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Numerical Solution Of Convection-Diffusion Equation Using Haar Wavelet Collocation Method With Neumann's Boundary Conditions

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Abstract-
To solve the convection-diffusion partial differential equations with Neumann's boundary conditions, Haar wavelet collocation method is proposed in this study. Algebraic equations with finite variables are used to solve HWCM's partial differential equations. Some examples are used to show how the Haar wavelet method can be applied and how it can be proven to be accurate. The exact solution is compared with the method's output to demonstrate its accuracy. To demonstrate the method's validity and application, examples are provided. Accurate solutions can be derived from this method's findings. It is the simplicity and low cost of execution that make these strategies so appealing to practitioners.

Keywords- CD equation, Partial differential equation, Haar wavelet collocation method

1. Introduction-

The convection-dissemination issues emerge in numerous significant applications in science and designing, for example, smooth movement, heat move, astronomy, oceanography, meteorology, semiconductors, power through pressure, toxin and residue transport, and substance designing. In writing, different mathematical strategies like limited contrasts, limited components, otherworldly systems, and the strategy for lines have been created and analyzed for addressing the one layered convection-dispersion condition with Dirichlets limit conditions. The vast majority of these methods depend on the two-level limited contrast approximations. In any case, less distinction plans have been created to tackle the convection-dispersion condition with Neumann's limit conditions, which are significantly more challenging to deal with than Dirichlet conditions.

Hout and Welfert (2007) thought about Alternating Direction Implicit (ADI) plans for the mathematical arrangement of introductory limit esteem issues for convection-dispersion conditions with cross subordinate terms. **Sun et al. (2011)** concentrated on the way of behaving of recurrence normal for cathode flows of the four-anode electrochemical cell. **Mittal and Jain (2012)** proposed two mathematical strategies to surmised the arrangements of the convection-dispersion halfway differential conditions with Neumann's limit conditions. The strategies depend on collocation of cubic B-splines over limited components with the goal that we have congruity of the reliant variable and its initial two subordinates all through the arrangement range. **Schmidt and Meurer (2013)** considered the direction arranging issue for coupled straight dispersion convection-response conditions utilizing a semi-mathematical methodology. **Ray and Gupta (2014)** applied a proficient mathematical plans in view of the Haar wavelet technique for observing mathematical arrangement of nonlinear third-request altered Kortewegde Vries (mKdV) condition as well as changed Burgers' conditions. **Zhou et al. (2015)** is utilized the customary limited distinction strategy or limited volume technique (FDM or FVM) for HTGR warm water powered estimation. **Liu et al. (2016)** considered mathematical limit conditions for high request limited distinction plans for settling convection-dispersion conditions on inconsistent calculation. **Kumar et al. (2017)** created Haar wavelet collocation component (HWCM) for acquiring the arrangement of higher request direct and nonlinear limit esteem issues. **Fendoglu et al. (2018)** zeroed in on the mathematical arrangement of the transient convection-dissemination response condition by changing it into adjusted Helmholtz condition through a remarkable sort change. **Casanova et al. (2019)** played out a mathematical report for the arrangement of ideal obliged enhancement issues for direct convection-dispersion PDEs by nearby and worldwide spiral premise work procedures. **Andallah and Khatun (2020)** introduced mathematical reproduction of one-layered shift in weather conditions dispersion condition. They concentrated on the scientific arrangement of shift in weather conditions dispersion condition as an underlying worth issue in endless and space and understand the subjective way of behaving of the arrangement as far as shift in weather conditions and dissemination co-productive and furthermore got the mathematical arrangement of this situation by utilizing express focused distinction plan and Crank-Nicolson conspire for endorsed beginning and limit information. **I'lsle and Owens (2021)** presented new superconsistent collocation plans for straight halfway differential conditions and show their utilization for the arrangement of two and three layered consistent convection-ruled convection-dispersion conditions, where the specific arrangements displays remarkable limit layers close to the outpouring limit of the convection speed field. **Safdari et al. (2022)** grew new mathematical plans for answer for nonlinear partial convection-dissemination conditions of order $\beta \in [1,2]$. They proposed the nearby spasmodic Galerkin techniques by taking on straight, quadratic, and cubic B-spline premise works and demonstrate solidness and ideal request of intermingling $O(h^{k+1})$ for the fragmentary dispersion issue.

2. Definition of Haar Wavelet-

Haar work was at first presented by Hungarian mathematician Alfred Haar in 1910. Later on, it is known as the easiest illustration of a symmetrical wavelet, which is characterized by a square wave work on the stretch [0,1]. The principal Harr wavelet is meant by

$$h_0(t) = \begin{Bmatrix} 1 & 0 \le t \le 1 \\ 0 & t > 1 \end{Bmatrix} \quad (1)$$

What's more, call the scaling capacity. The second Haar wavelet is

$$h_1(t) = \begin{Bmatrix} 1 & 0 \le t < \frac{1}{2} \\ -1 & \frac{1}{2} \le t < 1 \\ 0 & t \ge 1 \end{Bmatrix} \quad (2)$$

Likewise $h_1(t)$ is the essential square wave, or the mother wavelet which additionally ranges the entire stretch (0,1).

3. Governing Equation and Solution-

We consider the mathematical arrangement of the accompanying one dimensional convection-dispersion condition

$$\frac{\partial u}{\partial t} + \alpha \frac{\partial u}{\partial x} = \beta \frac{\partial^2 u}{\partial x^2}, 0 \le x < L, 0 \le t \le T \quad (3)$$

With initial condition

$$u(x,0) = f(x) \quad (4)$$

Neumann Boundary Conditions

$$\left(\frac{\partial u}{\partial x}\right)_{(0,t)} = g_1(t), \left(\frac{\partial u}{\partial x}\right)_{(L,t)} = g_2(t)$$

Let us approximate

$$u_{xxt}(x,t) = \sum_{i=1}^{2M} b_s(i) h_i(x) \quad (5)$$

For example, $b_s(i)$ is a coefficient of the Haar capacity that has not yet been settled, and $i = 1,2,3, \dots, N$

$$u_{xx}(x,t) - u_{xx}(x,t_s) = (t - t_s) \sum_{i=1}^{2M} b_s(i) h_i(x)$$

$$u_{xx}(x,t) = (t - t_s) \sum_{i=1}^{2M} b_s(i) h_i(x) + u_{xx}(x,t_s) \quad (6)$$

$$u_x(x,t) - u_x(0,t) = (t - t_s) \sum_{i=1}^{2M} b_s(i) q_i(x) + u_x(x,t_s) - u_x(0,t_s)$$

$$u_x(x,t) = u_x(0,t) + (t - t_s) \sum_{i=1}^{2M} b_s(i) p_i(x) + u_x(x,t_s) - u_x(0,t_s) \quad (7)$$

$$u(x,t) - u(0,t) = (t - t_s) \sum_{i=1}^{2M} b_s(i) q_i(x) + x[u_x(0,t) - u_x(0,t_s)] + u(x,t_s) - u(0,t_s)$$

$$u(x,t) = u(0,t) + (t - t_s) \sum_{i=1}^{2M} b_s(i) q_i(x) + x[u_x(0,t) - u_x(0,t_s)] + u(x,t_s) - u(0,t_s) \quad (8)$$

$$u_t(x,t) = u_t(0,t) + \sum_{i=1}^{2M} b_s(i) q_i(x) + x u_{xt}(0,t) \quad (9)$$

$\frac{\partial^2 u}{\partial x^2}, \frac{\partial u}{\partial x}$ and $\frac{\partial u}{\partial t}$ in equations (6, 7 and 8) respectively, are used in the formula (3). We are able to obtain

$$u_t(0,t) + \sum_{i=1}^{2M} b_s(i)q_i(x) + xu_{xt}(0,t) = \alpha[u_x(0,t) + (t - t_s)\sum_{i=1}^{2M} b_s(i)p_i(x) + u_x(x,t_s) - u_x(0,t_s)] = \beta[u_{xx}(x,t) = (t - t_s)\sum_{i=1}^{2M} b_s(i)h_i(x) + u_{xx}(x,t_s)\ ] \quad (10)$$

To find an approximate solution $u(x,t)$ to the system of equations (5) generated by 2M collocation points for $b_s(i)$'s (unknowns) at $t = t_{s+1}$, it is necessary to solve for the variables such that $0 \leq x \leq L,\ t_s \leq t \leq t_{s+1}$.

HWCM solution error would be computed based on the exact solution.

$L = max|u_{exact} - u_{HWCM}|$.

4. Numerical Experiments and Discussion-

Example 4.1: Allow us to think about the accompanying CD Equation

$$\frac{\partial u}{\partial t} + \alpha\frac{\partial u}{\partial x} = \beta\frac{\partial^2 u}{\partial x^2},\ 0 \leq x \leq 1, 0 \leq t \leq 1 \quad (11)$$

With $\alpha = 0.1, \beta = 0.02$

Initial condition $u(x,0) = f(x) = e^{\lambda x}$

And boundary conditions $\left(\frac{\partial u}{\partial t}\right)_{(0,t)} = \lambda e^{\mu t}, \left(\frac{\partial u}{\partial t}\right)_{(L,t)} = \lambda e^{\lambda + \mu t}$

The exact solution of (14) is given by $u(x,t) = e^{\lambda x + \mu t}$ (12)

As part of our calculation, we take into account

$\lambda = 1.17712434446770, \mu = -0.09$

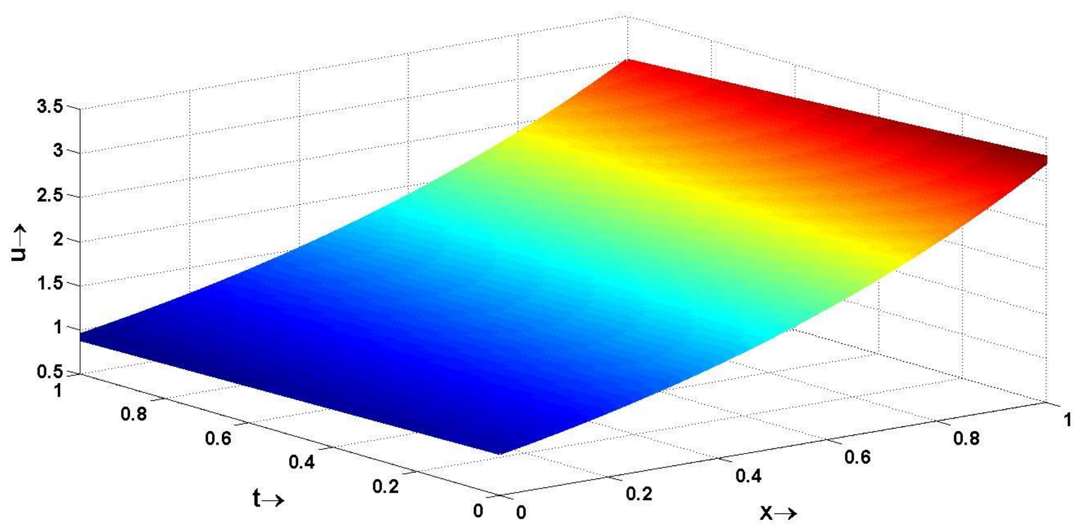

Figure 1: 3D Graphical representation of the solution of example (4.1) for distinct values of x and t

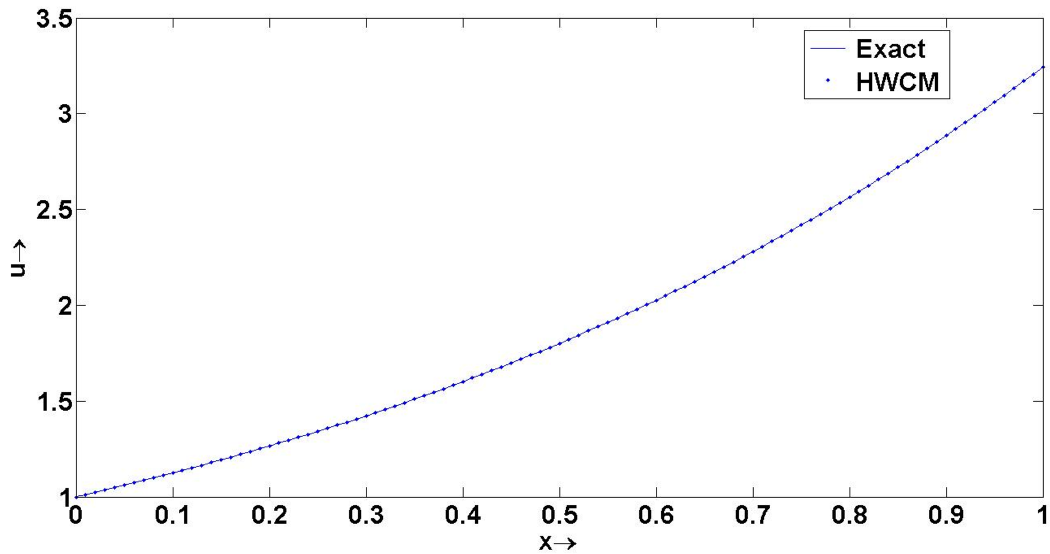

Figure 2: Comparison of the solution by Exact and HWCM of equation (4.1)

Example 4.2: Allow us to think about the accompanying CD Equation

$$\frac{\partial u}{\partial t} + \alpha \frac{\partial u}{\partial x} = \beta \frac{\partial^2 u}{\partial x^2}, \ 0 \leq x \leq 2, 0 \leq t \leq 2 \tag{13}$$

Initial condition $u(x, 0) = f(x) = sinx$

And boundary conditions $\left(\frac{\partial u}{\partial t}\right)_{(0,t)} = e^{-\beta t}\cos(\alpha t)$, $\left(\frac{\partial u}{\partial t}\right)_{(L,t)} = e^{-\beta t}\cos(2 - \alpha t)$

The exact solution of (14) is given by $u(x, t) = e^{-\beta t}\sin(\mathrm{x} - \alpha\mathrm{t})$

As part of our calculation, we take into account $\alpha = 1, \beta = 0.1$

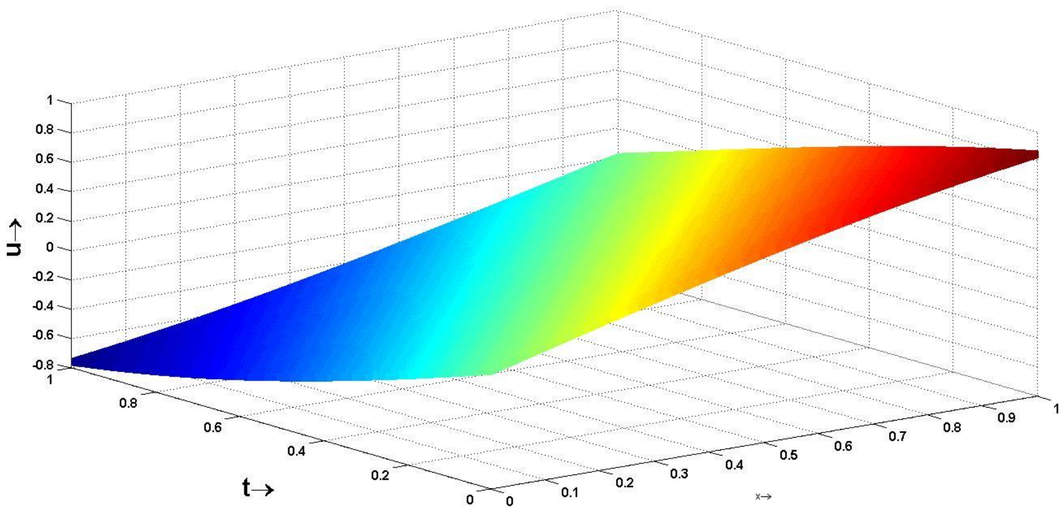

Figure 3: 3D Graphical representation of the solution of example (4.2) for distinct values of x and t

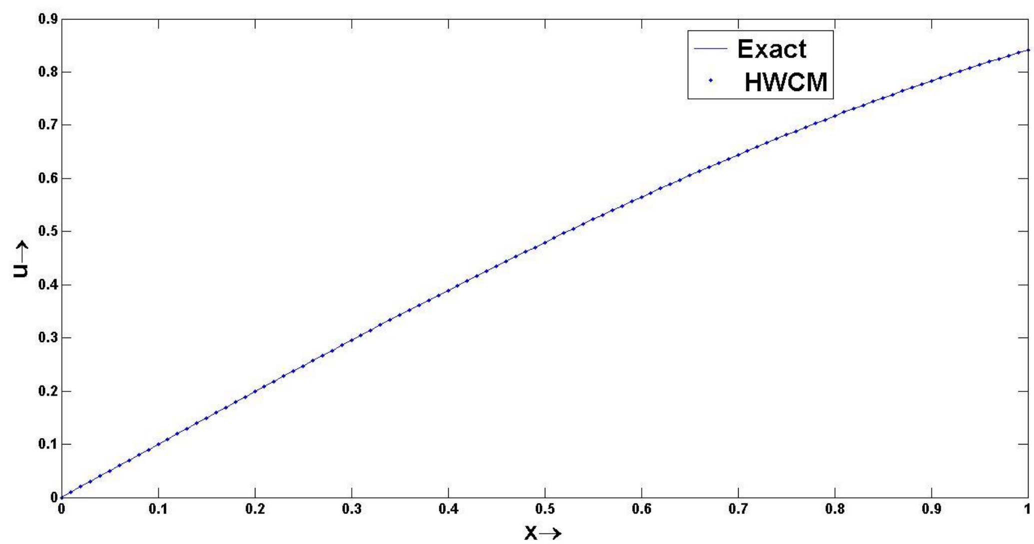

Figure 4: Comparison of the solution by Exact and HWCM of example (4.2)

Example 4.3: Allow us to think about the accompanying CD Equation

$$\frac{\partial u}{\partial t} + \alpha \frac{\partial u}{\partial x} = \beta \frac{\partial^2 u}{\partial x^2}, \; 0 \le x \le 1, 0 \le t \le 1 \tag{14}$$

Initial condition $u(x,0) = f(x) = \frac{1}{\sqrt{s}} \exp\left(-50\frac{x^2}{s}\right)$

And boundary conditions $\left(\frac{\partial u}{\partial t}\right)_{(0,t)} = \frac{100t}{s}\mathrm{u}(0,\mathrm{t})$, $\left(\frac{\partial u}{\partial t}\right)_{(L,t)} = \frac{-100(1-t)}{s}\mathrm{u}(1,\mathrm{t})$

The exact solution of (14) is given by $u(x,t) = \frac{1}{\sqrt{s}}\exp\left(-50\frac{(x-t)^2}{s}\right)$,

$\mathrm{s} = (1 + 200\beta t)$

We use the following calculations to arrive at our conclusion, $\alpha = 1, \beta = 1$

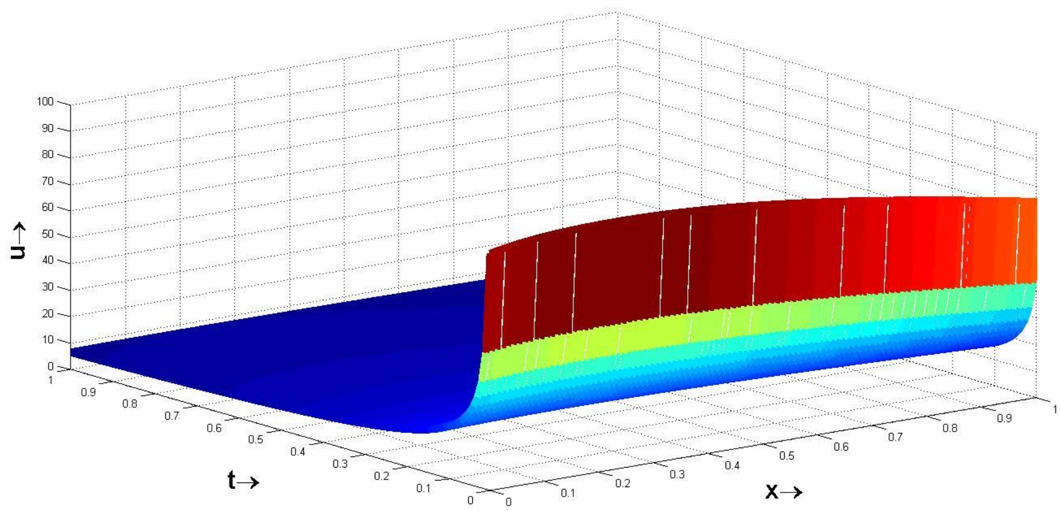

Figure 5: 3D Graphical representation of the solution of example (4.3) for distinct values of x and t

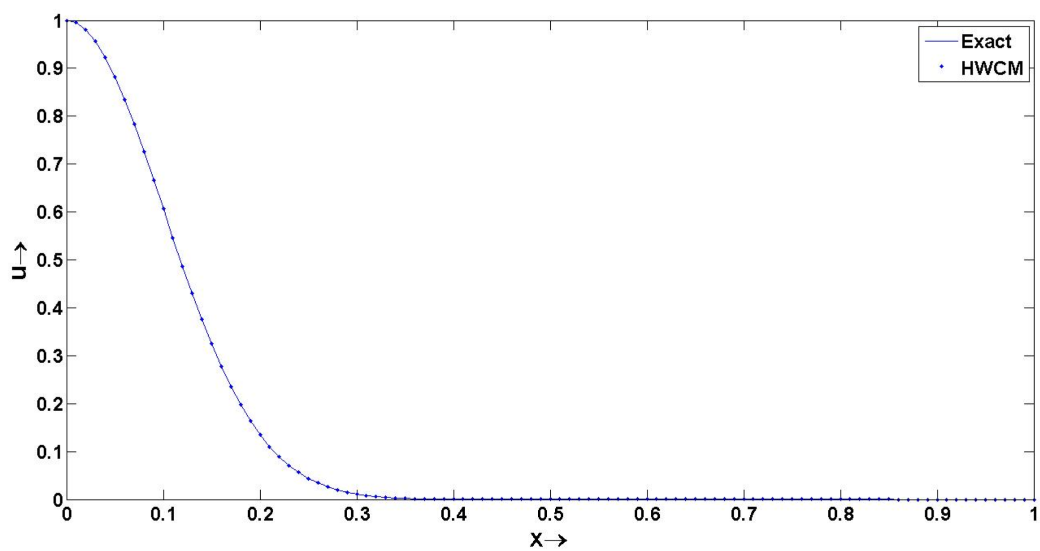

Figure 6: Comparison of the solution by Exact and HWCM of example (4.3)

5. Closing Comments-

This work manages the use of the Haar wavelet collocation strategy (HWCM) to accomplish the mathematical arrangement of one layered convection dissemination condition practically and proficiently with beginning as well as limit conditions. The arrangements of the illustrative models are acquired mathematically by Haar wavelet technique (HWCM) and are seen as exceptionally near definite arrangements of similar issues. The primary benefit of this strategy is its effortlessness and little calculation costs: it is because of the sparcity of the change networks and to the modest number of critical wavelet coefficients. This work affirmed the matchless quality of the Haar wavelet collocation technique in dealing with one layered CD condition. This strategy can be effectively reached out to track down the arrangement of different adaptations of CD condition. Consequently the proposed technique is easy to execute and exceptionally proficient to unravel convection as well as shift in weather conditions differential condition.

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