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Double Interpolation Method For Numerical Solution Of A One-Dimensional Heat Conduction Problem

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Abstract-
Utilizing the processes of mathematical methods and the limited distinctions, an attempt is performed in this piece to locate the configuration of a one-layered Heat condition complete with starting and limit conditions. We determined the values of $V(x, t)$at various cross section locations by applying the Bender-Schmidt repeat connection equation. We went on to make use of the double interpolation and identified the organization of the heat condition as a double interpolating polynomial. We then represented the solution graphically.

Keywords-Heat conduction equation, Bender Schmidt formula, Interpolation, Boundary value problem

1. Introduction-

Our economy will suffer as a direct result of the expansion of armed conflict in the Catatumbo neighbourhood. The current situation, which is reflected in the high rates of homelessness, demonstrates the necessity of putting forward solutions to this problem. It would be fascinating for the residents of Catatumbo, Colombia, to see government assistance produced as a result of the advancement of exploration revolving around new technological developments. The investigation of power-free devices is an important step in this direction, particularly in regions experiencing a poor turn of events financially. At the moment, one source of growing revenue is the investigation of different cooling methods and the effect that these techniques have on better locations.

Iyengar and Manohar (1988) applied the fourth-request distinction strategy for the purpose of arranging Poisson's situation in barrel-shaped facilitates. They expanded the method to settle the heat condition in two-layered configurations with polar directions and three-layered configurations with barrel-shaped arranges. Marwah and Chopra (1992) presented an absolutely remarkable scientific methodology that was utilized to determine the transient intensity conduction condition in a one-layered empty compound chamber with time-subordinate limit conditions. According to Sabaeian et al. (2008), the work on temperature conveyance is essential in the estimation, reenactment, and forecasting of warm impacts. Ciegis et al. (2010) create and validate numerical models as well as mathematical calculations for the purpose of recreating intensity movement in composite materials. Javed (2012) gained knowledge regarding sources of dry or wet intensity. The term "dry applications" refers to items such as electric cushions, boiling water bottles, and brilliant intensity. It is commonly believed that wet intensity is more penetrating than dry intensity; however, this perception is more likely due to the fact that water-soaked materials lose heat at a slower rate than dry ones. Shiferaw and Mittal (2013) took on the challenge of solving a three-layered Poisson's condition using a limited contrast strategy in a barrel-shaped organizing structure. When performing a mathematical reenactment of 2D convection-dispersion in barrel-shaped facilitates, Mori and Romo (2015) used the limited distinction technique. The Forward Time Central Space Scheme (FTCSS) was inferred for the intensity condition by Kafle et al. (2020). They also determined its mathematical arrangement by utilizing FTCSS, and they investigated the logical arrangement as well as the mathematical answer for a variety of homogeneous materials (for a variety of advantages of diffusivity α). Khatun et al. (2020) presented the findings of their investigation into the safety of a one-layered heat condition. A mathematical method was suggested by Maturi et al. (2020) in order to determine the correct answer for the intensity conduction condition of copper. They zeroed in on copper because it is exceptionally well suited to take the lead in terms of heat and electrical conductivity. For the purpose of addressing the two-layered Schrodinger condition in polar directions, Salehi and Granpayeh (2020) presented a limited distinction method as a potential solution. In one respect, Meyu and Koriche (2021) presented the essential treatment of the arrangement of intensity condition. Tsega et al. (2022) took on the challenge of a three-layered transient intensity conduction condition. They did this by approximating second-request spatial subordinates with five focal contrasts in tube-shaped facilitates. In addition to that, he addressed the matter of the strength condition.

2. Formulation of the Problem:

In this discussion, we will focus on the following boundary value problem associated with the one-dimensional heat equation. A partial differential equation (PDE) that is frequently defined by may tell you how hot or cold a rod is

$$\frac{\partial V}{\partial t} = K^2 \frac{\partial^2 V}{\partial t^2} \qquad (1)$$

Where $V(x, t)$ is the temperature of the pole estimated at position x at time t, and K is the warm diffusivity of the material, which estimates how well the bar can lead heat,

Dependent upon the accompanying limit conditions

$$V(0, t) = 0 \qquad (2)$$

$V(1, t) = 0$ (3)

$V(x, 0) = \sin\pi x$ (4)

Where $0 \leq x \leq 1$ and $t > 0$

For the solution of this problem, Let we take $K^2 = 1$

Analytic Solution of the above problem is

$V(x, t) = \sin\pi x \, e^{K^2\pi^2 t}$

The interval of differencing of x as 0.2 i.e. $h = 0.2$

From Bender Schmidt equation, the time span of t as

$$k = \frac{h^2}{2c^2} = \frac{(0.2)^2}{2} = 0.02 \quad (5)$$

Thus $x_0 = 0, x_1 = 0.2, x_2 = 0.4, x_3 = 0.6, x_4 = 0.8, x_5 = 1$

$t_0 = 0, t_1 = 0.02, t_2 = 0.04, t_3 = 0.06, t_4 = 0.08, t_5 = 0.1$

We have a total of 25 mesh points after drawing straight lines parallel to the coordinate axis (t, x).

x t	0	0.2	0.4	0.6	0.8	1
0	0	0.59	0.95	0.95	0.59	0
0.02	0	0.475	0.77	0.77	0.475	0
0.04	0	0.385	0.6225	0.6225	0.385	0
0.06	0	0.3113	0.504	0.504	0.3113	0
0.08	0	0.408	0.252	0.252	0.408	0
0.1	0	0.204	0.33	0.33	0.204	0

Table-1

S. No.	V_{1i}	$\Delta^{0+1}V_{1i}$	$\Delta^{0+2}V_{1i}$	$\Delta^{0+3}V_{1i}$	$\Delta^{0+4}V_{1i}$	$\Delta^{0+5}V_{1i}$
1	0.59	-0.115	0.025	-0.0087	0.1628	-0.788
2	0.475	-0.09	0.0163	0.1541	-0.6252	
3	0.385	-0.0737	0.1704	-0.4711		
4	0.3113	0.0967	-0.3007			
5	0.408	-0.204				
6	0.204					

Table-2

S. No.	V_{2i}	$\Delta^{0+1}V_{2i}$	$\Delta^{0+2}V_{2i}$	$\Delta^{0+3}V_{2i}$	$\Delta^{0+4}V_{2i}$	$\Delta^{0+5}V_{2i}$
1	0.95	-0.18	0.0325	-0.0035	-0.159	0.785
2	0.77	-0.1475	0.029	-0.1625	0.626	
3	0.6225	-0.1185	-0.1335	0.4635		
4	0.504	-0.252	0.33			
5	0.252	0.078				
6	0.33					

Table-3

S. No.	V_{3i}	$\Delta^{0+1}V_{3i}$	$\Delta^{0+2}V_{3i}$	$\Delta^{0+3}V_{3i}$	$\Delta^{0+4}V_{3i}$	$\Delta^{0+5}V_{3i}$
1	0.95	-0.18	0.0325	-0.0035	-0.159	0.785
2	0.77	-0.1475	0.029	-0.1625	0.626	
3	0.6225	-0.1185	-0.1335	0.4635		
4	0.504	-0.252	0.33			
5	0.252	0.078				
6	0.33					

Table-4

S. No.	V_{4i}	$\Delta^{0+1}V_{4i}$	$\Delta^{0+2}V_{4i}$	$\Delta^{0+3}V_{4i}$	$\Delta^{0+4}V_{4i}$	$\Delta^{0+5}V_{4i}$
1	0.59	-0.115	0.025	-0.0087	0.1628	-0.788
2	0.475	-0.09	0.0163	0.1541	-0.6252	
3	0.385	-0.0737	0.1704	-0.4711		
4	0.3113	0.0967	-0.3007			
5	0.408	-0.204				
6	0.204					

Table-5 Since both the First and the Last Column of Table 1 contain 0, this means that

$$\Delta^{0+1}V_{00} = \Delta^{0+2}V_{00} = \Delta^{0+3}V_{00} = \Delta^{0+4}V_{00} = \Delta^{0+5}V_{00} = 0 \quad (6)$$

And $\Delta^{0+1}V_{50} = \Delta^{0+2}V_{50} = \Delta^{0+3}V_{50} = \Delta^{0+4}V_{50} = \Delta^{0+5}V_{50} = 0$ (7)

From Table 2, we get

$$\Delta^{0+1}V_{10} = -0.115, \Delta^{0+2}V_{10} = 0.025, \Delta^{0+3}V_{10} = -0.0087, \Delta^{0+4}V_{10} = 0.1628, \Delta^{0+5}V_{10} = -0.788 \quad (8)$$

From Table 3,

$$\Delta^{0+1}V_{20} = -0.18, \Delta^{0+2}V_{20} = 0.0325, \Delta^{0+3}V_{20} = -0.0035, \Delta^{0+4}V_{20} = -0.159, \Delta^{0+5}V_{20} = 0.785 \quad (9)$$

From Table 4,

$$\Delta^{0+1}V_{30} = -0.18, \Delta^{0+2}V_{30} = 0.0325, \Delta^{0+3}V_{30} = -0.0035, \Delta^{0+4}V_{30} = -0.159, \Delta^{0+5}V_{30} = 0.785 \quad (10)$$

From Table 5

$$\Delta^{0+1}V_{40} = -0.115, \Delta^{0+2}V_{40} = 0.025, \Delta^{0+3}V_{40} = -0.0087, \Delta^{0+4}V_{40} = 0.1628, \Delta^{0+5}V_{40} = -0.788 \quad (11)$$

After carrying out the procedure described above for each row in table 1, we have

$$\Delta^{1+0}V_{00} = 0.59, \Delta^{2+0}V_{00} = -0.23, \Delta^{3+0}V_{00} = -0.13, \Delta^{4+0}V_{00} = 0.13, \Delta^{5+0}V_{00} = 0 \quad (12)$$

$$\Delta^{1+0}V_{01} = 0.475, \Delta^{2+0}V_{01} = -0.18, \Delta^{3+0}V_{01} = -0.115, \Delta^{4+0}V_{01} = 0.115, \Delta^{5+0}V_{01} = 0 \quad (13)$$

$$\Delta^{1+0}V_{02} = 0.385, \Delta^{2+0}V_{02} = -0.1475, \Delta^{3+0}V_{02} = -0.09, \Delta^{4+0}V_{02} = 0.09, \Delta^{5+0}V_{02} = 0 \quad (14)$$

$$\Delta^{1+0}V_{03} = 0.3113, \Delta^{2+0}V_{03} = -0.1186, \Delta^{3+0}V_{03} = -0.0741, \Delta^{4+0}V_{03} = 0.0741, \Delta^{5+0}V_{03} = 0 \quad (15)$$

$$\Delta^{1+0}V_{04} = 0.408, \Delta^{2+0}V_{04} = -0.564, \Delta^{3+0}V_{04} = 0.72, \Delta^{4+0}V_{04} = -0.72, \Delta^{5+0}V_{04} = 0 \quad (16)$$

$$\Delta^{1+0}V_{05} = 0.204, \Delta^{2+0}V_{05} = -0.078, \Delta^{3+0}V_{05} = -0.048, \Delta^{4+0}V_{05} = 0.048, \Delta^{5+0}V_{05} = 0 \quad (17)$$

The formula for determining the differences between two orders can be expressed generally as

$$\Delta^{m+n}V_{00} = \Delta^{m+0}V_{0n} - n\Delta^{m+0}V_{0n-1} + \frac{n(n-1)}{2!}\Delta^{m+0}V_{0n-2} - \cdots + (-1)^m\Delta^{m+0}V_{00} \quad (18)$$

$$\Delta^{n+m}V_{00} = \Delta^{0+n}V_{m0} - m\Delta^{0+n}V_{m-10} + \frac{m(m-1)}{2!}\Delta^{0+n}V_{m-20} - \cdots + (-1)^m\Delta^{0+n}V_{00} \quad (19)$$

$$\Delta^{1+1}V_{00} = \Delta^{1+0}V_{01} - \Delta^{1+0}V_{00} = 0.475 - 0.59 = -0.1150 \quad (20)$$

$$\Delta^{1+2}V_{00} = \Delta^{1+0}V_{02} - 2\Delta^{1+0}V_{01} + \Delta^{1+0}V_{00} = 0.385 - 2 \times 0.475 + 0.59 = 0.0250 \quad (21)$$

$$\Delta^{2+1}V_{00} = \Delta^{2+0}V_{01} - \Delta^{2+0}V_{00} = -0.18 - (-0.23) = -0.1150 \quad (22)$$

$$\Delta^{3+1}V_{00} = \Delta^{3+0}V_{01} - \Delta^{3+0}V_{00} = -0.115 - (-0.13) = 0.0150 \quad (23)$$

$$\Delta^{1+3}V_{00} = \Delta^{1+0}V_{03} - 3\Delta^{1+0}V_{02} + 3\Delta^{1+0}V_{01} - \Delta^{1+0}V_{00} = 0.3113 - 3 \times 0.385 + 3 \times 0.475 - 0.59 = -0.0087 \quad (24)$$

$$\Delta^{2+2}V_{00} = \Delta^{2+0}V_{02} - 2\Delta^{2+0}V_{01} + \Delta^{2+0}V_{00} = -0.1475 - 2 \times (-0.18) + (-0.23) = -0.0175$$

$$\Delta^{1+4}V_{00} = \Delta^{1+0}V_{04} - 4\Delta^{1+0}V_{03} + 6\Delta^{1+0}V_{02} - 4\Delta^{1+0}V_{01} + \Delta^{1+0}V_{00} \quad (25)$$

$$= 0.408 - 4 \times 0.3113 + 6 \times 0.385 - 4 \times 0.475 + 0.59 = 0.1628$$

$$\Delta^{4+1}V_{00} = \Delta^{4+0}V_{01} - \Delta^{4+0}V_{00} = 0.115 - 0.13 = -0.0150 \quad (26)$$

$$\Delta^{3+2}V_{00} = \Delta^{3+0}V_{02} - 2\Delta^{3+0}V_{01} + \Delta^{3+0}V_{00} = -0.09 - 2 \times (-0.115) + (-0.13) = 0.0100 \quad (27)$$

$$\Delta^{2+3}V_{00} = \Delta^{2+0}V_{03} - 3\Delta^{2+0}V_{02} + 3\Delta^{2+0}V_{01} - \Delta^{2+0}V_{00}$$

$$= -0.1186 - 3 \times (-0.1475) + 3 \times (-0.18) - (-0.23) = 0.0139 \quad (28)$$

Interpolating polynomials in two variables up to the difference of the fifth degree requires the following formula:

$$V(x,t) =$$

$$V_{00} + \left[\frac{(x-x_0)}{h}\Delta^{1+0}V_{00} + \frac{(t-t_0)}{k}\Delta^{0+1}V_{00}\right]$$

$$+\frac{1}{2!}\left[\frac{(x-x_0)(x-x_1)}{h^2}\Delta^{2+0}V_{00} + \frac{2(x-x_0)(t-t_0)}{hk}V_{00} + \frac{(t-t_0)(t-t_1)}{k^2}\Delta^{0+2}V_{00}\right]$$

$$+\frac{1}{3!}\left[\frac{(x-x_0)(x-x_1)(x-x_2)}{h^3}\Delta^{3+0}V_{00} + \frac{3(x-x_0)(x-x_1)(t-t_0)}{h^2k}\Delta^{2+1}V_{00} + \frac{3(x-x_0)(t-t_0)(t-t_1)}{hk^2}\Delta^{1+2}V_{00} + \frac{(t-t_0)(t-t_1)(t-t_2)}{k^3}\Delta^{0+3}V_{00}\right]$$

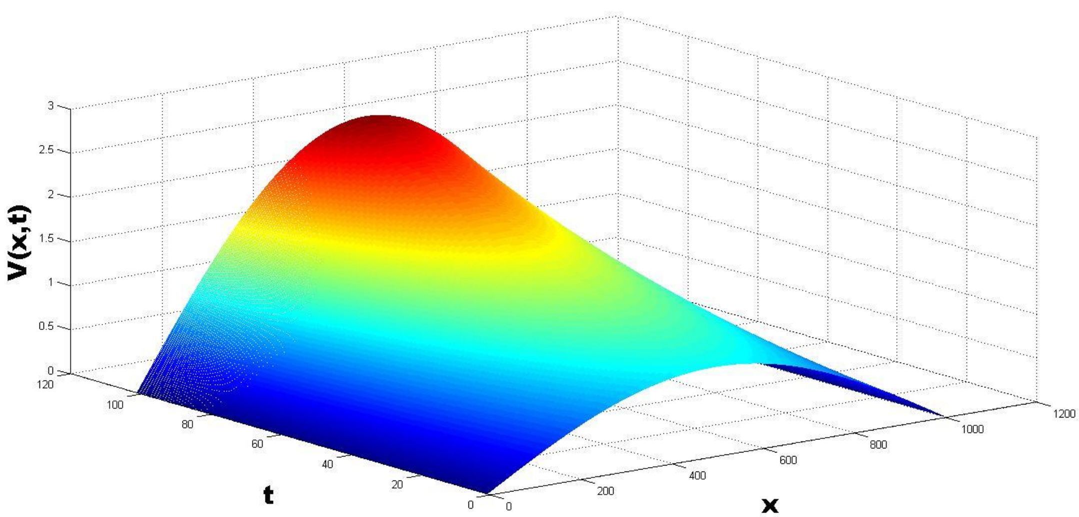
$$+\frac{1}{4!}\left[\frac{(x-x_0)(x-x_1)(x-x_2)(x-x_3)}{h^4}\Delta^{4+0}V_{00}+\frac{4(x-x_0)(x-x_1)(x-x_2)(t-t_0)}{h^3k}\Delta^{3+1}V_{00}+\frac{6(x-x_0)(x-x_1)(t-t_0)(t-t_1)}{h^2k^2}\Delta^{2+2}V_{00}+\frac{4(x-x_0)(t-t_0)(t-t_1)(t-t_2)}{hk^3}\Delta^{1+3}V_{00}+\frac{(t-t_0)(t-t_1)(t-t_2)(t-t_3)}{k^4}\Delta^{0+4}V_{00}\right]$$

$$+\frac{1}{5!}\left[\frac{(x-x_0)(x-x_1)(x-x_2)(x-x_3)(x-x_4)}{h^5}\Delta^{5+0}V_{00}+\frac{5(x-x_0)(x-x_1)(x-x_2)(x-x_3)(t-t_0)}{h^4k}\Delta^{4+1}V_{00}+\frac{10(x-x_0)(x-x_1)((x-x_2)(t-t_0)(t-t_1)}{h^3k^2}\Delta^{3+2}V_{00}+\frac{10(x-x_0)(x-x_1)(t-t_0)(t-t_1)(t-t_2)}{h^2k^3}\Delta^{2+3}V_{00}+\frac{5(x-x_0)(t-t_0)(t-t_1)(t-t_2)(t-t_3)}{hk^4}\Delta^{1+4}V_{00}+\frac{(t-t_0)(t-t_1)(t-t_2)(t-t_3)(t-t_4)}{k^5}\Delta^{0+5}V_{00}\right] \tag{29}$$

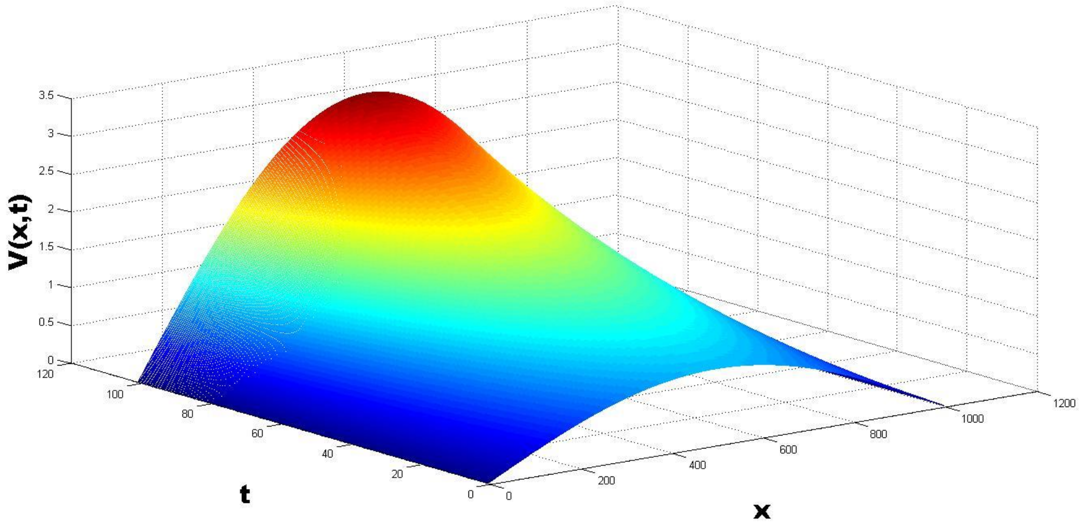
After changing the values of the various operators in equation (29) and simplifying the equation, we get the following:

$$V(x,t)=2.95x-2.875x(x-t-0.22)-x[2.7083(x-0.2)(x-0.4)+71.875t(x-0.2)-156.25t(t-0.02)]+[3.3854x(x-0.2)(x-0.4)(x-0.6)+15.6250xt(x-0.2)(x-0.4)-\ 273.4375x(x-0.2)(t-0)(t-0.02)-\ 906.2500xt(t-0.02)(t-0.04)]-[19.5312\mathrm{xt}(\mathrm{x}-0.2)(\mathrm{x}-0.4)(\mathrm{x}-0.6)-\ 260.4167\mathrm{xt}(\mathrm{x}-0.2)(\mathrm{x}-0.4)(\mathrm{t}-0.02)-3619.8\mathrm{xt}(\mathrm{x}-0.2)(\mathrm{t}-0.02)(\mathrm{t}-0.04)-211980\mathrm{x}(\mathrm{t}-0.02)(\mathrm{t}-0.04)(\mathrm{t}-0.06)] \tag{30}$$

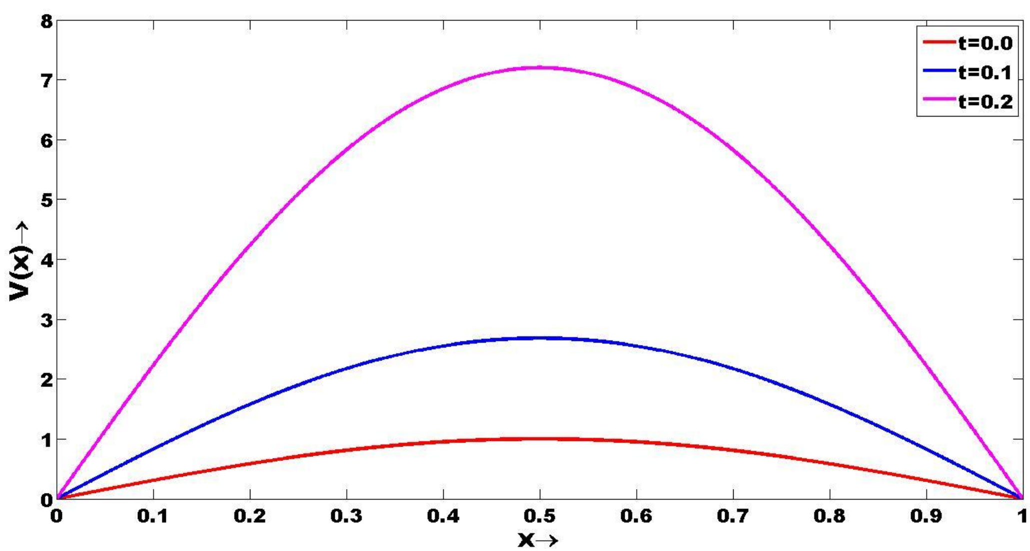
3. Graphical Solution:



Graph-1: The space-time graph of numerical solutions of heat conduction equation by double interpolation method for $\mathrm{m}=25$



Graph-2: The space-time graph of analytic solution of heat conduction equation



Graph-3: Numerical solution of Heat equation in different values of t

5. Closing Comments: In this research, a mathematical strategy known as the double interpolation method methodology is presented. This technique is used to approximate mathematical arrangements of important one-layered heat conditions. In the method that has been proposed, there are currently just a few lattices that focus on ensuring the necessary precision. Because the limit conditions are taken into consideration in a natural way, the technique is quite useful for addressing limit esteem difficulties. The proposed method is also quite easy to put into action, and the mathematical results demonstrate that it is highly successful for the mathematical arrangement of the cited problem. Additionally, the method may be applied to other circumstances involving fractional differentials.

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