH QUESTION

1.What are the underlying causes of mathematical errors involving fractions?

METHODOLOGY

The study employed a qualitative descriptive research design. Leary (2010) defines such design as one that includes all studies that purport to present facts concerning the nature and status of anything. This design is appropriate for the study since it seeks to provide a description of the conditions or errors of the respondents. Furthermore, the data collection involved a diagnostic examination. Interpretation was based on a combination of researcher and facilitator’s perspective and data collected with the aid of Newman’s Error Analysis Technique (NEA). This approach is apt for this study since it makes use of qualitative techniques to analyze errors. The population of this study was composed of Grade 7 students of the Zamboanga City National High School-West enrolled in the school year 2016-2017. The grade level has a total population of 1,997. Using Slovin’s formula, 333 participants were chosen to take the diagnostic test.

Table 1. Demographic Profile of Participants in Terms of Sex and Age

Demographics	Participants	
Sex	F	%
Male	130	39
Female	203	61
Total	333	100
Age	F	%
11 years old and below	18	5
12-15 years old	283	85
16-19 years old	30	9
20 years old and above	2	1
Total	333	100

For purposes of coding, each of the participants was assigned a code number. The first name that was picked was given a code number of Participant 1, the next Participant 2 and so on. As part of this study's ethical considerations, the researcher explained to the participants, in the presence of the mathematics teachers and guidance counselor, the purpose of the study and gave them the assurance that their identities shall remain confidential and that the results of the diagnostic examination will not affect their grades. The students agreed to participate in the study.

To answer the research question, the researcher utilized the Newman Error Analysis/Interview Protocol adopted by White (2005) and was used during the interview. The interview had 5 basic questions which correspond to the five possible mathematical error types, namely: reading error, comprehension error, transformation error; procedural or basic fact error; and encoding error. Also included were the causes of errors as proposed by Wijaya (2014). The instrument also has 5 basic questions, each of which correspond to five error types. The protocol is not a very structured tool. The researcher and the facilitators were free to ask probing or follow-up questions during the interview based on their observations and the responses of respondents. In addition, the Data Analysis Rubric was used to determine the prevalent error types and the causes of errors. It was adopted from Wijiya (2014). The rubric has three (3) components which are error types, causes of error and the explanation/s for each error.

RESULTS AND DISCUSSIONS

To further analyze the errors committed by the participants, the researcher looked into the error patterns in the solution transcripts. The solution transcript was chosen based on the type of error that was most prevalent in problem number 1, that is, transformation error; then, followed by procedural or basic fact error.

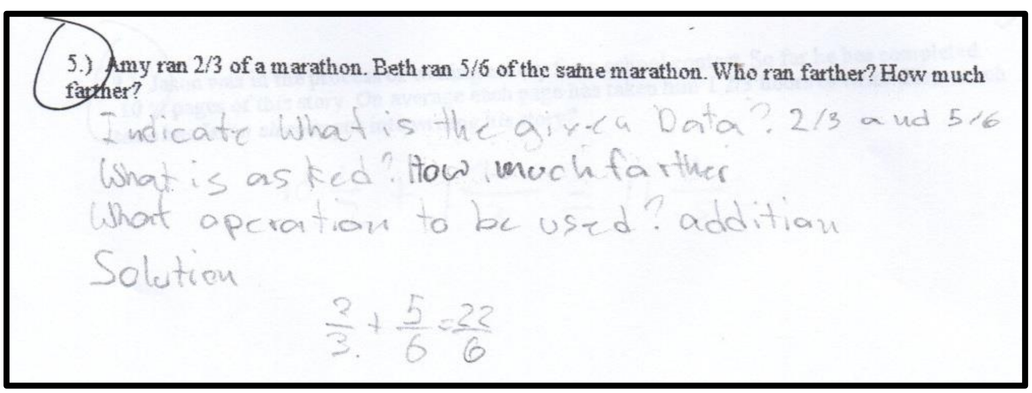

Figure 1. Solution of participant 203 to problem no. 5 illustrating transformation error in subtraction

Figure 1 shows that participant 203 was able to identify the given data, that is 2/3 that Amy ran in a marathon and 5/6 that Beth ran. The participant was also able to identify what was asked in the problem; he chose the wrong operation which is addition when the correct operation should have been subtraction. The participant also understood that he had to look for the lowest common denominator and correctly chose 6. However, he was not able to show how he arrived at 22 as the numerator of his final answer. He tended to use a mathematical procedure without analyzing whether or not it is needed. The participant, therefore, understood what the problem required but was unable to choose the right operation to solve the problem. This error is an error of the following kind: transformation error (Wijaya, 2014) and as quoted also in the study of White (2005), transformation error, which occurs when a learner understands what the problem wants him/her to find out but is unable to identify the appropriate operation or sequence of operations needed to solve the problem; procedural or basic fact error, which occurs when a learner is able to identify the appropriate operation or sequence of operations but does not know the necessary procedures to carry out the operations accurately.

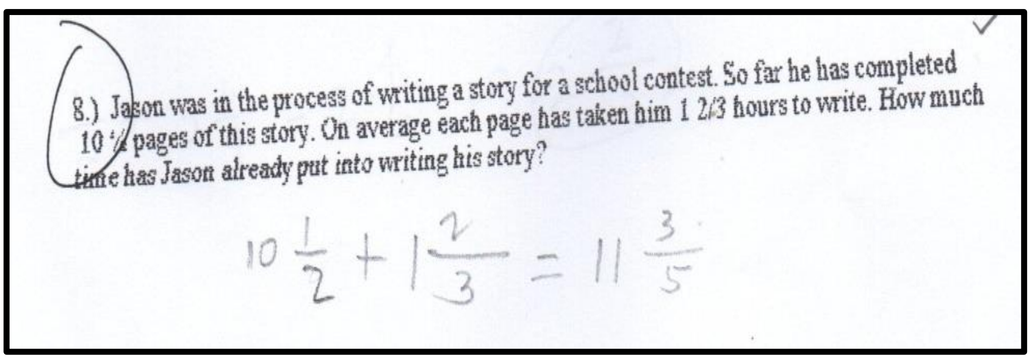

Figure 2. A solution of participant 205 to problem no. 8 illustrating transformation error in multiplication

Figure 2 illustrates that participant 205 was able to read the mathematical symbols contained in the problem, that is why she was able to identify 10 1/2 pages and 1 and 2/3 hours. However, she failed to see what was asked. She failed to see that 10 1/2 refers to the number of pages while 1 2/3 refers to time. She simply added the two numbers which led to the wrong because she did not know how to perform the operation.

Since she did not understand the problem, she used the wrong mathematical operations. Failure to choose the right operation is indicative of a problem in a comprehension and also of transformation error.

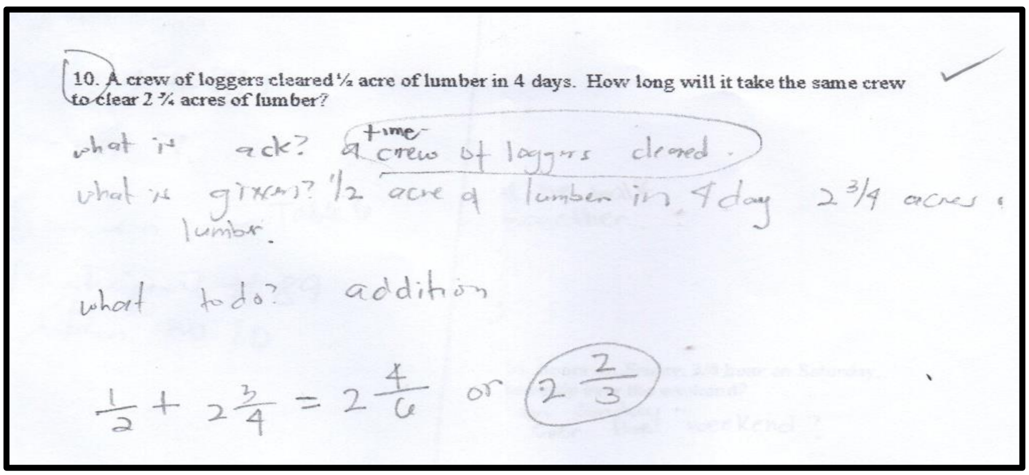

Figure 3. Solution of participant 1 to problem no. 10 illustrating transformation error in division

Figure 3 reveals that Participant 1 was unable to identify the right operation in solving Problem number 10. She was able to indicate what was given and what the problem was asking for, but in transforming the word problem into a mathematical equation, she chose the wrong operation, which is addition instead of division. This error type is classified as transformation error. In the case of participants who committed procedural or basic facts error, it is evident that they often have a problem implementing the correct operation, usually by missing a step or steps in the operation or using a step not associated with any operation. Figures 5 to 8 illustrate this.

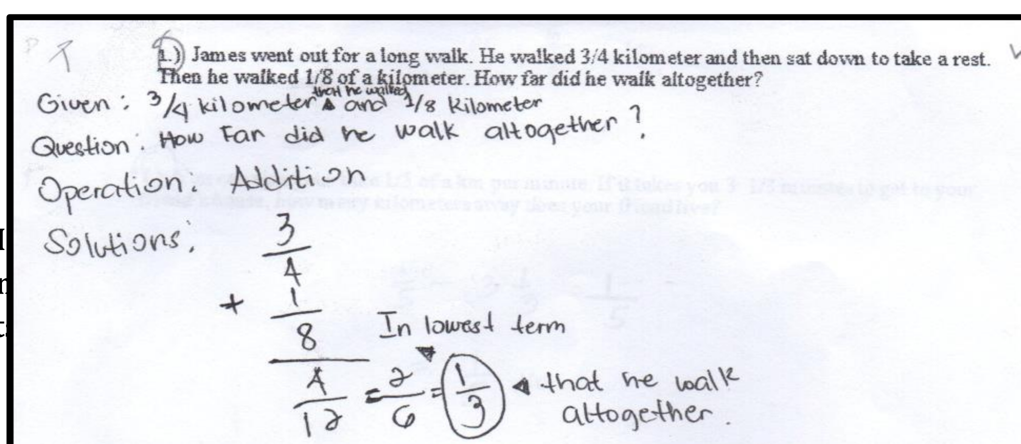

Figure 4. Solution of participant 6 to problem no. 1 illustrating procedural or basic facts error in addition

Figure 4 demonstrates how Participant 6 committed procedural or basic facts error involving the addition of fractions in her solution to Problem number 1. The participant was able to read and comprehend the mathematical symbols and terms, as well as identify the relevant information needed to solve the problem which was: ¾ kilometer and 1/8 kilometer. She was also able to determine what was asked by the problem and stated this in the test paper, "How far did he walk altogether?" Likewise, she was able to identify the appropriate operation which was addition, however, she failed to find the least common denominator which was 8. The equation then would have looked like this: 6/8 + 1/8 instead of this: 3/4 + 1/8. The wrong equation led her to arrive at the incorrect answer of 1/3 which should have been 7/8. Missing a step in the operation is a procedural or basic facts error.

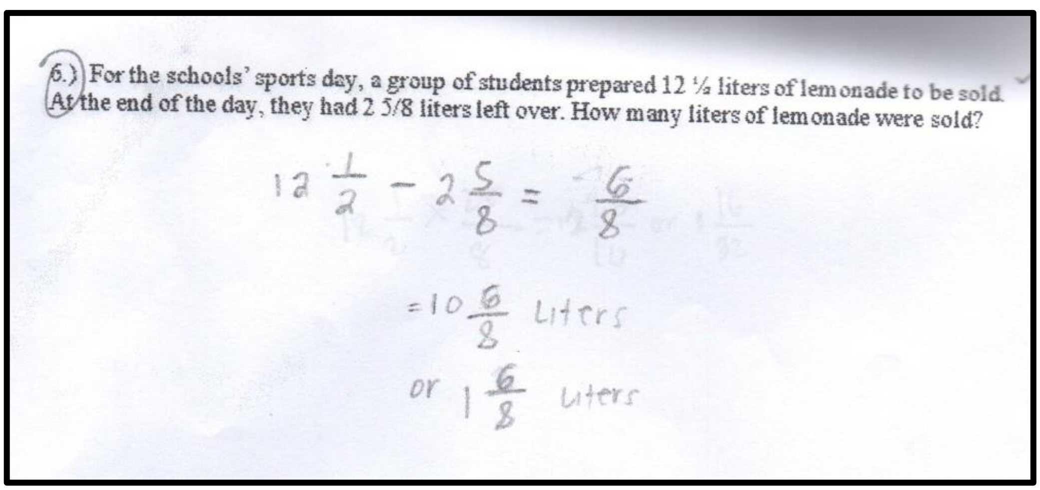

Figure 5. Solution of participant 211 to problem no. 6 illustrating procedural or basic facts error in subtraction

Figure 5 shows how Participant 211 committed procedural or basic facts error in solving Problem number 6 which requires the subtraction of fractions. Participant 211 obviously had no problem with reading and comprehending the problem and with identifying what was asked as well as the relevant information needed to solve the problem. The participant also identified the appropriate mathematical operation to be used which was subtraction. She was able to identify 12 ¼ liters of lemonade as the minuend and 2 5/8 the subtrahend. The error committed by the participant was in not being able to change the mixed number into fractions. She immediately proceeded to add the numerators and simply used the denominator of the subtrahend. In other words, the participant committed an error in calculation due to missed steps; changing the mixed number to improper fraction and finding the least common denominator. Likewise, she used a step that was not associated with any operation. Missing steps in the operation and using steps that are not associated with any operation is a procedural or basic facts error according to the classification of error based on the Data Analyses Rubric of Wijaya (2014) as profounded by White (2005).

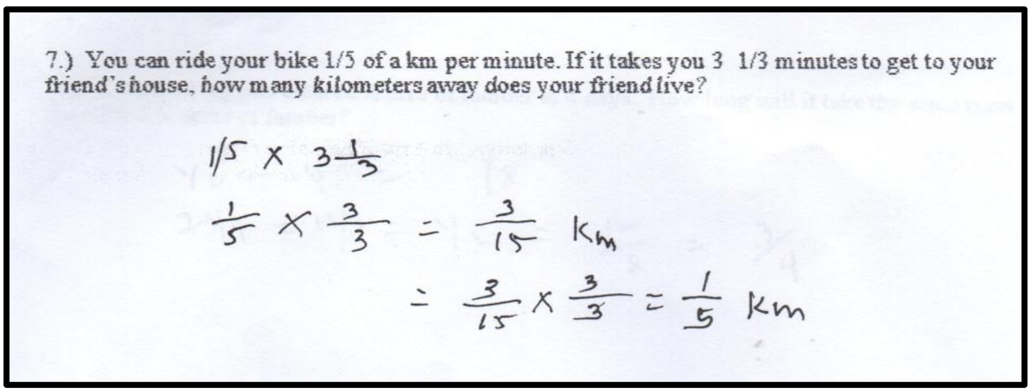

Figure 6. Solution of participant 317 to problem no. 7 illustrating procedural or basic facts error in multiplication

As shown in Figure 6, Participant 317 committed procedural or basic facts error in solving Test Item number 7. The participant read and understood the problem, was able to identify the relevant information needed to solve the problem which is 1/5 of a kilometer per minute and 3 1/3 minutes to get to a friend's house. The participant also identified the right mathematical operation which was multiplication, however, in proceeding with the multiplication, missed a step which was changing a mixed number to an improper fraction. What could have been done was to change 3 1/3 to 10/3 then multiply. Failing to do this led him/her to arrive at the wrong answer of 1/5 kilometer instead of 2/3. Missing steps in the operation, as committed by Participant 317 is a procedural or basic fact error based on the classification of Wijaya (2014).

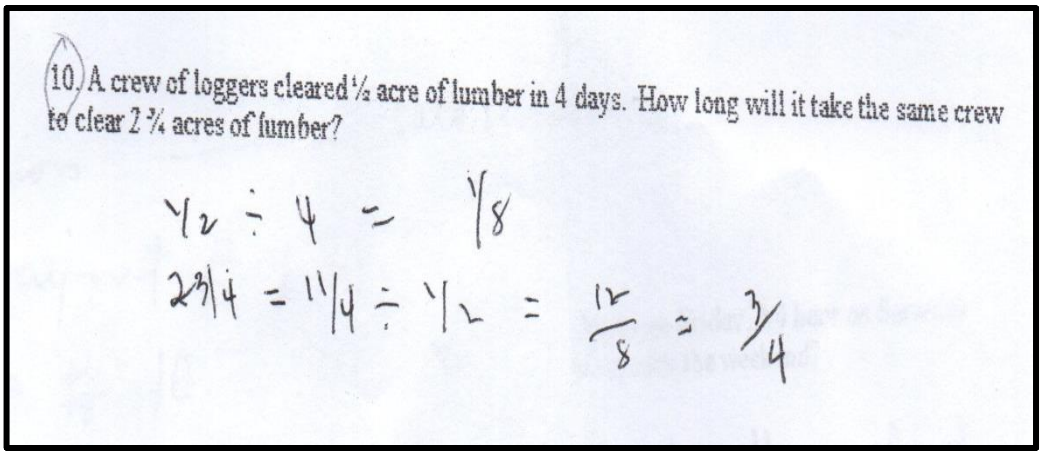

Figure 7. Solution of participant 89 to problem no.10 illustrating procedural or basic facts error in division

Figure 7 indicates that Participant 89 was able to identify what was given by and asked of the problem. She correctly chose the operation for Problem number 10, which is division. She intended to divide ½ by 4 but actually multiplied it by 4 which generated the answer 1/8. The participant did not invert the divisor. She changed the mixed fraction, 2 ¾ into improper fraction 11/4, then divided it by ½ which is 12/8 then reduced it to lowest term = ¾. As can be observed from the solution, she has mixed notions on dividing fractions. She lacks understanding of the reciprocal and the invert-and-multiply procedure. Having used steps not associated with the operation, she committed procedural or basic facts error.

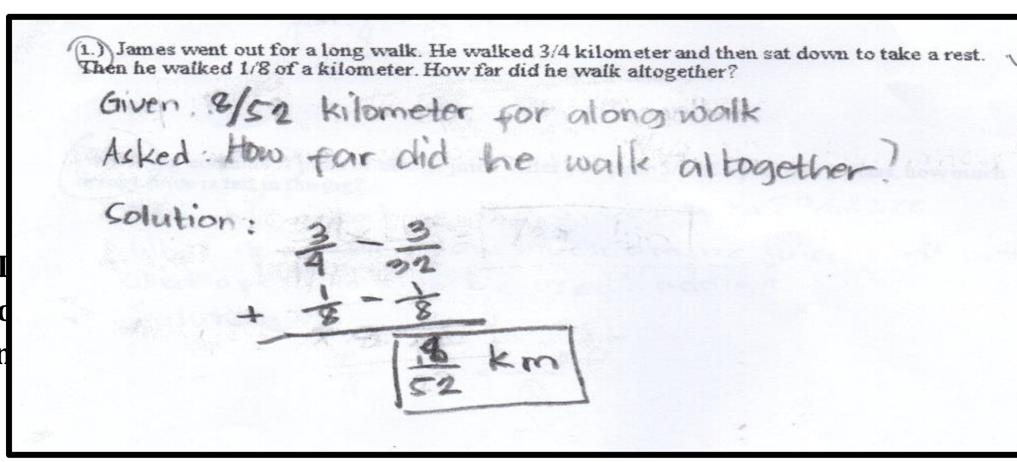

Figure 8. Solution of participant 5 to problem no. 1 illustrating comprehension error in addition

Figure 8 indicates that participant understood what was asked, but did not understand what was given in the problem. He assumed 8/52 and confused the given and the procedure. Thus, symbols were just placed together with no correlation since there is nothing in the problem that suggests subtracting 3/32 from 3/4. The answer, 3/32, seemed to come from nowhere. The plus sign meant that the participant was able to identify the right operation, but was unable to arrive at an answer. This type of error is classified in the Data Analysis Rubric adopted from Wijaya (2014) as comprehension error.

CONCLUSION

This study concludes that students faced difficulties in mathematic problem solving due to low competency in acquiring many mathematics skills and lacking in cognitive abilities of learning. Information skill was found to be the most critical mathematics skills. Although students acquired other mathematics skills, without the transfer of information skill, they could not understand and make effective connection of the information in the problems. This study indicates that students bring with them conceptions and preconceptions to the present Mathematics classroom that was not corrected previously. The teacher's pedagogical skills or competence also contribute to the errors committed by the students. The latter did not receive quality instruction, which could have prevented the errors and their patterns. Moreover, errors were also due to students' lapses in concentration, hasty reasoning, and lack of long-term learning. They appear to understand the concept of fractions at the end of the unit, but could not retain it. The students also had insufficient exposure to word problems. The aforementioned types of error exhibited by the different participants and respondents as analyzed from their scripts adhere to the findings of the studies of Wijaya (2014),

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