Ilkogretim Online - Elementary Education Online, 2021; Vol 20 (Issue 6): pp. 1793-1802
http://ilkogretim-online.org
doi: 10.17051/ilkonline.2021.06.173

Some Problem Situations For The Understanding Of The Notion Of Infinity.

Wilson Gordillo, Faculty of Environment, Universidad Distrital Francisco José de Caldas, Bogotá, Colombia

Wilson J. Pinzon, Faculty of Environment, Universidad Distrital Francisco José de Caldas, Bogotá, Colombia

Miguel A. Ávila A., Faculty of Engineering, Universidad Distrital Francisco José de Caldas, Bogotá, Colombia

ABSTRACT. This work presents some situations to approach the notion of infinity, which are framed in the relationship between competence, understanding and disposition. With the help of the tools proposed by the ontosemiotic approach to knowledge (OSA), primary mathematical objects are identified to validate the mathematical content to conclude that the proposed problem situations are determinants of the understanding of the notion of infinity.

Keywords: Infinity, notion, ontosemiotic approach, understanding.

I. INTRODUCTION

The notion of infinity is an intuitive mathematical concept [1], which has no determined empirical foundation and has a rigorous structure that has shown an axiomatic development. This evolution comes from Zeno to the present [2], this type of intuitive conception about infinity make connection with perceptions related to actions or situations in which this mathematical notion is presented, involving different representations [3], which are juxtaposed in problem situations when this notion is approached.

One way of approaching the concept [1], is a way of approaching intuitive thinking, primitive and opposed to scientific interpretations and conceptions, where intuition must prevail over practices in teaching [4], respecting the rigor and formalism of mathematics, so that students do not reach inadequate cognitive schemes [5].

II. THEORETICAL AND METHODOLOGICAL NOTIONS

For the development of this work, the theoretical framework known as the Ontosemiotic Approach (OSA) of mathematical cognition and instruction has been adopted [6][7]. OSA has emerged within mathematics education, with the purpose of articulating different points of view and theoretical notions about mathematical knowledge, its teaching and learning. To this end, it adopts a global perspective, considering the various dimensions or facets and the interactions between them.

Within the ontosemiotic approach (OSA) to mathematical knowledge and instruction, the notion of system of practices plays a central role from both epistemological and didactic points of view.

There are two basic ways of understanding: as a mental process or as competence [6]. These two points of view respond to divergent epistemological conceptions. Cognitive approaches in mathematics education understand understanding as a mental process. The pragmatist positions of EOS understand it as competence and not so much as a mental process (a subject is considered to understand a given mathematical object when he/she uses it competently in different practices), which implies conceiving it as knowledge and application of the rules that regulate the practice of mathematics.

The term knowledge is used in the sense of a general epistemic-cognitive construct that includes understanding, competence, and disposition [9]. Disposition, or capacity, is related to the notion of personal mathematical and didactic object, i.e., that which makes practice possible. Competence is related to the activation of the appropriate cognitive ontosemiotic configuration, suitably coupled to the epistemic ontosemiotic configuration (or ontosemiotic configuration of reference), and to the context in which the practice takes place. Understanding has to do with the relationships to be established among all the elements involved in the implementation of an epistemic and cognitive configuration suitable for a given context.

This work has a high emphasis on the characteristics of the qualitative methodology, since the interest is to link understanding, competence, and disposition through problem situations.

III. THE PROBLEM SITUATIONS

For this work, three criteria were taken into account that allow the construction of problem situations to investigate the notion of infinity, these are:

- Difference between finite and infinite.
- Infinite sequences (increasing and decreasing).
- Difference between actual and potential infinity.

Problem situation by plucking a flower:Figure 1 shows the problem situation with the objective of assigning cardinality in a set this assignment leads to difference between finite and infinite.

Observe the following image: it corresponds to a daisy flower shedding its leaves. With respect to the image, answer:

1. How many leaves should the flower have had before it started to leaf out?
2. In how many consecutive processes does it take to completely defoliate the flower?

Figure 1: Problem situation by plucking a flower, Source [10]

Problem situation the magic book: Figure 2 shows the problem situation. The objective is to associate an infinity to a countable set such as the Naturals, but then establish a relationship with a countable infinity such as the Rational.

Read carefully and answer the questions:

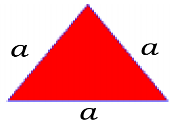
"Imagine that you have a book that is magic, the magic of the book makes that when you open the first page, the page numbering shows page 345, when you close it and open it again on the first page, the page numbering shows page 7545, when you close it and open it again on the first page, the page numbering shows page 34, so every time you close it and open the book again on the first page, it shows a different page numbering."

1. How many pages do you consider the book to have?
2. What do you think would happen if after opening the first page, you turn to the next page, what numbering would that second page have? (Remember that the book is magic)

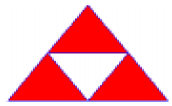
Figure2: Problem situation the magic book, Source [10]

Sierpinski's triangle problem situation: figure 3 shows the task, the objective is to activate the cognitive conflict situation.

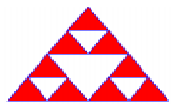
construct an equilateral triangle of side a



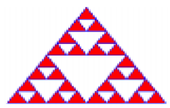
Join the midpoints of the sides



Repeat the process for each of the shaded triangles.



Repeat the process successively



with this procedure you built a triangle called the SIERPINSKI TRIANGLE.

answer the following questions according to the above procedure

1. How many shaded equilateral triangles are there in step 3?
2. If you continue the sequence of constructing equilateral triangles, can you describe when the sequence will end?

Figure3: Sierpinski's triangle problem situation, Source [10]

IV. ONTOSEMIOTIC ANALYSIS OF PROBLEM SITUATIONS

For each of the problem situations, a breakdown of the primary mathematical objects is performed to determine the mathematical content analysis.

(a) Problem situation by plucking a flower: Table 1 shows the breakdown of the primary mathematical objects identified.

Table 1: Problem situation by plucking a flower, source Authors.

PRIMARY MATHEMATICAL OBJECTS	MEANINGS
Linguistic Elements	Verbal, use of some expressions associated to the sequences with their respective graphic and symbolic representations.
Concepts	Finite set: a set that has a limited number of elements,

	generally its cardinal is represented by a natural number. Infinite set: a set that is not finite, and whose cardinal cannot be represented by a natural number or quantity. Even number: a natural number that is exactly divisible by 2. odd number: a number that is not even, that is, it is not exactly divisible by 2. Sequence: a sequence is a list of numbers or terms and there is always an order or pattern. Nth term: an expression that occupies an undetermined place in a series or sequence.
Propositions	If a set A is equipotent with the set of natural numbers ℕ , it is said to be numerable and is assigned the proper cardinal of ℕ. Any finite set is numerable because it is coordinable with a subset of ℕ.
Procedures	Association and identification of the mathematical elements presented in the problem through graphic and symbolic representation. Use of the numerical elements presented through the construction of sequences given from the definitions cited above.
Arguments	The proposed activity given from the objectives of the work pretends the visualization of the differences of the finite and the infinite, cardinality, sequences and nth term, the validations are subject to the definitions and propositions used with their relations to the term or the notion of infinity.

(b) Problem situation the magic book: Table 2 shows the breakdown of the primary mathematical objects identified.

Table 2: Problem situation the magic book, source Authors.

PRIMARY MATHEMATICAL OBJECTS	MEANINGS
Linguistic Elements	Verbal, use of some expressions associated with arithmetic, with their respective numerical representations.

Concepts	Magnitude: measure of something according to a given scale. Real number: rational or irrational number, it could also be defined as algebraic number or transcendent number. Indeterminate: something that does not have specific characteristics or lacks defined limits. Innumerable: something that cannot be counted because its elements do not have an established order. Indefinite: something that has no specific limits.
Propositions	The set of Real numbers is not countable. The set of transcendent Real numbers is not countable.
Procedures	Reading comprehension of the fragment of the magic book. Mental representation of the random events that occur with the pages of the book. Conjecture about the impossibility of ordering or numbering or counting the pages of the magic book. Order relations between the different synonyms associated with the notion of infinity, given in the mathematical context.
Arguments	The verification of the impossibility of establishing the order of the pages of the magic book involves working with random quantities that give another representation to the construction of the set of real numbers under the notion of infinity that it harbors. The notion of actual infinity is involved in this part of the activity. In the assignment of meanings or synonyms of infinity, characteristics are established from the mathematical point of view where the notions of potential and actual infinity are involved.

(c) Sierpinski's triangle problem situation: Table 3 shows the breakdown of the primary mathematical objects identified

Table 3: Problem situation the magic book, source Authors.

PRIMARY MATHEMATICAL OBJECTS	MEANINGS
Linguistic Elements	TRIANGLE: a rectilinear figure is one that is comprised of straight lines, trilinear those comprised of three. Figure: is that which is contained by any limit or limits. angle: an angle is the opening between two straight lines drawn from the same point. these straight lines are called sides of the angle and the common point, vertex. side: in a polygon, a side is a line segment whose ends are at two consecutive vertices of the polygon. Equilateral's triangle: a triangular figure, with sides of equal length. Nth term: an expression that occupies an undetermined place in a series or sequence. Succession: an ordered set of numbers that can be divided into arithmetic sequences and geometric sequences. Divergent: it is the separation or difference between 2 or more elements convergent: union of two or more elements that converge to the same point. Contoured: if a is a non-empty set, any application of $\mathbb{N}$ on a is called a succession of elements of a. in particular, a succession of real numbers is an application of $\mathbb{N}$ on $\mathbb{R}$
Concepts	Magnitude: measure of something according to a given scale. Real number: rational or irrational number, it could also be defined as algebraic number or transcendent number. Indeterminate: something that does not have specific characteristics or lacks defined limits. Innumerable: something that cannot be counted because its elements do not have an established order. Indefinite: something that has no specific limits.
Propositions	If A is a nonempty set, any application of $\mathbb{N}$ on A is called a succession of elements of A. In particular, a succession of real numbers is an application of $\mathbb{N}$ on $\mathbb{R}$.

	Let $\{x_n\}$ be a sequence of real numbers and let $x \in \mathbb{R}$. We say that $\{x_n\}$ converges to x, and we write $\{x_n\} \rightarrow x$, when, for every positive real number ε, a natural number m can be found such that one has $\lvert x_n - x \rvert < \varepsilon$ for any $n \in \mathbb{N}$ that verifies $n \geq m$. Thus, symbolically: $\{x_n\} \rightarrow x \Leftrightarrow \forall \varepsilon > 0, \exists m \in N : n \geq m \Rightarrow \lvert xn - x \rvert < \varepsilon$. Any convergent sequence is bounded.
Procedures	The action of bisecting segments of equilateral triangles originated from bisections of the previous sides of the previous equilateral triangles is performed, making it a repetitive process, and the relationship between the continuity of the figure and the elements that conform it is established. The type of succession or successions is characterized, according to what has been seen in the sequences and their behavior.
Arguments	The notion of current infinity is involved, given that when performing the process in a defined equilateral triangle, infinite geometric points or places are found that will originate more equilateral triangles that will approach, in this case the limit is a point where the terms associated to the elements of these equilateral triangles of an infinite sequence of processes given the nature of the problem situation progressively approach.

CONCLUSIONS AND RECOMMENDATIONS

These proposed problem situations can intuitively determine the approach to the notion of infinity, the different representations [3], and contexts in which the notion is approached, the validity through the ontosemiotic analysis tool [10] applied to each of the proposed situations, guarantees the mathematical content and the formality that leads to the fact that this notion can be worked in basic education students [11] enriching the curriculum and the formation of students, in the same way this proposal of problem situations in the notion of infinity collaborate in the construction of other concepts that form the calculus [10][12].

These problem situations for schoolwork in mathematics are directed towards the conception of mathematical systems and knowledge of the numerical universes given from logic and the conjunctive, particularities are given to the analytical systems and their representations, and although few indications were given in previously exposed works about the diversity of representations that each one of the mathematical objects associated to the notions of infinity have.

REFERENCES

[1] J. VEGA, "Workshop: Conceptions of current infinity: Software-mediated analysis". I CEMACYC. (2013): 1-11.

[2] I. D.BUITRAGO, C. GAVIRIAandC, MÁRQUEZ,"Some mathematical concepts associated with infinity". Ing. USMed, 5.1.(2014): 96-99.

[3] R. DUVAL, "Representation, vision and visualization: cognitive functions in mathematical thinking, basic issues for learning". PME 23 (1999): 3-26.

[4] Y. CHEVALLARD, "The didactic transposition of knowledge to knowledge taught". Buenos Aires: Aique. (1991)

[5] M. SOCAS, M. (1997): "Difficulties, obstacles and errors in the learning of Mathematics in Secondary Education.", cap. 5., in RICO, L. et al."Mathematics Education in Secondary Education". Ed. Horsori, Barcelona. (1997): 125-154.

[6] J.D. GODINO, C. BATANERO, AND V. FONT, "The onto-semiotic approach to research in mathematics education", ZDM Mathematics Education, Vol. 39 issue. 1, (2007): 127-135.

[7] P. LESTÓN,"Ideas prior to the construction of infinity in non-school scenarios". (PhD Thesis) IPN, México,(2007)

[8] L. R. PINO-FAN, V. FONT, W. GORDILLO, V. LARIOS, and A. BREDA, "Analysis of the Meanings of the Antiderivative Used by Students of the First Engineering Courses", International Journal of Science and Mathematics Education, vol. 16, issue. 6, (2017): 1091-1113. https://doi.org/10.1007/ s10763-017-9826-2

[9] L. PINO-FAN, J. D. GODINO, J. D., AND V. FONT," Didactic-mathematical knowledge on the teaching and learning of the derivative. Memories XIII Winter School in Educational Mathematics, InstitutoTecnológico y de EstudiosSuperiores de Monterrey, Monterrey, Nuevo León, México. (2010): 206-213.

[10] W. GORDILLO, M. ÁVILA, and W. PINZÓN. "A content analysis and validity for a problem situation associated with the antiderivative". Elementary Education Online, Vol. 20, Issue 6: (2021):1437-1445.https://doi.org/10.17051/ilkonline.2021.06.145.

[11] I. AYALA, "Analysis of the comprehension of the notion of infinity: a case study with elementary school students" (bachelor's degree thesis), Universidad Distrital, (2019).

[13]W. PINZÓN, W. GORDILLO, and M. ÁVILA,"Some analysis tools to identify difficulties in the teaching and learning of the integral concept". Elementary Education

Online, Vol. 20, Issue 6, (2021): 1428-1436.https://doi: 10.17051/ilkonline.2021.06.144