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Validation Of A Pedagogical Model Focused On The Construction Of The Robust Meaning Of Volatility For Higher Level Students

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ABSTRACT:
This article analyzes the results of a system of activities designed to establish the proper construction of meaning in students of administrative careers. The analysis is performed in four dimensions, the first consists of the statistical analysis first in descriptive form and later adapts the application of the Wilcoxon (1970) test. The second refers to the attitudes of students towards the general topic of statistics and then to the specific topic of volatility, at this stage the survey designed by Auzmendi (1992) is used. The third section analyzes the elements present in the construction of meaning proposed by Godino (1994). In the fourth and final section, it was analyzed the video evidence obtained in each session using the Planas (2006) model. The pedagogical model used, and the application of the activities took place at the Antonio Nariño University, Bogotá, Colombia, for two groups of second-year students in administrative careers.

Keywords: Construction of meaning, volatility, validation, challenge exercises.

INTRODUCTION:

To establish the proper construction of robust meaning of volatility, some activities were designed based on the realistic mathematics of Freudenthal (1986), gamification as described mainly in Huang (2013), the significant learning of Ausubel (1983); all this framed in an environment of solving challenging exercises in the classroom with the use of technology. The primary objective of this paper is to show the strategies that

were used to validate the model obtained in Osorio M.A.D. (2021) and the activities described in such work. The quantitative and qualitative analysis of the answers given to the exercises formulated in each of the activities described in the work of Diaz, M.A. (2019 p148.189), allowed to characterize the level achieved based to the construction of the robust[1] meaning of volatility measuring four dimensions of the experience in the classroom well defined as shown in the figure 1. The analysis of the data is carried out separately in two samples since the experiment was applied in two different locations of the Antonio Nariño University, Bogotá, namely: Sede sur and Sede Nicolás de Federman.

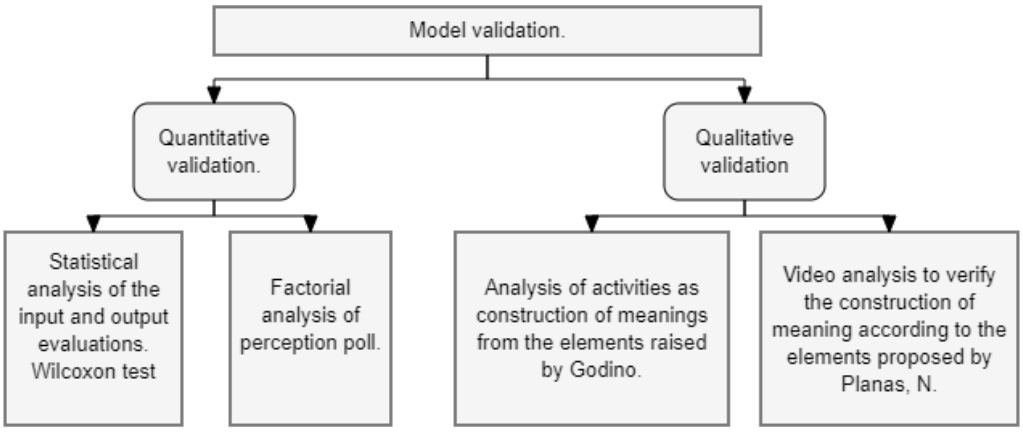


Figure 1: Diagram of the validation of the model in four dimensions. Own elaboration.

Description of dimensions revised

1. **Based on the test results.** The data are analyzed in a descriptive manner and a comparison is shown between the input and output evaluations using the Wilcoxon test in order to show if there really were significant changes against the appropriation of key concepts in the study of the volatility. Wilcoxon is used after finding that the results do not follow a normal distribution, parametric tests do not correctly reflect the results of the students before and after applying the model; the Wilcoxon test is chosen as a good option as it is a non-parametric contrast test.

2. **Based on attitudinal elements.** The results of two surveys concerning the attitudes of students towards the study of statistics and their usefulness are analyzed, using

[1] It is necessary to define "robust" as: "the construction of conceptual networks developed by the student to provide solutions to non-routine exercises", Pérez (2016 p11).

the survey designed by Auzmendi (1992). In addition, another survey was designed to measure the attitudes of students regarding the specific topic of volatility. These two perception surveys are implemented because such surveys include fun-oriented questions through learning, oriented to the perception of the usefulness of statistics, to the student's sense of security in solving a problem, and these are the theoretical bases for the proposed activities, gamification, problem solving, meaningful learning and realistic mathematics. These two tests are subjected to reliability analysis using the Cronbach Alpha internal consistency index and other analyses that affirm or reject the possibility of manipulating their results by factorial analysis.

3. **Based on the elements that evidence the construction of meaning found on the tests.** An example is shown of the methodology used for the analysis of the construction of meaning, through the elements proposed by Godino (2002), regarding the solutions provided by the students. Once all the tests have been analyzed, a summary matrix is prepared, analyzed, and concluded.

4. **Based on the video evidence obtained in each work session.** Finally, the videos obtained are analyzed using the methodology proposed by Planas (2006), who designed a model of video analysis for the study of processes of construction of mathematical knowledge, showing a small example referring to the methodology used, the overall results are summarized in a results matrix and concluded.

METHODS

1. **Analysis of beginning and ending exams of academic period using the Wilcoxon test.**

Two entrance tests were carried out to measure the general knowledge regarding the topic of statistical volatility, at the beginning of the experiment in the classroom and another after the output test, that can be displayed in Diaz, M.A. (2019 p148, 189), the Wilcoxon sign range test is compared with the results of the input test using a contrast analysis. This test relates the different measures of central tendency and verifies if there were significant changes in them by posing the alternative hypothesis in this same sense. The objective of the test was to demonstrate the relationship between two variables, one qualitative and the other quantitative, as follows:

Qualitative: Improvement in the proper conception of meanings related to volatility.

Quantitative: The results of input and output tests.

This raises the following hypotheses:

H_0: "The application of the teaching model does not affect the performance of students in topics related to the subject of volatility"

H_1: "The application of the teaching model affects the performance of students in topics related to volatility".

In the case of Sede sur, the probationary statistician: $\min\{T_+, T_-\}$ results 22,5; outside the range (59-66) for a two-tailed test with $n = 22$.

In the case of Nicolas de Federman, in the test statistician: $\min\{T_+, T_-\}$ obtained 18, result outside the range (30-35), which suggest the critical values of the Wilcoxon test $n = 18$, The analyses in both cases, Sede sur and Sede Nicolás de Federman, are elaborated with a reliability of 95%, this indicates that the results obtained in tests of entry and exit are significantly different. The test of the signs was carried out as follows: exit test less entry test; thus, it is concluded that the difference besides being significant is positive, which suggests the noticeable improvement in the appropriation of the meaning of volatility in both cases. For this reason, H_0 is rejected and concludes: "There is sufficient statistical evidence to state that the application of the model significantly and positively affects the appropriation of the robust concept of volatility".

2. Factor analysis of the perception survey versus statistics.

Starting with the survey designed by Auzmendi (1992), referring to the perception of students versus statistics and their usefulness, once the data is obtained, a factorial analysis of main components of Varimax rotation is made, as recommended in Izquierdo (2014), same process used by Auzmendi.

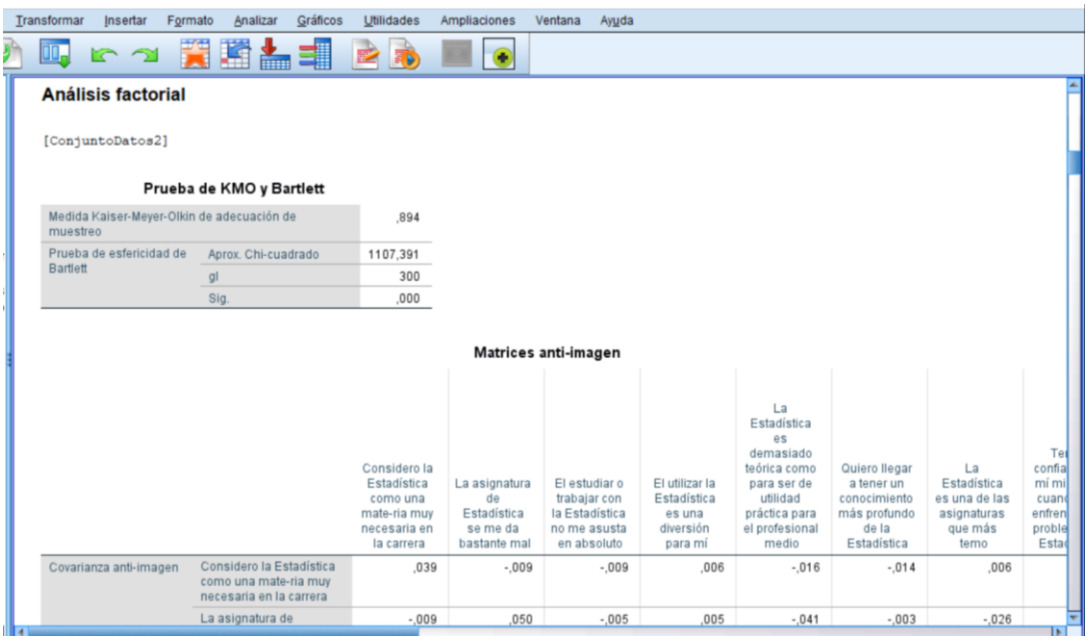


Figure 2: Start of the statistical report for the perception survey. Own elaboration in SPSS.

Figure 2 shows the consistency of the survey since the index (KMO Kaiser-Meyer-Olkin) equals 0.894. This index varies between 0 and 1 if it is close to zero; the results do not show sufficient factor correlation and indicate that it should not be analyzed by factor correlation. If it is close to 1, the data should be analyzed by factorial methods as shown in Dziuban (1974). The result is about 0.9 and a p-value of zero, with the hypothesis that the variables are not related to each other, indicating that is appropriate to work the data with the Varimax rotation method.

Another appropriate indicator in terms of the consistency of the test is the total variance explained, in factor 6 it already accumulates 86.44% of the explained variance, very good for these analyses and indicates that there are strong correlations between the factors analyzed.

Varianza total explicada

Componente	Autovalores iniciales			Sumas de cargas al cuadrado de la extracción			Sumas de cargas al cuadrado de la rotación		
	Total	% de varianza	% acumulado	Total	% de varianza	% acumulado	Total	% de varianza	% acumulado
1	17,428	69,714	69,714	17,428	69,714	69,714	5,458	21,832	21,832
2	1,170	4,679	74,393	1,170	4,679	74,393	4,129	16,515	38,347
3	,992	3,968	78,361	,992	3,968	78,361	3,186	12,745	51,092
4	,745	2,980	81,341	,745	2,980	81,341	2,956	11,823	62,915
5	,647	2,586	83,927	,647	2,586	83,927	2,950	11,800	74,715
6	,628	2,513	86,440	,628	2,513	86,440	2,931	11,725	86,440
7	,478	1,911	88,351						
8	,404	1,618	89,969						

Figure 3: Total explained variance of the Auzmendi test. Own elaboration in SPSS.

However, the most relevant data from the analysis is the strong correlation between the rotated factors shown below, there, although the totality of the data is not displayed, in figure 4, the factors with higher correlations and that explain much of the above-mentioned figures are the questions:

Question 9: I enjoy talking to others about statistics.

Question 8: I am self-confident when I faced with a statistical problem.

Question 14: Statistics is enjoyable and stimulating to me.

Question 19: I would like to have an occupation in which I would have to use statistics.

Question 4: Using statistics is fun for me.

Matriz de componente rotado[a]

	Componente					
	1	2	3	4	5	6
Me divierte el hablar con otros de Esta-dística	,771	,173	,441	,127	-,268	-,001
Tengo confianza en mí mismo/a cuando me enfrento a un problema de Estadística	,717	,268	,007	,268	-,175	,404
La Estadística es agradable y estimulante para mí	,707	,329	,281	,229	,012	,280
Me gustaría tener una ocupación en la cual tuviera que utilizar la Estadística	,663	,355	,253	,342	-,322	,089
El utilizar la Estadística es una diversión para mí	,626	,352	,282	,326	-,260	,380
Trabajar con la Estadística hace que me sienta muy nervioso/a	-,570	-,226	-,078	-,282	,317	-,471
El estudiar o trabajar con la Estadística no me asusta en absoluto	,553	,380	,319	,300	-,484	,218
Quiero llegar a tener un conocimiento más profundo de la Estadística	,545	,482	,355	,328	-,114	,316
Considero la Estadística como una mate-ria muy necesaria en la carrera	,512	,504	,457	,383	-,188	,135

Figure 4: Analysis of main components. Rotation method: Varimax with Kaiser normalization. Own elaboration in SPSS.

Figure 4 shows the factor saturation of the major components, and they can be grouped into those five questions for the first and most important factor. The survey

analyzed corresponds to the attitudes towards the statistics shown by the students under investigation, therefore it is concluded that there is a strong correlation between the factors analyzed in the five questions mentioned above.

It can be said that the apprentices studied correlate these 5 factors in a strong way and in a positive way as they show such results. For the above mentioned it is considered that this perception of the student in relation to the statistics is consistent with the theoretical framework of the experiment in the classroom under analysis; the effect that gamification had in the classroom, realistic mathematics, meaningful learning, and problem solving can be evidenced because they are precisely the questions that inquire directly about the referents of the theoretical framework as follows:

Question 9: I enjoy talking to others about statistics. (Attributable to the implementation of gamification in the classroom.)

Question 8: I am self-confident when faced with a statistical problem (Attributable to the implementation of the problem-solving methodology.)

Question 14: Statistics is enjoyable and stimulating to me. (Attributable to the implementation of gamification in the classroom.)

Question 19: I would like to have an occupation in which I would have to use statistics. (Attributable to the implementation of meaningful learning and realistic mathematics.)

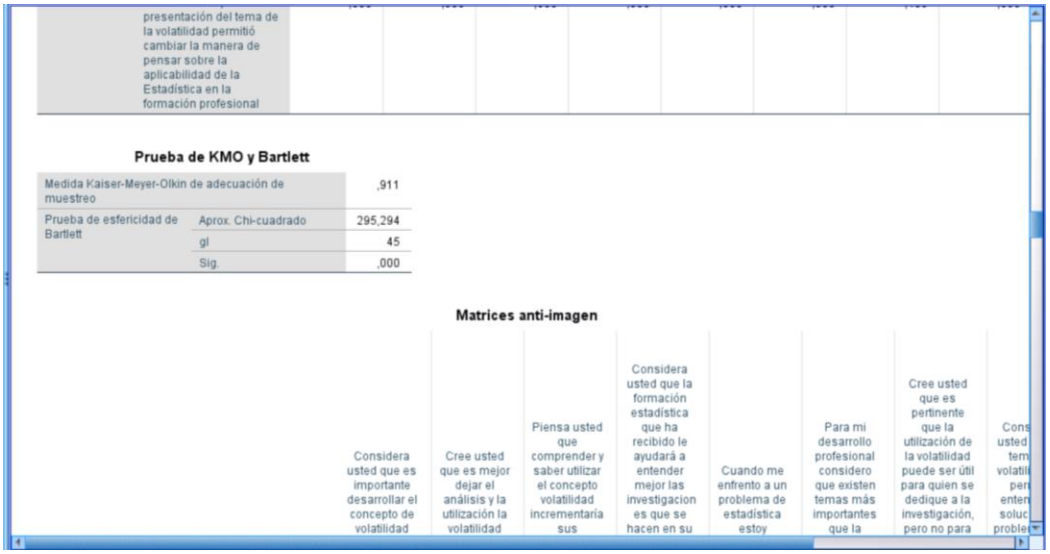
Question 4: Using statistics is fun for me (Attributable to the implementation of gamification in the classroom.)

These favorable perceptions on the part of students towards the general topic of statistics indicate that the main factors throughout the survey are those directly related to the theoretical framework of the work of Diaz, M.A. (2019 p34), it is concluded that the passage through the course of statistics II, incorporating the proposed model, leaves good perception in front of the students in relation to the theoretical referents of the model.

2.1. Factor analysis of the volatility perception and satisfaction survey.

After analyzing the Auzmendi survey, an own survey is designed also with Likert scale, which is shown in Diaz, M.A. (2019 p187) to measure the student's perception of the subject of volatility in order that, once the data is complete, a factorial analysis of the main components of Varimax rotation is performed.

The first analysis is made to verify the Kaiser-Meyer-Olkin Sampling Adequacy Measure that indicates whether it is possible to analyze reducing test factors, obtaining a 0.911 and a p-value of zero, as shown in figure 5.



presentación del tema de la volatilidad permitió cambiar la manera de pensar sobre la aplicabilidad de la Estadística en la formación profesional

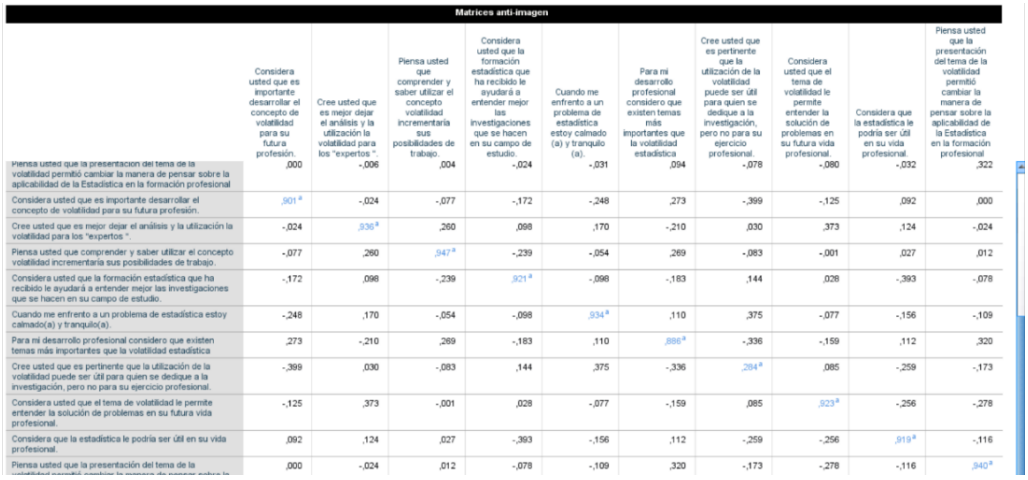
Prueba de KMO y Bartlett

Medida Kaiser-Meyer-Olkin de adecuación de muestreo		,911
Prueba de esfericidad de Bartlett	Aprox. Chi-cuadrado	295,294
	gl	45
	Sig.	,000

Matrices anti-imagen

	Considera usted que es importante desarrollar el concepto de volatilidad	Cree usted que es mejor dejar el análisis y la utilización la volatilidad	Piensa usted que comprender y saber utilizar el concepto volatilidad incrementaría sus	Considera usted que la formación estadística que ha recibido le ayudará a entender mejor las investigacion es que se hacen en su	Cuando me enfrento a un problema de estadística estoy	Para mi desarrollo profesional considero que existen temas más importantes que la	Cree usted que es pertinente que la utilización de la volatilidad puede ser útil para quien se dedique a la investigación, pero no para	Cons usted tem volatil per enten soluc proble

Figure 5: Analysis for the perception survey about volatility. Own elaboration in SPSS.



Matrices anti-imagen

	Considera usted que es importante desarrollar el concepto de volatilidad para su futura profesión.	Cree usted que es mejor dejar el análisis y la utilización la volatilidad para los "expertos ".	Piensa usted que comprender y saber utilizar el concepto volatilidad incrementaría sus posibilidades de trabajo.	Considera usted que la formación estadística que ha recibido le ayudará a entender mejor las investigaciones que se hacen en su campo de estudio.	Cuando me enfrento a un problema de estadística estoy calmado (a) y tranquilo (a).	Para mi desarrollo profesional considero que existen temas más importantes que la volatilidad estadística	Cree usted que es pertinente que la utilización de la volatilidad puede ser útil para quien se dedique a la investigación, pero no para su ejercicio profesional.	Considera usted que el tema de volatilidad le permite entender la solución de problemas en su futura vida profesional.	Considera que la estadística le podría ser útil en su vida profesional.	Piensa usted que la presentación del tema de la volatilidad permitió cambiar la manera de pensar sobre la aplicabilidad de la Estadística en la formación profesional
Piensa usted que la presentacion del tema de la volatilidad permitió cambiar la manera de pensar sobre la aplicabilidad de la Estadística en la formación profesional	,000	-,006	,004	-,024	-,031	,094	-,078	-,080	-,032	,322
Considera usted que es importante desarrollar el concepto de volatilidad para su futura profesión.	,901[a]	-,024	-,077	-,172	-,248	,273	-,399	-,125	,092	,000
Cree usted que es mejor dejar el análisis y la utilización la volatilidad para los "expertos ".	-,024	,936[a]	,260	,098	,170	-,210	,030	,373	,124	-,024
Piensa usted que comprender y saber utilizar el concepto volatilidad incrementaría sus posibilidades de trabajo.	-,077	,260	,947[a]	-,239	-,054	,269	-,083	-,001	,027	,012
Considera usted que la formación estadística que ha recibido le ayudará a entender mejor las investigaciones que se hacen en su campo de estudio.	-,172	,098	-,239	,921[a]	-,098	-,183	,144	,028	-,393	-,078
Cuando me enfrento a un problema de estadística estoy calmado(a) y tranquilo(a).	-,248	,170	-,054	-,098	,934[a]	,110	,375	-,077	-,156	-,109
Para mi desarrollo profesional considero que existen temas más importantes que la volatilidad estadística	,273	-,210	,269	-,183	,110	,886[a]	-,336	-,159	,112	,320
Cree usted que es pertinente que la utilización de la volatilidad puede ser útil para quien se dedique a la investigación, pero no para su ejercicio profesional.	-,399	,030	-,083	,144	,375	-,336	,284[a]	,085	-,259	-,173
Considera usted que el tema de volatilidad le permite entender la solución de problemas en su futura vida profesional.	-,125	,373	-,001	,028	-,077	-,159	,085	,923[a]	-,256	-,278
Considera que la estadística le podría ser útil en su vida profesional.	,092	,124	,027	-,393	-,156	,112	-,259	-,256	,919[a]	-,116
Piensa usted que la presentación del tema de la	,000	-,024	,012	-,078	-,109	,320	-,173	-,278	-,116	,940[a]

Figure 6: Correlation's perception survey versus volatility. Own elaboration in SPSS.

However, there is an item that is far below what is expected, as shown in figure 6, in the matrix of anti-imagen correlations, results are very close to one on the entire diagonal, except in the factor 7, with a 0.284 corresponding to the question:

Do you think that the use of volatility can be useful for those engaged in research, but not for their professional practice?

Therefore, it is decided to remove from the analysis since it does not contribute anything in the correlation with other factors, as suggested by factorial analysis manuals for example shown in Lozares (1991). Once the item has been removed, the

reliability and internal consistency of the surveys are measured using the Cronbach alpha; the new alpha value is shown in figure 7.

Estadísticas de fiabilidad

Alfa de Cronbach	Alfa de Cronbach basada en elementos estandarizados	N de elementos
,934	,932	10

Figure 7: Cronbach's Alpha of the Volatility Perception Survey. Own elaboration in SPSS.

Once the reliability of the survey with a high alpha has been reassessed, the factorial analysis and the Kaiser-Meyer-Olkin measurement of sampling adequacy are again performed, improving from 0.911 to 0.943 as shown in figure 8.

Análisis factorial

[ConjuntoDatos6]

Prueba de KMO y Bartlett

Medida Kaiser-Meyer-Olkin de adecuación de muestreo		,943
Prueba de esfericidad de Bartlett	Aprox. Chi-cuadrado	282,516
	gl	36
	Sig.	,000

Figure 8: Kaiser-Meyer-Olkin measure for the survey of perception about volatility. Own elaboration in SPSS.

Matrices anti-imagen

	Considera usted que es importante desarrollar el concepto de volatilidad para su futura profesión.	Cree usted que es mejor dejar el análisis y la utilización la volatilidad para los "expertos ".	Piensa usted que comprender y saber utilizar el concepto volatilidad incrementaría sus posibilidades de trabajo.	Considera usted que la formación estadística que ha recibido le ayudará a entender mejor las investigaciones que se hacen en su campo de estudio.	Cuando me enfrento a un problema de estadística estoy calmado(a) y tranquilo(a).	Para mi desarrollo profesional considero que existen temas más importantes que la volatilidad estadística	Considera usted que el tema de volatilidad le permite entender la solución de problemas en su futura vida profesional.	Considera que la estadística le podría ser útil en su vida profesional.	Piensa usted que la presentación del tema de la volatilidad permitió cambiar la manera de pensar sobre la aplicabilidad de la Estadística en la formación profesional
Considera usted que es importante desarrollar el concepto de volatilidad para su futura profesión.	,974[a]	-,013	-,120	-,126	-,116	,161	-,100	-,013	-,076
Cree usted que es mejor dejar el análisis y la utilización la volatilidad para los "expertos ".	-,013	,935[a]	,263	,095	,171	-,212	,372	,137	-,019
Piensa usted que comprender y saber utilizar el concepto volatilidad incrementaría sus posibilidades de trabajo.	-,120	,263	,950[a]	-,230	-,025	,257	,006	,005	-,002
Considera usted que la formación estadística que ha recibido le ayudará a entender mejor las investigaciones que se hacen en su campo de estudio.	-,126	,095	-,230	,933[a]	-,165	-,145	,016	-,372	-,054
Cuando me enfrento a un problema de estadística estoy calmado(a) y tranquilo(a).	-,116	,171	-,025	-,165	,961[a]	,270	-,118	-,066	-,048
Para mi desarrollo profesional considero que existen temas más importantes que la volatilidad estadística	,161	-,212	,257	-,145	,270	,919[a]	-,139	,028	,282
Considera usted que el tema de volatilidad le permite entender la solución de problemas en su futura vida profesional.	-,100	,372	,006	,016	-,118	-,139	,928[a]	-,243	-,268
Considera que la estadística le podría ser útil en su vida profesional.	-,013	,137	,005	-,372	-,066	,028	-,243	,943[a]	-,170
Piensa usted que la presentación del tema de la volatilidad permitió cambiar la manera de pensar sobre la aplicabilidad de la Estadística en la formación	-,076	-,019	-,002	-,054	-,048	,282	-,268	-,170	,952[a]

Figure 9: Survey for the perception of volatility once reduced. Elaborated in SPSS

In Figure 9, the correlation indicators improve a lot, and it is evident with all the diagonal, which was highlighted in red, all over 0.9; and the matrix of rotated components shows a very strong correlation between all the factors that does not allow for another solution since the components cannot be rotated as shown in figure 10.

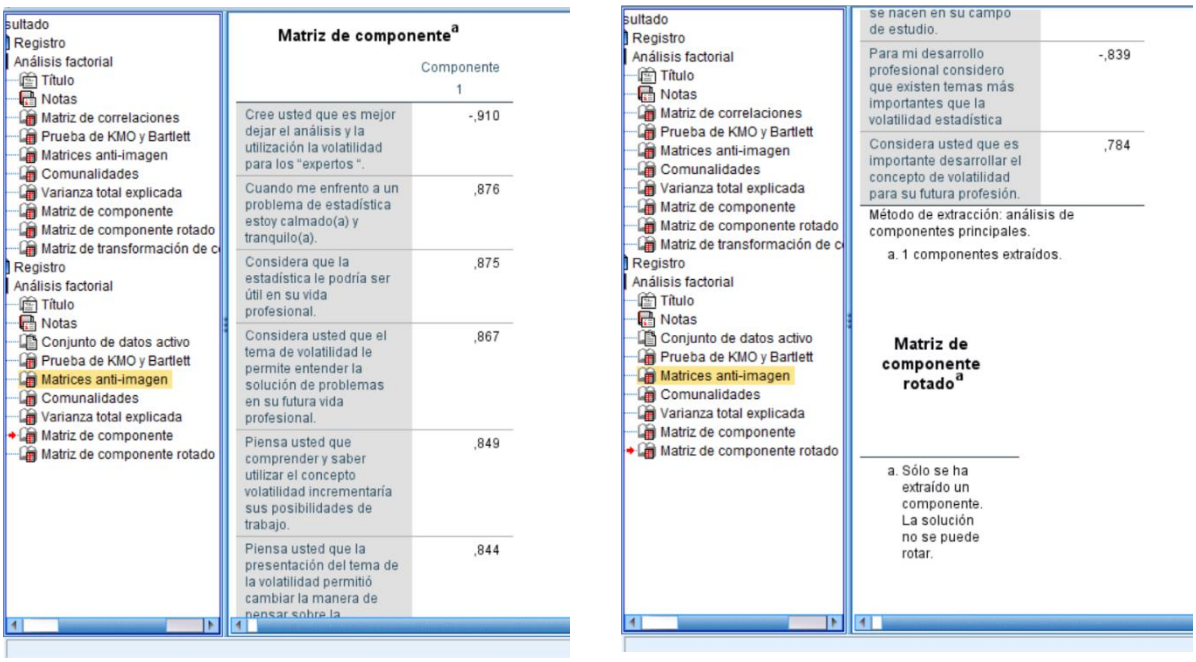


Figure 10: Factor rotation matrix for the volatility perception survey. Prepared in SPSS

From the previous factorial analysis, it can be concluded that the students correlate in a strong and positive way the elements that are displayed in each of the questions.

3. Analysis of activities through the elements of meaning

Each of the answers of the students to the problems proposed in the activities, were submitted to the analysis by means of the elements of meaning proposed by Godino (2002), however, for lack of space the totality of the analysis is not shown, only one example of the methodology used; It is clear that each of the answers for all students, must have a detailed analysis, so it is impossible to record everything in this article.

Once the example is finished, a summary table is shown with the results of each of the activities. The example above is shown below.

Example: Analysis of the third activity: "Simulation: Brownian Movement"

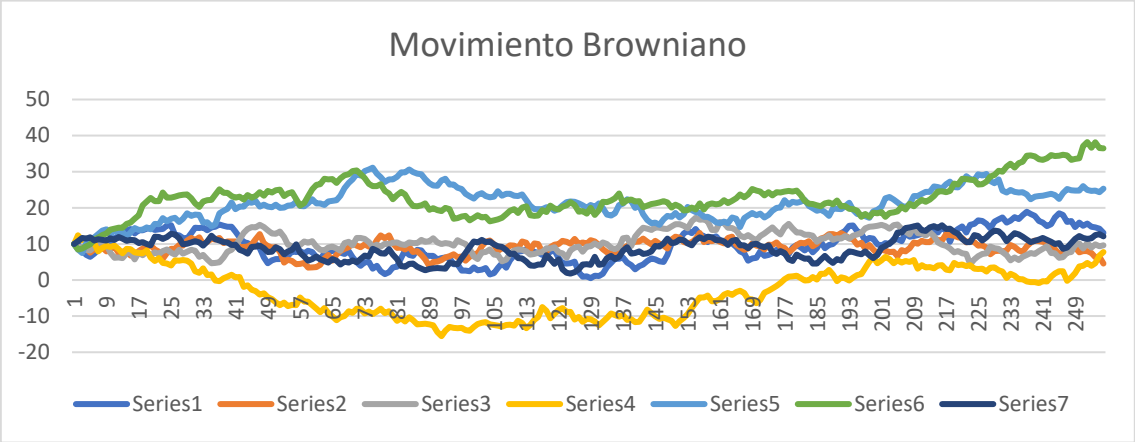
This third activity can be displayed in Díaz, M.A. (2019 p170), this consists of 4 exercises, where the student makes an approach to the topic of random walks in a simple way, with the strong motivation of already being immersed in a game called Market Watch[2]. For simulations, students place the initial price of an action and make different scenarios where they verify the effects of variance on random routes. At the beginning the students sensed that "luck" by generating random numbers distributed normally from Excel, can be manipulated in their favor; in this way they make several times the generation in order to get a better scenario. One of the objectives of the experiment is that the student must build from scratch his own simulator, only the model is provided, both of Brownian arithmetic Movement and of the geometric. Once the answers to the exercises proposed in the activity are given, the individual analysis is made through the elements present in each of them, the elements from each answer are the following: Extensive, Ostensive, Actuative, Intensive and Validative as proposed in Godino, J. D., & Batanero, C. (1994).

The example of the analysis is shown below, a student is chosen at random from among the participants and the results are recorded in a summary table with the elements present or not present in each of the activities, the answers of the student chosen for the example are shown in quotation marks.

Activity: 3. Student A. Archives 2.

1. The graph below is a simulation of the price of a share.

[2] https://www.marketwatch.com/



After analyzing the graph, what can be said about the simulation in series 4?

Answer of the student A. "It can be said that series 4 reaches negative values that are unrealistic. It is unrealistic that there are negative values in those simulations, if I buy an share, the lowest value you can take is zero".

a. Are random walks geometric or arithmetic? Justify your answers.

Answer of the student A. "Random walks are arithmetic because they have negative values in the graph, while the geometric has no negative values. "

Elements in question 1:

Extensive: Present element, since the models to be used are provided, thus the student constructs the simulator, identifies the situation of the exercise, analyzes the different scenarios, and concludes correctly. The idea of answering properly makes to take the necessary time to program the spreadsheet.

Ostensive: Present Element, properly identifies $((\boldsymbol{\mu} - \boldsymbol{\sigma^2}/2)\Delta\mathbf{t} + \boldsymbol{\sigma}\sqrt{\Delta\mathbf{t}}\,\mathbf{Z})$ and is evident in the proper programming of the spreadsheet, each element in the expression means to achieve a good programming of your worksheet, figure 11 shows how to program the spreadsheet to solve question 1.

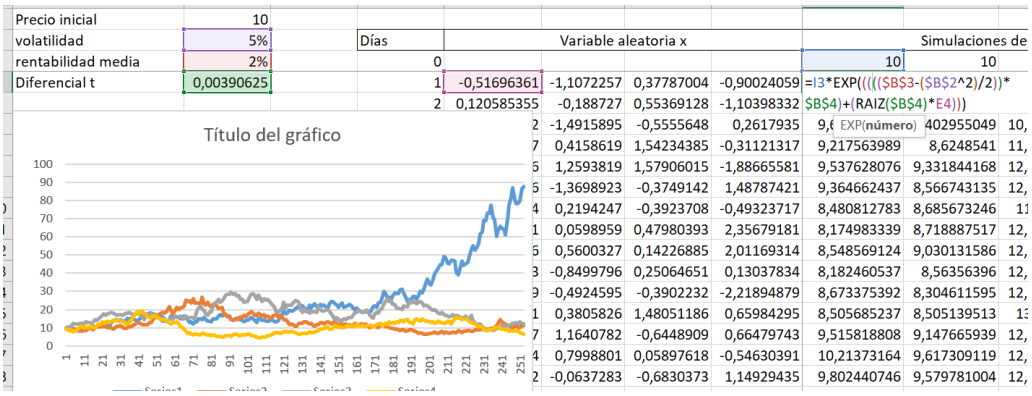


Figure 11: Proper handling of mathematical syntax, as an Ostensive element. Written by the student A.

Actuative: Present element. The student sends her solution in Word text editor but uses the Excel tools and displays two types of scenarios, shown in figure 12.

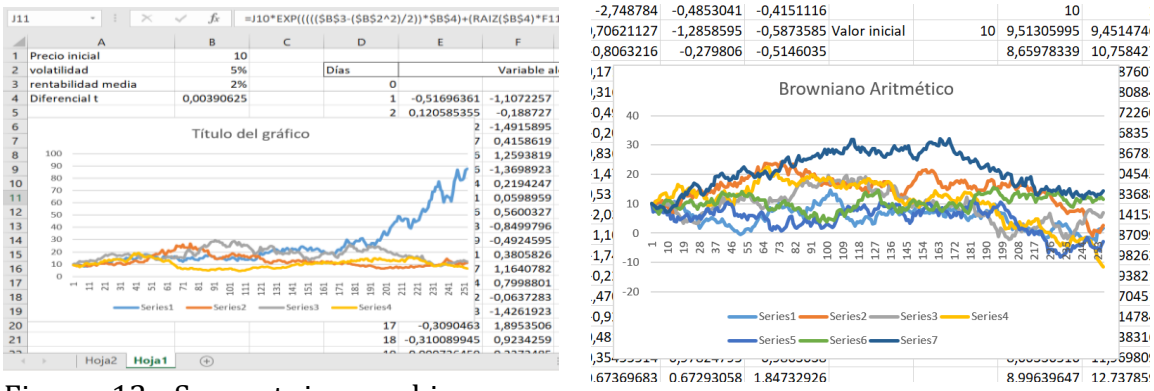
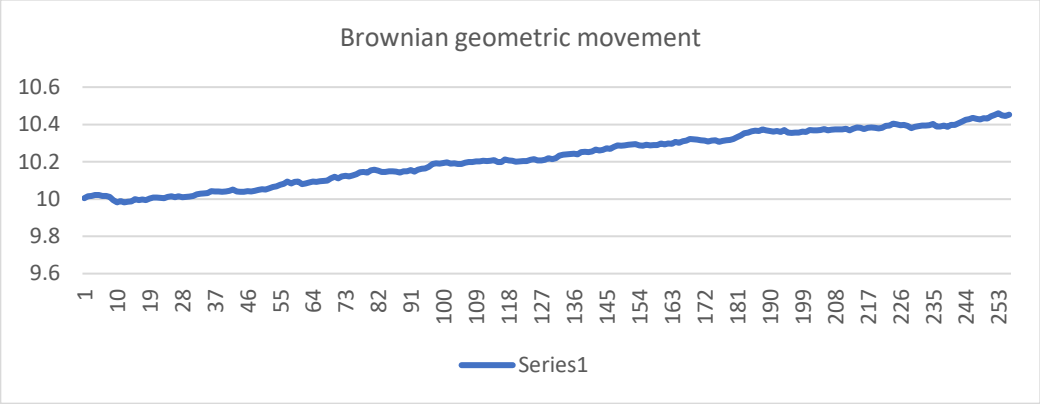


Figure 12: Support in graphics as an actuative element. Elaboration: Student A.

Intensive: Present Element: The student identifies those negative values cannot be possible, she relates finance in a spontaneous way with the Brownian movement. In the classroom the students had the same doubt, the teacher must intervene stating that the application of these models is not exclusive to finance and that there are different random variables that can take negative values such as temperature. Finally, students conclude that for geometric Brownian motion it would not be possible for negative values to exist by the exponential form of the model and relate it to the domain of an exponential function.

Validative: Present element: The student checks in different scenarios the possibility that there are negative values in the geometric, achieving correct conclusions.

2. The graph below shows a 253-day simulation of an action with an initial value of $10.



Do you consider that the bullish trend is only due to the "good luck" of the values taken by the random component in the simulations? Numerically justify your answers.

Answer of the student A. "When the variance is greater, and the measure is small the graph has an upward trend".
Elements in question 2:

Extensive: Present Element. For answering the student had to program the spreadsheet and do simulations.

Ostensive: Not present

Actuative: Element Present, takes the initiative to simulate in different scenarios and creating the worksheet.

Intensive: Not present. The student does not correlate correctly and concludes incorrectly.

Validative: Not present. The student does not validate her answers and concludes incorrectly.

3. What relationship there is between μ y σ^2 and the growth or decrease of S(t).

Answer of the student A. "The relationship between measure and variance is that having a large amplitude in variance the graph will be decreasing, on the other hand, if the measure and variance are small the graph will have an increasing behavior because the random numbers being greater the mean and the variance will be larger which increases the risk of decrease of a share".

a. Simulate S(t) for the following parameter values with a t-spread of $\frac{1}{256}$ and observe the behavior of S(t)relative to S(0).
b. Using the spreadsheet, count in each simulation the number of times that the amount $(\mu - \sigma^2/2)\Delta t + \sigma\sqrt{\Delta t}\,Z$ is positive or negative for the following three cases:

i. $\mu = 0.1\%$ y $\sigma = 10\%$ **Answer of the student A.** "507 are negative and 505 are positive".
ii. $\mu = 2\%$ y $\sigma = 10\%$ **Answer of the student A.** "506 are negative and 506 are positive".
iii. $\mu = 1\%$ y $\sigma = 40\%$ **Answer of the student A.** "508 are negative and 504 are positive".

What does the proportion of negative values mean in each case?

Answer of the student A. "Negative value means the decline of the share price".
Repeat these simulations for $N = 20$ and calculate in each case the measure and variance of S(256). Compare the average obtained in each case with S(0).
Elements in question 3:

Extensive: Element Present, to give answer elaborates its spreadsheet, to make the simulations.

Ostensive: Present Element, identifies and understands the notation used and correctly reproduces it in the spreadsheet.

Actuative: Present element, includes functions in Excel like "Count if" and the displayed of the graphs that had already elaborated, conditions their simulations to verify what is asking.

Intensive: Present Element: The student correctly concludes after preparing different simulations and identifies that the form of the exponential function is what gives the upward or downward trend of the share being simulated.

Validative: Present. Sends a file of the simulations with the functions and graphs used to validate their conclusions.

4. If you had to compare 2 assets knowing their average return and volatility, what criteria would you use? Tip: look at the random variables $\mathbf{X} = ((\boldsymbol{\mu} - \boldsymbol{\sigma^2}/\mathbf{2})\Delta\mathbf{t} + \boldsymbol{\sigma}\sqrt{\Delta\mathbf{t}}\,\mathbf{Z}$

Answer of the student A. "She doesn't answer."

Once the analysis of each of the questions in the files submitted by the 39 students is completed, a summary table is prepared about the elements present in each of the students' answers. As an example, the summary table of the elements present in activity 3 is shown. It should be noted that for the eight proposed activities the same procedure is performed and with each student.

	Activity 3				Total
	Answer 1	Answer 2	Answer 3	Answer 4	
Extensives:	90%	75%	70%	70%	76%
Ostensives:	90%	75%	70%	70%	76%
Actuatives:	98%	93%	94%	69%	89%
Intensives:	90%	95%	95%	85%	91%
Validatives	80%	80%	95%	85%	85%
					83%

Figure 13: Statistical summary of the meaning analysis with the elements proposed by Godino (2002). For the activity 3. Own elaboration.

The activity in which students performed best in the construction of robust meaning displayed on activity 3: "Simulation: Brownian Movement," believed to be motivated by the first two activities, added to the fact of the applicability of these interesting concepts in the game of the stock exchange. In contrast, the activity in which there was least construction of robust meaning was the activity "Challenge your creativity", Díaz, M.A. (2019 p182), it is considered that this reflects the impotence of

students to realize some exercises added to the fact that these exercises belong to the last academic court and students at this time are trying to recover other subjects in which they do not have good notes.

4. Example of the analysis of the construction of meaning evidenced on video graphic tests.

Planas (2006), proposes an analysis model for the study of processes of construction of mathematical knowledge, which is applied in video analysis. Due to the large amount of video material collected throughout the academic cycle.

The model proposed in Planas (2006) for the analysis of videos is summarized as follows:

1. General study, description of the video.
2. Identification of episodes of revision of mathematical meanings.
3. Search for processes of reworking these meanings.
4. Characterization of social interactions.
5. Elaboration of explanatory stories
6. Comparison of review episodes.

The details of the application methodology of each of the stages of the proposed model for the analysis of videos are not explained in this document, as they are not central to the research, they are left to the reader in the reference: Planas (2006). In this document it is impossible to systematically record all video episodes recorded in the classroom, setting an example of the application of the model for the analysis of the question:

"If the covariance between two sets of discrete consecutive random variables greater than zero is $\mathrm{Cov}_{x,y} = 2$ if one of the values of a set is 12 and one of the averages matches 12, ¿what is the minimum value that takes the random variable of the two sets?"

i. Overview, description of the video:

Each intervening student is denoted as A1, A2…An; in dialogue with the teacher.

Transcription of the main dialogue:

A1: "... and down is 13... Ahhhh! (Exclamation of surprise)"

Teacher: Is it maintained? (Referring to the average calculated previously 12).

A1: Three times?

A2: No, the ones we want, because, if we increase one up and one down, the average must be constant.

A1: Ahhh, for example, if I put here a 10...

A2: No, no... if you add a 15 now you must add a nine... if you add a 16 you must add an 8 ... (A2 Corrects his partner and has the main idea for the development of the problem).

A1: Ahhh already I understand teacher...!

Teacher: But there are two data sets! so here you would have to start playing for covariance to equal 2.

A2: But we don't even know what the other data set is!

A3: And the question tells us: what should be the minimum value that the variable takes in the two sets?

Teacher: Exactly! So, when you have the conditions given, then you look for the smallest.

A1: So, should we invent the other data set?

Teacher: Invent… no, take into account the conditions.

A3: and the covariance is taken out with respect to the two sets.

Teacher. Yes, always.

A3: But how should we know that the covariance is going to be 2, if we don't have the other set...

Teacher: (Exclamation: That's the goal of the problem, NONVERBAL)

A3: Ah, yes ... just... hmmm... forget it!... it was a stupid question...

Teacher: No, there is not stupid questions, it was fine; but if you must have two sets of data... (Returning to the problem and not to the situation) and keep in mind that they are consecutive.

A3: Okay, then if they are consecutive, it can be 16, 17, 18...! ahhhh then I already did it! (Mistakenly thinks it was the result.)

A2: If you have 10 data, the multiplication should give 20... (Student 2 begins to spin about the ideal conditions for guessing a more analytical response, rather than "testing" values in Excel.)

Teacher: Yes, ma'am, because you must give 2 and it's about n.

A1: (Analyzes the consecutive description of the data and tries to explain to the student 4, 5 and 6.) ... If one decreases, the other increases... and answer this question that I don't understand...

A4: and what is the minimum? (Thinks that it is already close to the solution, and it is not so)

A5: and how do you know...

With the student 3.

A3: It's wrong! Must the sets be the same size?

A7: Obviously!

... Three minutes of doubt.

With the student 8.

... Conjecture that does not lead to any advancement, but the student understands the concept.

With the student 8.

... conjectures that approach to a solution, without success.

With the student 2.

... This is like a square, okay? 3 squared, 2 squared, so that doesn't make sense... then we start playing by placing one more number on each side...(each end of the two datasets) already with 9 if we gave... because there are 14 data, $n = 14$ and the answer of the squares is 28... (Explaining her strategy, the student recognizes that, if they are consecutive, the product $(\bar{x} - x_i)(\bar{y} - y_i)$, it's like a "square" because those differences are always going to be the same.) and 28 divided 14 is 2, which is covariance.

Teacher: Good! do it with covariance... (In order to validate with Excel, the answer) ... the COVAR function is shown.

A2: Teacher this is too difficult!

Teacher: It's with n, they're 14 and it must be 7...

... Comments about the number of data...

A2: Then I must remove data... set the database and find the result. Ah then it is 10...

Teacher: (Checks the answer) Congratulates the student however, everyone leaves very quickly because the class had already finished 25 minutes ago.

Description of the episode:

Describing the population protagonist of this video episode, in addition to everything related to the construction of meaning; it is important to recognize the motivation of studying that some students have, for example student number 1 (A1), from an indigenous community NASA from Putumayo Colombia; the Antonio Nariño University grants them study facilities and therefore it is very common to see them in regular courses, despite its limitations in terms of communication, the lack of adequate academic foundations and extreme shyness due to its cultural features; it is important to highlight how the model helped with their self-confidence, actively participating in the solution of each problem; not necessarily reaching a solution to the problem gives

a degree of motivation, Being an active part of the class makes the student build self-confidence. It is worth highlighting in the first session with this dialog:

Teacher: Okay guys, we are not going to learn statistics... we are going to do statistics! Open Excel and let's start with....

After about 20 minutes...

A1: Teacher... what is Excel?

This conversation arises with the student 1 on the first day of class, so it is important to show the active way in which she faces problems and the motivation to learn.

Regarding student 2, she is a very disciplined student, but she succeeds in routine problems, an opinion she gave at the beginning of the course was:

"I just need a formula... I must give results and you must tell us if it is right or wrong..."

Student 2 had a hard time adjusting to the methodology, but it is undeniable that she is an extremely skillful student with numbers, in other videos she smoothly solves bigger problems such as the following:

1. Suppose you enter "Default" or bankruptcy for an investor, if it happens that que $V_t < B$, with B a certain value. Show that:

$$P(V_t < B) = \Phi\left(\frac{\ln B - \ln V_0 - (\mu - \sigma^2/2)t}{\sigma\sqrt{t}}\right)$$

Where Φ is the distribution function of a standard normal. From the above formula discuss the effect of the increase of σ on the probability of "default"

The students (A3) up to (A7), all participate actively, guessing at each stage of the process and these guesses are leading the group to obtain the solution in a very respectful, competitive environment.

ii. Identification of episodes of revision of mathematical meanings.

As stated by the author of the Planas model, "...at this stage the student recognizes doubts about the meaning and asks other participants to make their interpretations explicit". Planas (2006 p45)

Element shown in the example video and in each of the analyzed videos. Sometimes some mistake is made in the interpretation of the meaning, but in turn shows interest in finding the source of the error and how to overcome it, the author is quoted again in this sense:"... It expresses a certain block around the meaning, but it does not fail to heed the conversations of other participants about this meaning" Planas, (2006 p55). Once this behavior is characterized by the author of the model, it is

interesting to verify how this element systematically emerges in each of the videos captured in class on different occasions; This type of situation is usually overlooked in class and is really an element to consider as it has too much pedagogical relevance. It is the way in which the student manifests himself in front of the error and according to his character he confronts the situation; in this way in the same classroom before a problem that everyone is shown in the same way; multiple situations are generated according to the way each of them must face the difficulty.

In the video described above, this element is shown at every moment, as each of the students launches their hypothesis and tries to validate it in front of their classmates. Others give their positions to support or reject such statements, this learning approach correlates perfectly with the philosophical foundations of the study, evidencing the quasi-empiricism applied in the model the philosophical elements of the pedagogic activity reflected in Diaz, M.A. (2019p 77).

iii. Search for processes of reworking these meanings.

In the video transcribed above, it is inferred that the reworking of the meaning of covariance was present in the following dialogue:

A2: If you have 10 data, the multiplication should give 20... (Student 2 begins to spin about the ideal conditions for guessing a more analytical answer, rather than "testing" values in Excel.

A2: ... This is like a square, okay? 3 squared, 2 squared, so that doesn't make sense... then we start playing by placing one more number on each side...(each end of the two datasets) already with 9 if we gave... because there are 14 data, $n = 14$ and the answer of the squares is 28... (Explaining her strategy, the student recognizes that, if they are consecutive, the product $(\bar{x} - x_i)(\bar{y} - y_i)$, it's like a "square" because those differences are always going to be the same.) and 28 divided 14 is 2, which is covariance.

This reworking of meanings is precisely what is sought through the model, that the student gets closer and closer to its meaning of volatility using the means offered to him in the classroom, the problems, the computer resources, the academic discussions, the real databases, the stock market game and so on. Fortunately, in all the videos this element emerges as protagonist.

iv. Characterization of social interactions.

It can be clearly seen, in the formulation of guiding questions for a student involving 5 more students, social interaction makes the meaning emerge more naturally and spontaneously within the group. All the videos show the social interaction as an element that validates or rejects the hypotheses launched by the students in front of

some problematic situation, these interactions confirm that the model favors the construction of the meaning of volatility in each stage.

v. Elaboration of explanatory stories.

The video shows how student 2 explains her similarities in a well-versed way, student 1 tries to explain to students 4, 5 and 6, but what they really want is an approval to continue along the same line of solution. This element is very important in the construction of the robust meaning of volatility, the student looking for the proper way to express what he is visualizing in his solution, for having the approval of his classmates and the teacher, builds a language associated with robust meaning and, as a result of those explanatory stories, builds robust meaning.

vi. Comparison of review episodes

Different videos are compared and there are constantly repetitive episodes, such as:

a. The fact of working in a group seeking the approval of their colleagues.
b. Reluctance to work before the teacher's orientation.
c. The motivation to see important results. It brings the following statement from a student: "Teacher, the truth is that I never thought that I could do this"
d. The desire to "challenge" the teacher with ever deeper questions.
e. Students try to identify a leader and look for different groups to feel approved by others.
f. The spontaneous association of students who consider that they have found an advance with similar characteristics.
g. Evaluation integrated to motivation because the student feels "capable".

CONCLUSIONS

In conclusion; 100% of the videos analyzed have some element of meaning generation, everything was not recorded, because it would be inoperative to have so many cameras recording four hours of class per week, however the sample is representative and points out in some way many elements that the written proof leaves without analysis, for example, expressions of emotion, their mistakes, the different paths it takes erroneously and correctly; the spontaneous formation of working groups, the way in which they inquire themselves and the way in which they ask for help from the teacher.

It is very important that students to whom the model proposed in Osorio, M. A. D (2021) applied have sufficient trust between themselves and their teacher. Consequently, with the model, it is necessary for the teacher to provide the possibility

of participation to all his students, for practicing the conceptual wealth of a class in which the construction of meanings is the protagonist.

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