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# DEVELOPMENT OF ROBUST CONTROL USING H∞ TECHNIQUE FOR DC/DC CONVERTERS

**Kunal Diwakar,** Department of Electrical Engineering, Delhi Technological University Delhi, India kunaldiwakardtu99@gmail.com
**LalitBansal,** Department of Electrical Engineering, Delhi Technological University Delhi, India lalitbansal0012@gmail.com

**Abstract:** The emphasis of this paper is to develop robust control using H∞ control method for DC/DC converters namely buck, boost, buck-boost and cuk. Since DC/DC converters are nonlinear in nature classical control approaches fails to provide the robust behavior. In this paper a generalized and feasible H∞ algorithm is designed and implied to DC/DC converters. The developed H∞ controller is compared with sliding mode controller and PI based controller. The controller performance and stability were tested through extensive simulation of models on MATLAB.

**Keywords: Buck, Boost, Buck-Boost, Cuk, state space average, small signal model, sliding mode control, PI control, robust H∞ control**

## I. INTRODUCTION

The DC/DC converters are of the important pillars of the modern world industrial systems. So, to stabilize these converters an important aspect in the design of switch mode power supplies; a feedback control is used to achieve the required performance. Ideally the circuit is in steady state, but actually the circuit is affected by line and load variations (disturbances), as well as variation of the circuit component (robustness). These parameters have a severe effect on the behavior of switch mode power supply and may cause instability. Design of controller for these converters is a major concern in power converters design [1]. Classical/conventional controllers can give steady state results only when we have the accurate mathematical model of the system. But a real time system has disturbances, parasitic quantities and noise which makes the development of an accurate mathematical model difficult. The main goal of robust control is to take uncertainties into account when analyzing a control system or when designing a controller for it. In order to do so, one has to arrive at a mathematical description of the uncertainties. These dynamics were analyzed and controllers were designed such that the closed loop system was at least stable and showed some desired performance H∞ control technique can serve as a solution for the problem. The first work on H∞ based optimized control strategy was with a conference paper by Zames [4,5]. The revolution in the solution of the H∞ control method began in 1988 which led to the development of feasible algorithm for finding optimal H∞ controllers. This paper proposed a generalized method for designing a H∞ controller in the presence of system disturbances and noises. The input disturbances is due to effect of voltage variations at input due to ripples in input stage the rectifier since the voltage we get after rectification is never pure and hence contains some ripples leading to harmonic contents which is usually called as Total Harmonic Distortion. The output disturbances is caused because of output current variations and hence leads to finally variations in output voltage. The uncertainties are considered as additive uncertainty. The two uncertainities include parametric and LTI dynamic uncertainties. Parameter uncertainities are due to variations in real parameters of a system like resistance, capacitance, input voltage, inductance etc.

## II. DC/DC CONVERTERS DYNAMICS & MATHEMATICAL MODELING

### A. Buck Converter

A Buck converter works as a converter which steps down the input voltage. It comprises of a diode and a mosfet device for controlling inductor. The switching device mosfet moves continuously between on and off positions due to which inductor and capacitor get charged and discharged with each cycle. When switch in the given figure 1 is closed, the potential difference on the inductor is VL = Vinput — Voutput. The current on the inductor begins to rise. The diode present prevents the reverse flow of the current. When the switch in the given fig 1 is open circuited, the diode starts to conduct so voltage is VL = —Voutput across inductor. The current across the inductor that was increasing when the switch was closed now starts to decrease.

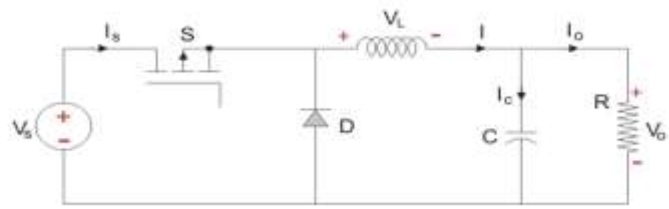


**Fig. 1.** ***Buck Converter***

The Buck Converter circuit equations when the switch is closed: -

Applying Kirchhoff Voltage law,

$v_{in} — L(dI_L/dt) —V_c = 0$ (1)

Applying Kirchhoff Current law,

$V_c/R + c(dV_c/dt) — I_L = 0$ (2)

The State space formed: -

$$\begin{bmatrix} \frac{dI_L}{dt} \\ \frac{dV_c}{dt} \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{c} & -\frac{1}{Rc} \end{bmatrix} \begin{bmatrix} I_L \\ V_c \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} v_{in} \quad (3)$$

The Buck Converter circuit equations when the switch is open: -

Using Kirchhoff Voltage law (KVL),

$V_c + L(dI_L/dt)$(4)

Using Kirchhoff Voltage law (KCL),

$I_L — (V_c/R) — c(dV_c/dt) = 0$(5)

The State space formed: -

$$\begin{bmatrix} \frac{dI_L}{dt} \\ \frac{dV_c}{dt} \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{c} & -\frac{1}{Rc} \end{bmatrix} \begin{bmatrix} I_L \\ V_c \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} v_{in} \quad (6)$$

After deriving the state space equations for buck converter, the need to find the average $A$ and $B$ with taking the effect of duty cycle $D$ into account.

$A = A_{closed} D + A_{opened} D'$(7)

$B = B_{closed} D + B_{opened} D'$(8)

Where,

$D' = (1-D)$ (9)

So,

$$A = \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & \frac{-1}{RC} \end{bmatrix} D + \begin{bmatrix} 0 & \frac{-1}{L} \\ \frac{1}{C} & \frac{-1}{RC} \end{bmatrix} (1 - D) \quad (10)$$

$$A = \begin{bmatrix} 0 & \frac{-1}{L} \\ \frac{1}{C} & \frac{-1}{RC} \end{bmatrix} (11)$$

$$B = \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} D + \begin{bmatrix} 0 \\ 0 \end{bmatrix} (1 - D) (12)$$

$$B = \begin{bmatrix} \frac{D}{L} \\ 0 \end{bmatrix} (13)$$

From equation (11) and (13) the overall system equations are represented in (14).

$$\begin{bmatrix} \frac{dI_L}{dt} \\ \frac{dV_c}{dt} \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{c} & -\frac{1}{Rc} \end{bmatrix} \begin{bmatrix} I_L \\ V_c \end{bmatrix} + \begin{bmatrix} \frac{D}{L} \\ 0 \end{bmatrix} v_{in} (14)$$

The output equation for $v_c$ and $i_L$ is shown in (15).

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} I_L \\ V_c \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} v_{in} \ (15)$$

The small signal model of the above averaged model is represented as follows

$$\begin{bmatrix} \frac{d(I_L + iL)}{dt} \\ \frac{d(V_c + vc)}{dt} \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{c} & -\frac{1}{Rc} \end{bmatrix} \begin{bmatrix} I_L \\ V_c \end{bmatrix} + \begin{bmatrix} \frac{D+d}{L} \\ 0 \end{bmatrix} v_{in} (16)$$

where the variables,

are the small signal constraints.

iL = small signal variable of inductor current

vc = small signal variable of capacitor voltage

d = small signal variable of duty ratio

**TABLE I. BUCK CONVERTER SYSTEM PARAMETERS**

| **BUCK CONVERTER SYSTEM PARAMETERS** | **VALUES** |
|---|---|
| **$v_{in}$ –** Input Voltage | **12 V** |
| **L –** Inductor | **100e-9 H** |
| **C –** Capacitor | **800e-6F** |
| **R –** Load Resistance | **100Ω** |
| **$V_d$ –** Output desired voltage | **1.2 V** |

### B. Boost Converter

A boost converter steps up the applied voltage. On the output capacitors are applied so as to remove the ripples from the voltage.When the switch in the given figure 2 is closed the inductor, current starts to increase.When the switch is opened, the diode starts to conduct and the inductor current starts to flow through the combination of capacitor and load. Due to this the energy stored in the capacitor during the on-period transfers in the off condition.

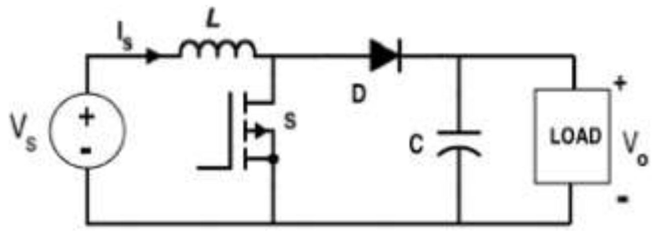


**Fig 2.*Boost Converter***

The Boost converter circuit equations when the switch is closed: -

Using Kirchhoff Voltage law (KVL),

$v_{in} — L(dI_L/dt) = 0$(17)

Using Kirchhoff Current law (KCL),

$(V_c/R) + c(dV_c/dt) = 0$ (18)

The State space formed: -

$$\begin{bmatrix} \frac{dI_L}{dt} \\ \frac{dV_c}{dt} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & -\frac{1}{Rc} \end{bmatrix} \begin{bmatrix} I_L \\ V_c \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} v_{in} \quad (19)$$

The Boost converter circuit equations when the switch is open: -

Applying Kirchhoff Voltage law,

$v_{in} — V_c — L(dI_L/dt) = 0$ (20)

Applying Kirchhoff Current law,

$I_L — (V_c/R) — c(dV_c/dt) = 0$(21)

The State space formed: -

$$\begin{bmatrix} \frac{dI_L}{dt} \\ \frac{dV_c}{dt} \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{c} & -\frac{1}{Rc} \end{bmatrix} \begin{bmatrix} I_L \\ V_c \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} v_{in} \quad (22)$$

After deriving the Boost converter state space matrix equation, we need to find A and $B$ taking the effect of duty cycle $D$ into account.

$A = A_{on} D + A_{off} D'$

$B = B_{on} D + B_{off} D'$

Where,

$D' = (1-D)$

So,

$$A = \begin{bmatrix} 0 & 0 \\ 0 & \frac{-1}{RC} \end{bmatrix} D + \begin{bmatrix} 0 & \frac{-1}{L} \\ \frac{1}{C} & \frac{-1}{RC} \end{bmatrix} (1-D) (23)$$

$$A = \begin{bmatrix} 0 & -\frac{(1-D)}{L} \\ \frac{(1-D)}{c} & -\frac{1}{Rc} \end{bmatrix} (24)$$

$$B = \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} D + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} (1-D) (25)$$

$$B = \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} (26)$$

From equation (24) and (26) the overall system equation is represented as (27).

$$\begin{bmatrix} \frac{dI_L}{dt} \\ \frac{dV_c}{dt} \end{bmatrix} = \begin{bmatrix} 0 & -\frac{(1-D)}{L} \\ \frac{(1-D)}{c} & -\frac{1}{Rc} \end{bmatrix} \begin{bmatrix} I_L \\ V_c \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} v_{in} (27)$$

The output equation for $v_c$ and $i_L$ is shown in (28).

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} I_L \\ V_c \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} v_{in} (28)$$

The small signal model of the above averaged model is represented as follows

$$\begin{bmatrix} \frac{d(I_L+iL)}{dt} \\ \frac{d(V_c+vc)}{dt} \end{bmatrix} = \begin{bmatrix} 0 & -\frac{((1-D)+d)}{L} \\ \frac{((1-D)+d)}{c} & -\frac{1}{Rc} \end{bmatrix} \begin{bmatrix} I_L + iL \\ V_c + vc \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} v_{in} (29)$$

where *iL, vc, and d* are the small signal parameters of inductor current, capacitor voltage and duty ratio respectively.

**TABLE II. BOOST CONVERTER SYSTEM PARAMETERS**

| BOOST CONVERTER SYSTEM PARAMETERS | VALUES |
|---|---|
| $v_{in}$ – Input Voltage | **25 V** |
| **L –** Inductor | **310 µH** |
| **C –** Capacitor | **2300 µF** |
| **R –** Load Resistance | **64 – 164 Ω** |
| $V_d$ – Output desired voltage | **48 V** |

### C. Buck-Boost Converter

A Buck-Boost converter can both increase and decrease the applied voltage. By adjusting the duty ratio of switch the output voltage can be controlled. When switch in fig 3 is closed, the energy gets stored in the inductor which results in the capacitor supplying energy to the load. When the switch in fig 3 is opened the energy stored in the inductor is supplied to the capacitor and resistor load.

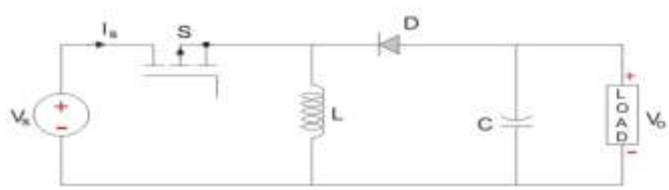


**Fig 3.** ***Buck-Boost Converter***

The Buck-Boost Converter circuit equations when the switch is closed: -

Using Kirchhoff Voltage law (KVL),

$v_{in} — L(dI_L/dt) = 0$ (30)

Using Kirchhoff Current law (KCL),

$(V_c/R) + c(dV_c/dt) = 0$ (31)

The State space formed: -

$$\begin{bmatrix} \frac{dI_L}{dt} \\ \frac{dV_c}{dt} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & -\frac{1}{Rc} \end{bmatrix} \begin{bmatrix} I_L \\ V_c \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} v_{ln} (32)$$

The Buck-Boost converter circuit equations when the switch is open: -

Applying Kirchhoff Voltage law,

$— V_c + L(dI_L/dt) = 0$(33)

Applying Kirchhoff Current law,

$I_L + (V_c/R) + c(dV_c/dt) = 0$(34)

The State space formed: -

$$\begin{bmatrix} \frac{dI_L}{dt} \\ \frac{dV_c}{dt} \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{L} \\ -\frac{1}{c} & -\frac{1}{Rc} \end{bmatrix} \begin{bmatrix} I_L \\ V_c \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} v_{ln} \ (35)$$

After deriving the Buck-Boost converter state space equation, we need to find its average $A$ and $B$ with taking the effect of duty cycle $D$ into account.

$A = A_{closed} D + A_{opened} D'$

$B = B_{closed} D + B_{opened} D'$

Where,

$D' = (1-D)$

So,

$$A = \begin{bmatrix} 0 & 0 \\ 0 & \frac{-1}{RC} \end{bmatrix} D + \begin{bmatrix} 0 & \frac{1}{L} \\ \frac{-1}{C} & \frac{-1}{RC} \end{bmatrix} (1 - D) \ (36)$$

$$A = \begin{bmatrix} 0 & \frac{(1-D)}{L} \\ -\frac{(1-D)}{c} & -\frac{1}{Rc} \end{bmatrix} (37)$$

$$B = \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} D + \begin{bmatrix} 0 \\ 0 \end{bmatrix} (1 - D) (38)$$

$$B = \begin{bmatrix} \frac{D}{L} \\ 0 \end{bmatrix} (39)$$

From equation (37) and (39) the overall equations are represented as (40).

$$\begin{bmatrix} \frac{dI_L}{dt} \\ \frac{dV_c}{dt} \end{bmatrix} = \begin{bmatrix} 0 & \frac{(1-D)}{L} \\ -\frac{(1-D)}{c} & -\frac{1}{Rc} \end{bmatrix} \begin{bmatrix} I_L \\ V_c \end{bmatrix} + \begin{bmatrix} \frac{D}{L} \\ 0 \end{bmatrix} v_{ln} (40)$$

The output equation for $v_c$ and $i_L$ is shown in (41).

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} I_L \\ V_c \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} v_{in} \ (41)$$

The small signal model of the above averaged model is represented as

$$\begin{bmatrix} \frac{d(I_L+iL)}{dt} \\ \frac{d(V_c+vc)}{dt} \end{bmatrix} = \begin{bmatrix} 0 & \frac{(1-D)+d}{L} \\ -\frac{((1-D)+d)}{c} & -\frac{1}{Rc} \end{bmatrix} \begin{bmatrix} I_L + iL \\ V_c + vc \end{bmatrix} + \begin{bmatrix} \frac{D+d}{L} \\ 0 \end{bmatrix} v_{ln} (42)$$

**TABLE III. BUCK-BOOST CONVERTER SYSTEM PARAMETERS**

| BUCK-BOOST CONVERTER SYSTEM PARAMETERS | VALUES |
|---|---|
| $v_{in}$ – Input Voltage | **15 V** |
| **L –** Inductor | **550 µH** |
| **C –** Capacitor | **330 µF** |
| **R –** Load Resistance | **64 – 164 Ω** |
| $V_d$ – Output desired voltage | **30 V** |

**D. CuK Converter**

This DC/DC converter provide changing output voltage. The current flowing through the inductor in Cuk converter is continuous due to which it is more attractive in comparison with other converters. The Cuk converter is comprised of two inductances, the first in continuation with input source and the second in continuation with load. A capacitor of appropriate rating is located between the two inductors and may divide into two capacitors coupled by a transformer, if electrical isolation between source and load is desired. Both the input and output currents are non-pulsating and have a current source characteristic. The Cuk converter is naturally inverting, as the Buck-Boost. In addition, it can also have DC gain greater or smaller than 1, i.e. the output voltage can be greater or smaller than input voltage. One advantage of the Cuk converter is that it uses a capacitor for energy transfer, unlike other topologies rely, to some extent, on inductive energy transfer with its losses due continually circulating currents. Capacitors also offer higher energy density per unit volume, resulting in smaller and lighter converters.

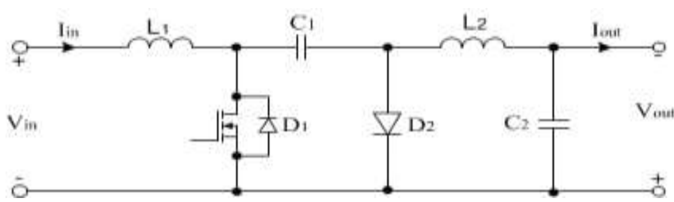


**Fig 4.*Cuk Converter***

When the switch is closed

$L_i(di_{Li}/dt) — v_i = 0$(43)

$L_o(di_{Lo}/dt) — v_{ci} — v_{co} = 0$(44)

$c_i(dv_{Li}/dt) + i_{Lo} = 0$(45)

$c_o(dv_{Lo}/dt) + i_{Lo} + (v_{co}/R) = 0$(46)

The State space formed,

$$\begin{bmatrix} \dot{i}_{Li} \\ \dot{i}_{Lo} \\ \dot{v}_{Ci} \\ \dot{v}_{Co} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & (1/L_o) & (-1/L_o) \\ 0 & (-1/C_i) & 0 & 0 \\ 0 & (1/C_o) & 0 & (-1/RC_o) \end{bmatrix} \begin{bmatrix} i_{Li} \\ i_{Lo} \\ v_{Ci} \\ v_{Co} \end{bmatrix} + \begin{bmatrix} (1/L_i) \\ 0 \\ 0 \end{bmatrix} [v_i]$$

When the switch is opened

$L_i(di_{Li}/dt) — v_i + v_{ci} = 0$(48)

$L_o(di_{Lo}/dt) + v_{co} = 0$(49)

$c_i(dv_{Li}/dt) — i_{Li} = 0$(50)

$c_o(dv_{Lo}/dt) — i_{Lo} + (v_{co}/R) = 0$(51)

The State space formed,

$$\begin{bmatrix} \dot{i}_{Li} \\ \dot{i}_{Lo} \\ \dot{v}_{Ci} \\ \dot{v}_{Co} \end{bmatrix} = \begin{bmatrix} 0 & 0 & (^{-1}/_{L_i}) & 0 \\ 0 & 0 & 0 & (^{-1}/_{L_o}) \\ (^{-1}/_{C_i}) & 0 & 0 & 0 \\ 0 & (^{1}/_{C_o}) & 0 & (^{-1}/_{C_o R}) \end{bmatrix} \begin{bmatrix} i_{Li} \\ i_{Lo} \\ v_{Ci} \\ v_{Co} \end{bmatrix} + \begin{bmatrix} (^{1}/_{L_i}) \\ 0 \\ 0 \\ 0 \end{bmatrix} [v_i] \quad (52)$$

The output equations are,

During closed switch condition

$y = C_1x + E_1u$(53)

During opened switch condition

$y = C_2x + E_2u$(54)

$$\begin{bmatrix} v_o \\ i_i \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_{Li} \\ i_{Lo} \\ v_{Ci} \\ v_{Co} \end{bmatrix} \quad (55)$$

Where,

$v_o$ = Output voltage

$i_i$ = Input current

$C_1 = C_2$

$E_1 = E_2 = [0]$

From equation (55) the $i_i$ output can be used for calculating the input impedance.

To obtain the averaged model of the above equations,

$A = A_{closed}\, d + A_{opened}\, (1-d)$

$B = B_{closed}\, d + B_{opened}\, (1-d)$

$C = C_{closed}\, d + C_{opened}\, (1-d)$

$E = E_{closed}\, d + E_{opened}\, (1-d)$

From equation (44) and (49) the averaged model is represented as (53).

$$\begin{bmatrix} \dot{i}_{Li} \\ \dot{i}_{Lo} \\ \dot{v}_{Ci} \\ \dot{v}_{Co} \end{bmatrix} = \begin{bmatrix} 0 & 0 & (^{(d-1)}/_{L_i}) & 0 \\ 0 & 0 & (^{d}/_{L_o}) & (^{-1}/_{L_o}) \\ (^{(1-d)}/_{C_i}) & (^{-d}/_{C_i}) & 0 & 0 \\ 0 & (^{1}/_{C_o}) & 0 & (^{-1}/_{C_o R}) \end{bmatrix} * \begin{bmatrix} i_{Li} \\ i_{Lo} \\ v_{Ci} \\ v_{Co} \end{bmatrix} + \begin{bmatrix} (^{1}/_{L_i}) \\ 0 \\ 0 \\ 0 \end{bmatrix} [v_i]$$

TABLE IV. CUK CONVERTER SYSTEM PARAMETERS

| CUK CONVERTER SYSTEM PARAMETERS | VALUES |
|---|---|
| **$v_{in}$** – Input Voltage | **24 V** |
| **$L_i$** – Input inductor | **3 mH** |
| **$L_o$** – Output inductor | **1.9 mH** |
| **$C_i$** – Input Capacitor | **110 µF** |
| **$C_o$** – Output capacitor | **47.5 µF** |
| **R** – Load Resistance | **64 – 164 Ω** |
| **$V_d$** – Output desired voltage | **22 V** |

## III. CONTROL METHOS

To imply the control strategies to DC/DC converters to maintain the required output a negative feedback loop is used which eliminated the disturbances from the output and preserves the converter from various changes in the real time world. The converters are subjected to sudden variation in load to check their robustness.

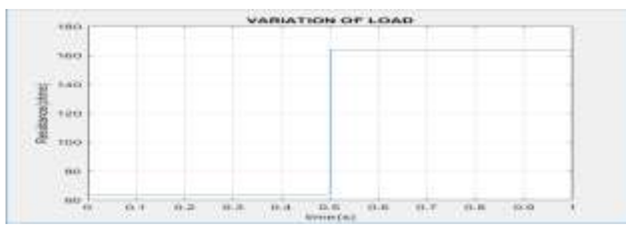


**Fig 5.*Generalized Plant Model with Weight and disturbance signal***

### A. PI Control

PI which stands for proportional integral control is a control method which takes the advantages of both types of control and solves individuals' disadvantages. Although the proportional gain constant can change the integral gain constant but we can adjust the integral gain independently. The integral feature effectively provides a 'reset' of the zero-error output after variation in the system occurs. Whenever there is a change in the system an error is produced. To accommodate the error produced the controller should be provided with the error. The output of the controller is a sum of both proportional and integral exploit which reduces the error towards zero with each cycle.

The transfer function for PI control is given by;

Kp + (KI/s)

The open loop action of DC/DC converter is not immune to the changes and disturbances in the circuit so, the closed loop operation is implied. In closed loop operation we use a negative feedback signal from the output and the PI controller compares the output signal with reference or desired signal. The error thus obtained is reduced by the PI control in each cycle and it goes on to till the error signal becomes zero. When the system is stabilized an efficient and accurate control is achieved [3]. .

### B. Sliding Mode Control

Sliding mode control can be defined "as the variable structure control system which is considered by a discontinuous feedback control topology that switches as the system crosses firm multiple in the state space toforce the system state to reach, and subsequently to remain on a specified surface within the state space" as stated in [3]. "The sliding surface is a function of the states and represents a relationship between the state variables. The system dynamics when restricted to the sliding surface is a perfect sliding motion and signifies the controlled behavior, which results in reduced order dynamics with respect to the original system. This reduced order dynamics gives striking advantages such as robustness to parameter variations and matched reservations and disturbances, making it a perfect control method strategy for robust control. The strategy of sliding mode control law entails of the construction of an appropriate sliding surface so that the dynamics of the system are to the sliding multiple produces a desired behavior, and the construction of a discontinuous control law which forces the system trajectory to the sliding manifold and maintains it there" [2] "The sliding mode control design consists of two steps, the construction of the desired sliding surface and the sliding mode enforcement capable of driving and confining the system motion on the sliding surface. Hence, the control input consists of two components, a discontinuous component Nn(t) to drive the system states on to the sliding surface and a continuous component Neq(t) which ensures the motion of the system on the sliding surface whenever it is on the surface to force the error variables to the origin" [2].

N(t) = Nn(t) + Neq(t)

But it exhibits a chattering phenomenon which results in decreased control, wear and tear of movable parts and high-power loss.

$$N = \begin{matrix} 1 & S < 0 \\ 0 & S > 0 \end{matrix}$$

$N = T_1'(V_o + V_d) + T_2\,(i_l + I_L)$

where,

$T_1' = T_1 — (T_2V_d)/RV_{in}$

## IV. DESIGNING OF H INFINITY

For best performance of transfer function G(s), the H∞ control law is given by,

$||G(s)||_\infty = ά\{G(j\omega)\}$(57)

Where ά is the maximum singular

The value of ά is calculated from matrix A

$ά(A)= \{\lambda_{max}(A^*A)\}^{1/2}$

$\lambda_{max}$ represents the maximum value of the eigenvalues

If G(s) represents a SISO system then equation (57) can be represented as

$||G(s)||_\infty = max\ |G(j\omega)|$(58)

This H∞ represents the maximum gain that G(s) can achieve.

Also,

$||G(s)||_\infty < 1$

$=> |G(j\omega)|_\infty < 1$

In designing process, it is important to understand the robust stability and performance.

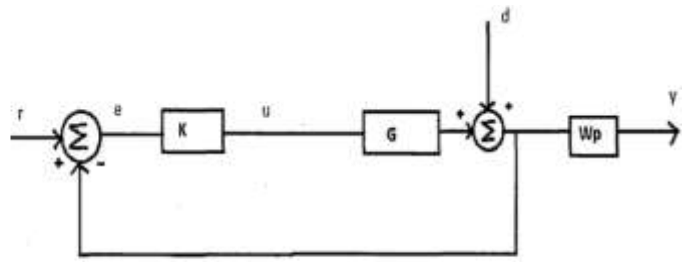


**Fig 6.** ***Generalized plant model with Weight and disturbance signal***

r= the reference input

y= output

u= control signal

e= error signal

d= disturbance signal

From fig 6;

$y= (I+GK)^{-1}GKr + (I+GK)^{-1}d$ (59)

For a better rejection of disturbance signal we need to minimize the given term;

$||(I+GK)^{-1}||_\infty$ (60)

Let,

Q= $(I+GK)^{-1}$ as the sensitivity function

T= $(I+GK)^{-1}GK$ as the complement function of Q

So, the criterium for performance would be;

$||Wp(I+GK)^{-1}||_\infty < 1$ (61)

$||(I+GK)^{-1}||_\infty < 1/Wp$ (62)

## A. Uncertainities

There are number of uncertainties occur control engineers encounter while designing the control law for a non-linear system. These uncertainties can be because of number of factors for exampleparasitic elements, disturbance signal and unmodelled parameters of the system.So, for taking the effect of all the uncertainties into account while designing a robust control law we make a upper linear fraction transform given as

$\Delta= diag\ [\mu_1 I_{r1},\ldots\ldots\mu_2 I_{rs}, \Delta_1,\ldots\ldots\Delta_f] : \mu \in C, \Delta j \in C^{mj \times mj}$

The scalars $\mu_i$ has to satisfy the following condition to be H∞ norm bounded function,

$|\mu_i| \leq 1$

This kind of uncertainty is defined as the structured uncertainty.

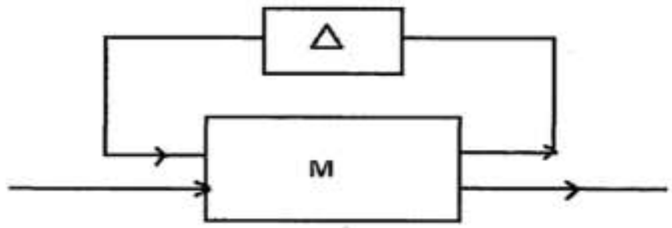


**Fig 7**. ***Generalized plant model with Uncertainty***

The upper linear fractional transform that represents the structured uncertainty is given by the formula

$$F_u(M,\mu_m) = M_{22} + M_{21}\mu_m(I-M_{11}\mu_m)^{-1}M_{12}$$

Where M is given as

$$M = \begin{pmatrix} M11 & M12 \\ M21 & M22 \end{pmatrix}$$

Also,

if $M= M'(1+P_m\mu_m)$

then

$$M= \begin{pmatrix} 0 & M' \\ Pm & M \end{pmatrix}$$

So from these we can find the parameters of uncertainty.

## B. Design methodology for Weighting functions

The weighting functions is selected so as to give better transient response characteristics. The sensitivity function Q should have low gain at low frequencies for good tracking performance and high gain at high frequencies to limit overshoot. As we know that disturbance signal is significant ar low frequencies. From equations (61) and (62) it can be depicted that the weighting function is inversely proportional to the sensitivity of the system.

For Designing the weighting function for the controller, the general formula is,

$Wp = ((s/M) +Wb)/(s+WbA)$

Wb= allowable bandwidth

A= allowable steady state error

M= is allowable high frequency error

For our design the values of these parameters

M= 1.75, Wb= 50, A=1.e-4

Assuming the matrix Δ a transfer function matrix which satisfies the norm

$||\Delta||_\infty < 1$

In figure 6 assuming the reference signal to zero gives the standard block diagram for the H∞ control design

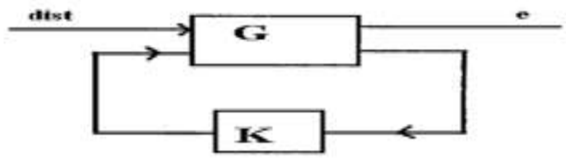


**Fig 8.** ***Standard H∞ control model***

With addition of additive uncertainty into the generalized plant model we get the following model,

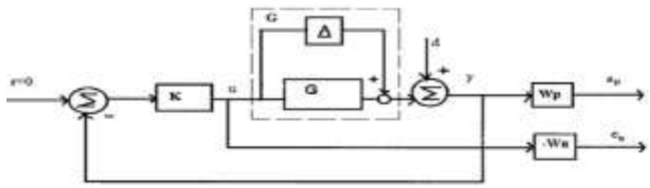


**Fig 9.** ***Plant Model with the Addition of Additive uncertainty***

$$\begin{matrix} ep \\ eu \end{matrix} = \begin{pmatrix} WpQ \\ WuQ \end{pmatrix} d$$

From previously discussed performance condition we know that,

TF from d to $e_p$ and $e_u$ should be small for norm $||..||_\infty$ for all uncertainties matrix

The values of Wp and Wu show the significance of performance criteria over different frequency ranges.

### C. Genralized Algorithm for Working of H∞ Controller

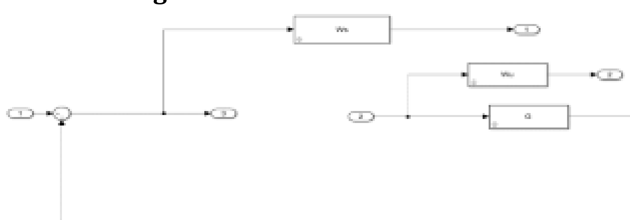

**Fig 10.** ***SIMULINK plant model for extracting transfer function and weighting funtions for H∞ Synthesis***

- Find the values for μl, μh such that; μl< ||M(s)||< μh
- Assume μ= (μl+ μh)/2 and do verification whether the matrices X=XT and Y=YT satisfies Riccati equations.
- If the condition satisfies put μh= μ, else put μl= μ
- Put $€_0$ = μh- μl, check whether $€_0$> 0
- If the condition satisfies go to step 2
- Now, μ= μh and define the state space equations for controller K(s)

This can be calculated using the "hinfsyn" function from MATLAB robust control toolbox as stated below

[K clp] = hinfsyn (p, nmeas, ncon, glow, ghigh, tol);

Where,

P= SYSTEM MATRIX

Nmeas= Measurement numbers

Ncons= Number of controls

Glow= Lower bound of bisection

Ghigh= Upper band of bisection

Tol= Toleration

CLP= closed loop system matrix

## V. RESULT AND DISCUSSIONS

This H∞ control (represented in blue) algorithm is applied to Buck, Boost, Buck-Boost and Cuk converter and after rigorous testing on MATLAB/SIMULINK their step responses were obtained. The same converters models were implemented through PI controller (represented in purple) and sliding mode controller (represented in black)

**Buck Converter**

The step response of the controllers applied to Buck converters is tested.

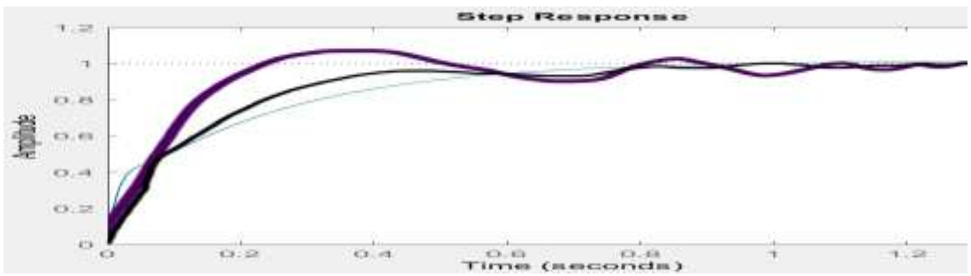


**Fig 11.** ***Unity step response of H∞ control for Buck Converter***

**TABLE V. MEASURED VALUES**

| CONTROLLERS | SETTLING TIME (sec) | OVERSHOOT |
|---|---|---|
| **PI** | 1.2 | 0.15 |
| **SLIDING MODE CONTROLLER** | 0.9 | - |
| **H∞ CONTROLLER** | 0.7 | - |

**Boost Converter**

The step response of the controllers applied to Boost converters is tested.

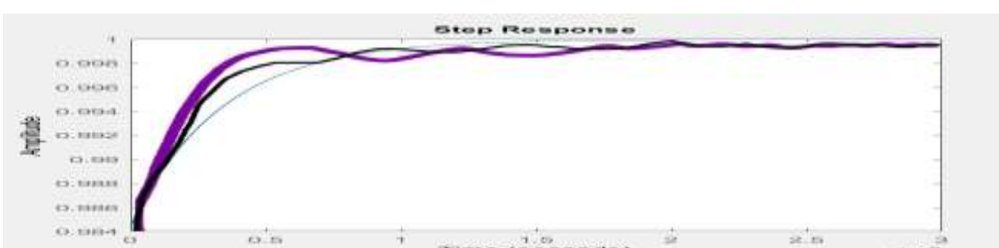


**Fig 12.** ***Unity step response of H∞ control for Boost Converter***

**TABLE VI. MEASURED VALUES**

| CONTROLLERS | SETTLING TIME (sec) | OVERSHOOT |
|---|---|---|
| PI | 1.6 | - |
| SLIDING MODE CONTROLLER | 1.46 | - |
| H∞ CONTROLLER | 1.2 | - |

**Buck-Boost Converter**

The step response of the controllers applied to Buck-Boostconverters is tested.


**Fig 13.** ***Unity step response of H∞ control for Buck-Boost Converter***

**TABLE VII. MEASURED VALUES**

| CONTROLLERS | SETTLING TIME (sec) | OVERSHOOT |
|---|---|---|
| PI | 1.75 | 0.40 |
| SLIDING MODE CONTROLLER | 1.3 | - |
| H∞ CONTROLLER | 1.06 | 0.21 |

**Cuk Converter**

The step response of the controllers applied to cukconverters is tested.


**Fig 14.** ***Unity step response of H∞ control for Cuk Converter***

**TABLE VIII. MEASURED VALUES**

| CONTROLLERS | SETTLING TIME (sec) | OVERSHOOT |
|---|---|---|
| **PI** | 1.9 | 0.39 |
| **SLIDING MODE CONTROLLER** | 1.7 | 0.22 |
| **H∞ CONTROLLER** | 1.5 | 0.25 |

## VI. CONCLUSIONS

On studying and analyzing the measured data and comparing the graphical results for all the converters it can be deduced that PI controller has the poorest results for overshoot and settling time.On comparing the results obtained from sliding mode control and H∞ control, we get a better overshoot response in sliding mode control but the sliding mode controller suffers from chattering phenomenon which must be eliminated.The H∞ control has a good settling time and overshoot response and is free from any kind of chattering phenomenon, which makes it the ideal controller out the three controllers used.

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