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Fractal Approach and Evaluation of Mathematical Model of Covid-19 in Tamil Nadu

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Abstract- The Epidemic of the newest Coronavirus infection broke get in December 2019 in Wuhan city, Hubei province and has spread across the world to 97 countries all over the world. The aim of this work is evaluate COVID-19 in tamilnadu region based on Fractal Dimension of confirmed cases in March to October. Its use two Mathematical model to compare the method of estimating and predicting the growth of the pandemic: The Logistic growth model and Gompertz growth model. The investigation was carried out using data on cases of infection and confirmed death to analysis the accuracy model. To verify the consistency of the templates its use RAMSE and R^2. For both model (R^2> 0.99), the experimental findings have encouraging modification errors. This model offers a simple reliable framework for tracking the growth of pandemics that can be helpful to health system.

Keywords: COVID-19, Fractal Dimension, Gompertz model, Hurst value, Mathematical models.

I. Introduction

1.1 Fractal

Fractal object science uses a particular non-Euclidean geometric mathematical model object. The word *fractal* is used to refer to objects that are identical to themselves and have the same features on different scales [1]. Fractal properties consist independence of size, self-similarity, complexity, and an infinite length scale,whereas single time scale cannot be characterize time series. Fractal processes traditionally, biologist use Euclidean images of nature objects or sequence. Including example, describing cardiac rates as time waves, coniferous trees as cones, animal environments as simples region [2].

1.2 Mathematical Model

Mathematical modelling is the use of mathematics to explain and investigate real world problems. It's were used in recent years to validate experimentally based hypotheses and simultaneously to design and evaluate those models to predictions [3]. Infectious diseases can be shown to explain the possible effects of an outbreak and to inform public health measures through Mathematical models. For the determination of various parameters of infectious diseases and for measuring the impact of different measure such as mass vaccination initiatives, the methods used simple assumptions or statistics obtained along with mathematics [4].

1.3 COVID-19

The virus is now known as the severe acute respiratory syndrome coronavirus 2 (SARS-CoV-2). The disease it causes is called coronavirus disease 2019 (COVID-19) in Fig.1.COVID- 19 is an outbreak that has spread to over 200 countries infecting more than 40 lakhs in the rest and is a consequences of SARS-CV-2 VIRUS [5][6]. The WHO confirmed the outbreak of this disease after a first case report on December 31, 2019, in Wuhan china and pandemic on March 11, 2020. In India 30 January 2020, the pandemic was first identified in Trissur.

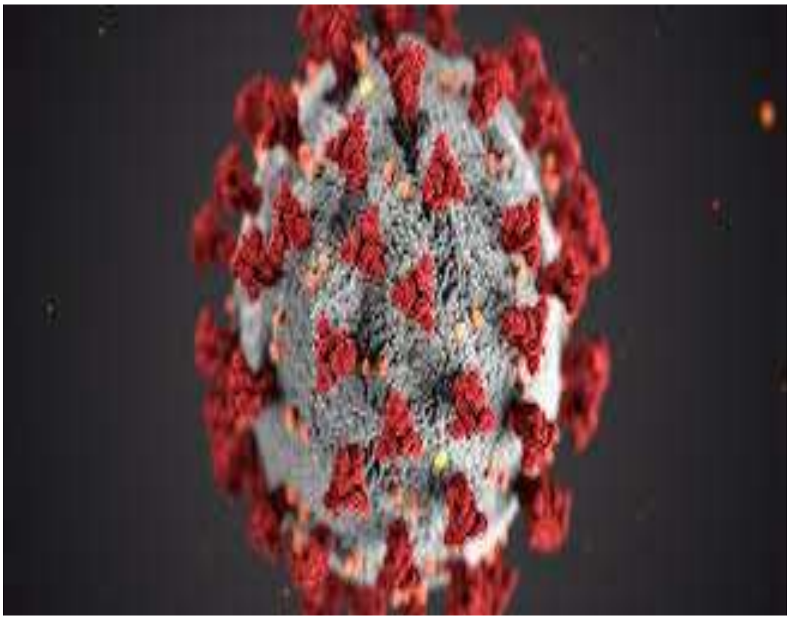

Figure 1: Coronavirus

In Section 2 Estimating and Analysis of Fractal Dimension and comparing the both model with Statistics model. In Section 3 the results are explained.

II. Methods

The propagation mechanism of the COVID-19 is confirmed to the expansion direction of Fractal theory. To obtain the H index of different regions, evaluate the Fractal dimension of the model and numerical data at over time the Rescaled range analysis value is time series Interval.

2.1 Notation

R/S	Rescaled range analysis
$\bar{x}$	Mean
R	Range
x(t,n)	Cumulative dispersion
S	Mean square error
$\boldsymbol{\gamma}$,c	Constant proportionally
T	Length of time series
R_x	Stretched out of spread
S_x	Standard deviation
D	Fractal Dimension
H	Hurst Exponents
$\boldsymbol{\beta_0}$	Initial population
t	Time
$\boldsymbol{\alpha}$,b	Final population
P,v	Infection and death case at time

2.2 Rescaled Range Analysis (R/S Analysis)

The Rescaled range analysis is used in this article its new coronavirus transmission mechanism conforms to the Fractal expansion route. At this time, the newly defined rate of change is being used by Tamil Nadu region. The R/S value is reduce linearly on the basis of determine the Fractal component of the time series period to obtain the H index of various region [7].

Find the mean of the time series

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \qquad (1)$$

Find Cumulative dispersion as

$$x(t,n) = \sum_{i=1}^{n} (x_i - \overline{x_n}) \tag{2}$$

Find the Range as

$$R(j) = \max x(t,n) - \min x(t,n) \tag{3}$$

Find Mean Square error of the time series

$$S(j) = \sqrt{\frac{1}{t+j-1} \sum_{i=1}^{t+j-1} (x_i - \overline{y_i})^2} \tag{4}$$

Find the Rescaled Range analysis as

$$\left(\frac{R}{S}\right) = \frac{R_x}{S_x} \tag{5}$$

2.3 Hurst Exponents And Fractal Dimension

The study of the Rescaled range analysis spends on the corresponding connection between the proportions of most stretched out spread (R) and standard deviation (S) to an intensity of the information length (T). A model may be the literature todescribe as[8].

$$\frac{R_x}{S_x} \alpha T_x^H \tag{6}$$

Equation can be written as

$$\frac{R_x}{S_x} = k T_x^H \tag{7}$$

Where *k* = constant of proportionality. We obtain the logarithm of both side of the equation by taking

$$\log \frac{R_x}{S_x} = \log(k) + H \ \log(T_x) \tag{8}$$

The graph is a straight line on a graph of a log-log with H is a slop of the graph from the equation. Interpretation of the follows, the Hurst exponent is

- H = 0.5; is the Hurst exponents value for uncorrelated time series data or random walk.
- H > 0.5; is the Hurst exponents value for positively correlated time series data or persistence.
- H < 0.5; is the Hurst exponents value for negatively correlated time series data or anti-persistence.

By plotting the estimation of log (N) and log (R/S) the Hurst exponents are calculated and the Hurst exponents are obtained as the slope of the curve. Its directly related to the Fractal dimension which based on the asymptotic behavior of the method, measure the smoothness of the time series. The correlation between the Fractal Dimension D and the Hurst exponent H is given by:

$$D = 2 - H \tag{9}$$

2.4 Logistic Growth Model

A statistical function that can be seen in many contexts is the logistic growth model. Rapidly increasing growth in the beginning but decreasing growth later, as it is closer to the limit, are characteristic for logistical growth [9]. The justification for using rational growth in the simulation process is that the outbreak of the virus has been studied by epidemiologists and that the first epidemic has exponential growth and can be formed as a logistic growth model for the whole time [10].
The Logistic equation it can be in the form:

$$\frac{dp}{dt} = \alpha.\beta\left(1 - \frac{\beta}{\alpha}\right) \tag{10}$$

It form of differential equation and Integrate as

$$\int \frac{d\beta}{\beta\left(1 - \frac{\beta}{\alpha}\right)} = \int \alpha dt \tag{11}$$

Then the equation (11) becomes

$$\ln|\beta| - \ln|\alpha-\beta| = \alpha t + \gamma \quad (12)$$

$$\ln\left|\frac{\alpha-\beta}{\beta}\right| = -\alpha t - \gamma \quad (13)$$

$$e^{\ln\left|\frac{\alpha-\beta}{\beta}\right|} = e^{-\alpha t-\gamma} \quad (14)$$

$$\left|\frac{\alpha-\beta}{\beta}\right| = e^{-\alpha t}.e^{-\gamma} \quad (15)$$

Where $A = \pm e^{-\gamma}$ solve equation (15), we get

$$\frac{\alpha-\beta}{\beta} = A.e^{-\alpha t} \quad (16)$$

$$\frac{\alpha}{\beta} = 1 + A.e^{-\alpha t} \quad (17)$$

Thus the solution of the Logistic Growth model equation

$$\beta = \frac{\alpha}{1+A.e^{-\alpha t}} \text{where} A = \frac{\alpha-\beta_0}{\beta_0} \quad (18)$$

2.5 Gompertz Growth Model

The Gompertz and Logistic models are the most regularly utilized sigmoid function and the writing on these models is broad [11]. As a whole, the cumulative number of death and cases by COVID-19 presents and unbalanced sigmoidal growth curve [12]. In this manner apply a fitting growth curve can sigficantly affect forecasting.

The Gompertz equation it can be in the form:

$$\frac{dv}{dt} = a(\ln b - \ln v)v \quad (19)$$

It can write the equation in differential equation in differential form and integrate as

$$\int \frac{dv}{v\left[\ln\left(\frac{b}{v}\right)\right]} = \int a.dt \quad (20)$$

$$-\ln\left|\ln\left(\frac{b}{v}\right)\right| = at + c \quad (21)$$

where $C = \pm e^{-c}$ is an arbitrary constant

$$\ln\left(\frac{b}{v}\right) = C.e^{-at} \quad (22)$$

$$\frac{b}{v} = e^{c.e^{-at}} \quad (23)$$

Thus the solution of the Gompertz Growth model

$$v = be^{-c.e^{-at}} \quad (24)$$

2.6 Statistical Model

According to RMSE, tocompare the model accuracy of the various regressions, which forecast measures that calculate the size of the absolute error in percentage terms, giving us a relative error measure [13]. The functions used for measuring precision are as follows.

$$RMSE = \left|\frac{1}{t} = \sum_{t=1}^{t}(u_p - u_0)^2\right|^{1/2} \quad (25)$$

III. Results

Table 1. Represent the original data which was collected from the Tamil Nadu health department. The results indicate that the area of Tamil Nadu is closed until the end of the pandemic. Table 1 indicates the findings of death and infections both in the daily cumulative case.

Table 1: Estimated Real Cumulative number of infected case and Deaths by month in Tamil Nadu

MONTH	TOTAL INFECTED CASE	TOTAL DEATH
March	124	5
April	2234	27
May	22856	193
June	90167	1201
July	245859	3935
August	428041	7322
September	597602	9520
October	724522	11122

The data collected over March to October time intervals with various sample size of Hurst value and Fractal dimension are listed in Table 2. Its shows graphical ways in Figure 2 of Hurst value help of Originpro software. The R/S analysis its calculated Hurst of newly confirmed cases in Tamil Nadu, the H estimation of in initial stage was found to be 0.0574, that is greater than 0.5 demonstrating that the future time series has a positive correlation fractal attributes. The H esteem in the subsequent stage is 1.368 with is greater than 1 which is more prominent than 1 which shows that the future epidemic circumstance is totally not preventable.

Table 2: Hurst Exponent value and Fractal Dimension of COVID-19 by month in Tamil Nadu

MONTH	HURST VALUE	FRACTAL DIMENSION
March	0.677	1.323
April	0.529	1.471
May	0.444	1.556
June	0.111	1.889
July	0.066	1.934
August	0.377	1.623
September	0.601	1.399
October	0.739	1.261

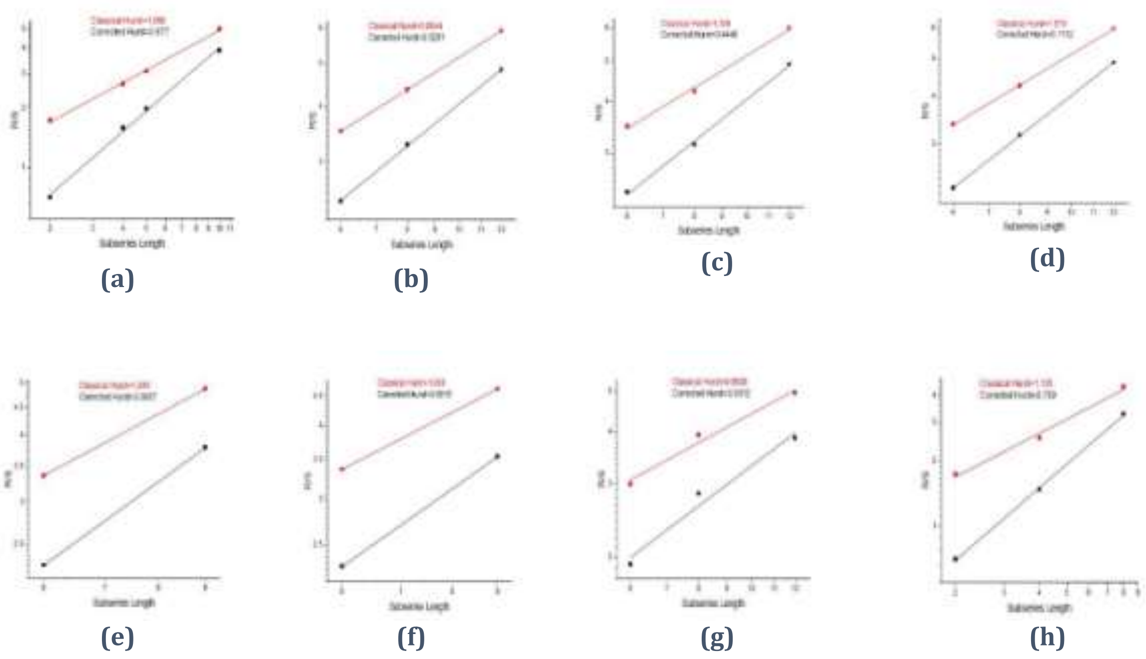

Figure 1: Hurst Exponent of Covid-19 by Month in Tamil Nadu

The predicted parameters for the logistic growth model are summarized in Table 3. Considering that the same condition prevails in the state and the current pace of development, the approximate overall number of people infected and death in Tamil Nadu is estimated. The parameters for the Gompertz growth model are estimates in Table 4.

Table 3: Logistic Growth Model Cumulative number of infected case and Deaths by month in Tamil Nadu

MONTH	TOTAL INFECTED CASE	TOTAL DEATH
March	124	5
April	2234.17	26.3
May	22170.20	186.46
June	89674.19	1196.76
July	235561.23	364.07
August	436189.05	7365
September	506816.34	9526.02
October	768192.36	10376.56

Table 4: Gompertz Growth Model Cumulative number of infected case and Deaths by month in Tamil Nadu

MONTH	TOTAL INFECTED CASE	TOTAL DEATH
March	124	5
April	2145.23	27.6
May	22674.32	190
June	88765.21	1185.65
July	236567.64	3987.2
August	428697.74	8624.2
September	510289.63	9571.82

October	767225.95	10594.10

The approximately overallNumber of infected and death cases respectively. In Table 5 and 6 both models, the correlation index is nearer than 0.999 in all cases. RMSE apply in the two Mathematical model such as Gompertz model and Logistic model. In Logistic model gets the infected rate error reaches the value 1 and Gompertz model gets the Death rate error reaches the value 1.

Table 5: Description of the obtained error of the Infected and Death

LOGISTIC GROWTH MODEL	INFECTED	DEATH
RMSE	0.074	0.078
R^2	0.9998	0.9994

Table 6: Description of the obtained error of the Infected and Death

GOMPERTZ GROWTH MODEL	INFECTED	DEATH
RMSE	0.083	0.097
R^2	0.9996	0.9998

Also it's verified that morality forecasts have greater accuracy with the Gompertz and Logistic growth model is more accurate for modelling the number infected and death in Tamil Nadu.

IV. Conclusion

This article sets forward a rescaled range analysis method to analyses and forecast the outbreak. The obtained H index is located at various intervals. Use this consequence, we can see that there are also variations in the growth paths between similar disease hours in Tamil Nadu. This study demonstrates the validity of the model based on the biological growth function of Gompertz growth model and logistic growth model to describe the pandemic growth of COVID-19 both in terms of the number of infections and deaths dynamic progression to predict the point of the trends change. This articles will be help and health and political authorities during the difficult times of this global pandemic outbreak.

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