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Gesture As A Semiotic Means Of Objectifying The Slope, In A Calculus Course

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ABSTRACT. A case study is presented with students entering university, in order to determine semiotic means of objectification and gestures that emerge when they analyze a curve to interpret a variation phenomenon. Three group tasks are designed and applied, with the purpose of identifying cultural knowledge that promote and strengthen mathematical ideas. The productions are analyzed through three proposed categories with which some notions and gestural relations to which the students resort in their arguments are concluded.

Keywords: Gesture, slope, semiotic means of objectification.

I. INTRODUCTION

Mehrabian [1] states that in communication only 7% of a message is based on words, while 38% comes from the tone of voice and 55% from body language. In other words, the body speaks louder than the voice and words, also suggesting that emotions are one of the main referents of communication. Then, the International Group for the Psychology of Mathematics Education-2005 recognized the importance of gesture in the learning processes of mathematics, in [2] they analyzed how gesture contributes to the construction of meaning of mathematical concepts and, as Vergel [3] states, the importance of the study of gesture lies in recognizing that through it is possible to materialize intentions, besides being an integral element, not peripheral, in students' ways of thinking, as a semiotic means of objectification, it is relevant in the expression of students' intentions and in their conceptualization process.

Research [4] [5] establishes that gesture is often used when the circumstances in which communication takes place make it difficult or impossible for verbal words to be received, it is capable of communicating ideas or mental representations that are impossible to communicate with speech. Thus, gesture becomes a form of communication as important as speech that does not depend on verbal sentence.

For Edwards [2] gestures are part of the solution of a mathematical problem; they are not restricted to being simple illustrations of the objects referred to in verbal explanations. The use of bodily expressions can create cognitive mechanisms for mathematical abstraction, states [6], for his part, Radford [7], determines that thought is not only produced in the mind, but also through a sophisticated coordination of language, body, gestures, symbols and tools; where [8] the gesture is presented with a mediating character in the processes of problem solving, allowing to realize the conceptual aspects that, due to their own generality, cannot be completely shown in the concrete world.
Miranda [9] proposes that the analysis of the learning process should take into account the relationship of gesture with other semiotic systems, in order to deepen the Vygotskian idea of the mind as a unity of material and ideational elements, where the enrichment of one implies the enrichment of the other, in a dialectical relationship.

Carrasco and Díaz [10] approach the study of figuration practices, understood as the use of axes or planes in dimensions to represent a variation phenomenon -among students compared to Newton's figuration practices- with respect to the study of the movement of a mobile, with the purpose of establishing links between them. These practices involve sociocultural, cognitive and mathematical aspects, i.e., they are social constructions that are studied within the framework of socioepistemology and the line of variational thinking [11]. From this background and research [12] [13], the central question of this paper is: Can gesture be a semiotic means of objectifying the slope in an engineering course?

II. METHODOLOGY

This research is framed within a qualitative research approach, of a descriptive and interpretative type that aims to determine whether gesture can be a semiotic means of objectification of slope, in a calculus course when approaching tasks involving the idea of slope. Following the ideas of the Theory of Objectification (TO), within a multimodal analysis [2]; that is, relating the various semiotic systems mobilized during an activity of an initial calculus course. The bet is then, to analyze gestures, movement, written and audiovisual records simultaneously. The subjects of this study were engineering students taking calculus for the first time. The students' data were obtained through worksheets and the recording of a video in the performance of three proposed tasks, in small group work, where the episodes are selected for analysis, in which the interaction of the answers given by the students is used.

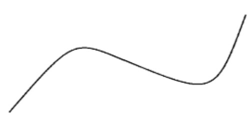
III. CONJECTURES

Each of the tasks was designed to investigate in the students the gestures that emerge in the activity when they interact with their peers and evolve in their complexity, seeking to observe the transformation of the gestures.

Three tasks were posed to the groups of students, which were designed within the cultural knowledge of the group and it is not that all students address all the tasks proposed here, but that each group develops them as far as they can with the collaboration of the teacher, within the TO methodology. Through the task, the student works in small groups, with their respective recording, where it is seen that the student needs this outside to be able to develop, which is why the resources they use to position themselves in the classroom and the discussions between students, groups and the teacher are important, These discussions seek the activation of several semiotic resources synchronically and of the semiotic means of subjectivation, where the gestures that they use to be able to position themselves in the classroom are seen, being of interest the activities linked to the social character strata that are made when generating those mathematics subject to the way a culture looks at its knowledge and the function that this knowledge develops or plays in that culture.

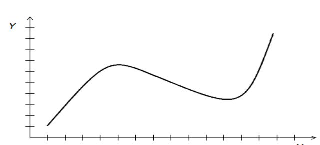
Each of the tasks is designed in a cultural context specific to the students' knowledge and seeks to find out what gestures emerge in the activity with their peers.

Table 1: Homework 1 proposed to the students., source Authors.
Homework 1:

	A mobile is moving on the track represented by the attached graph, where does the mobile have the greatest degree of inclination? Justify your answer.

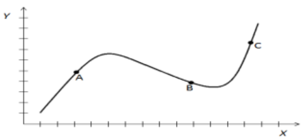
This homework aims to explore gestures, with the tracing of Cartesian axes for orientation, location of points or straight lines in a way that represents the inclination of the curve, as well as it is expected that in the dialogue with his peer, he interacts in speech and involuntary movement of arms or hands to explain a concept to his partner.

Table 2: Homework 2 proposed to students, source Authors.
Homework 2:

	A mobile is moving on the track represented by the attached graph, where does the mobile have the greatest degree of inclination? Justify your answer.

This homework aims to explore gestures, on the location of points or straight lines, in a way that represents the inclination of the curve, just as it is expected that in the dialogue with their peer, they interact in speech and involuntary movement of arms or hands to explain a concept to their partner.

Table 3: Homework 3 proposed to the students., source Authors.
Homework 3



A mobile is moving on the track represented by the attached graph, where does the mobile have the greatest degree of inclination? Justify your answer.

This homework seeks to explore gestures, tangent lines at the given points in a way that represents the inclination of the curve, as well as it is expected that in the dialogue with their peer, they interact in speech and present involuntary arm or hand movements to explain a concept to their partner.

IV. RESULTS

The students' responses in the activity are presented below.

Homework 1:

Group 1

Transcription of the justification

"Judging by the appearances of the curve the final section has the steepest slope. In addition, the angle formed by the curve compared to a vertical straight line is clearly smaller in the final section."

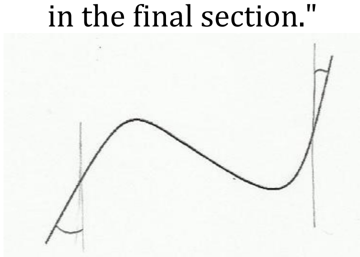


Figure1. Homework answer to worksheet 1, group 1

Group 2

Transcription of the justification

“One of the two thinks that slope is the notion of slope and the other thinks that it is where you get more slope for the first case the points 1 and 2 and for the second case we have the point 3”

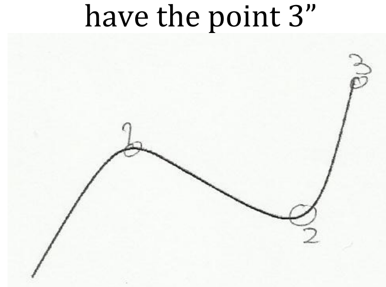


Figure2. Homework answer to worksheet 1, group 2.

Homework 2:

Group1 Justification transcript	Group 2 Justification transcript
“By means of a not very regular measurement of the coordinates and taking as a reference the advance of the space on the OY axis with respect to a space on the OX axis, we can verify that in the final section of the curve is the greatest slope.”	“We can see that where it is more inclined depends on the reference point we take, because if we take as reference the X axis, it will be more inclined where the slope is greater and if we take as reference the Y axis, it will be when the slope is greater.”

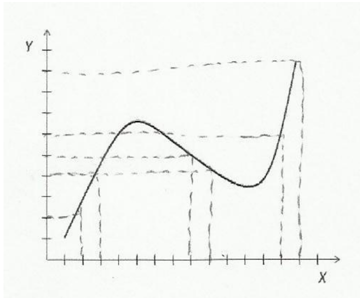


Figure3. Homework answer to worksheet 2, group 1

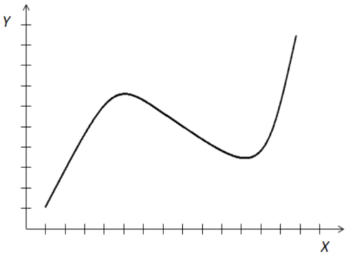


Figure4. Homework answer to worksheet 2, group 2

Homework3:

Group 1	Group 2
Justification transcript	Justification transcript
"Guided by the appearance of the curve, we analyze the coordinates of each section of the curve, finally finding that the slope at point C is greater than the slope of points A and B."	"A is impossible to be steeper since its slope is between those of B and C, now if we take as reference the X axis, C will be the steeper one, and if we take as reference the Y axis, B will be the steeper one."

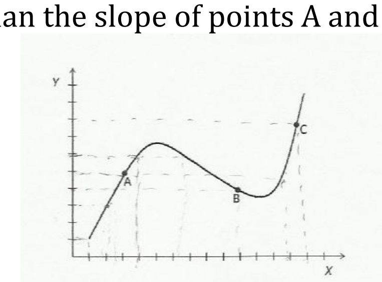


Figure5. Graph 3. Homework answer to worksheet 3, group 1

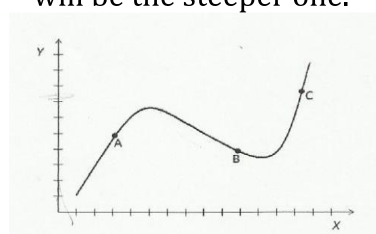


Figure6. Graph 3. Homework answer to worksheet 3, group 2

V. CONTRASTING THE ANSWERS WITH THE CONJECTURES

To analyze and contrast the answers given by the groups, categories of analysis are elaborated to detect elements of the study:

- The figuration: additional traces on the curve presented (traces, straight lines, tangents, signs, symbols).
- Justification: How the answer to the question was justified (form to describe variation phenomena).
- The gesture: movement of hands and arms accompanied by argumentative speech to the question (analysis of the video in the activity).

Homework1, category figuration

Group 1:	Group 2:

This group traces vertical lines on the curve making angles location on the curve to try to measure inclinations in comparison with angles in order to determine the inclination of the curve, this trace denotes a figuration of location without resorting to the Cartesian. (Graph 1)	This group indicates with small circles, numbered points of inflection of the curve, this figuration of signaling, shows that the slope analysis must be in these points of inflection, there is no allusion to the concavity. (Graph 2)
Homework 1, category justification	
Group 1: The variation is determined by the angle plotted on the graph, where the comparison between them involves this process	Group 2: They describe their discrepancy, referring to the variation, with the use of the word slope
Homework 2, category figuration	
Group 1: The graph, was approached with extension of dashed lines starting from the axes, in order to mark some points on the curve for referencing, inclination. (Graph 3)	Group 2: There are no marks on the graph, it was only visualized. (Graph 4)
Homework 2, category justification	
Group 1: Reference is made to the visual difference, because there are no calculations to the contrary, implying that this variation is given by this difference between the axes.	Group 2: Reference is made to the Cartesian plane and the axes, to introduce the word slope, onlydescribing that the slope is greater depending on where you look at the graph with respect to the axes.
Homework 3, category figuration	
Group 1: There are dotted lines from the axes to the given points, in order to determine position of the points with respect to the plane and the curve. (Graph 5)	Group 2: There are no marks on the graph, only visualized. (Graph 6)
Homework3, category justification	
Group 1: Dotted lines make identify slopes at the points, determined that point C is the steepest point.	Group 2: They use the word "impossible" to make comparisons between the three given points and the X and Y axes.
Homework 1, 2 and 3, gesture category	

The analysis of the video shows, in some of the episodes, the use of gestures with the arm to refer to slopes or inclination of straight lines.

Figure7. Gesture for positive slope

Figure8. Gesture for negative slope

Figure9. Gesture for a constant slope

Figure10. Gesture for zero slope

VI. CONCLUSIONS

According to the categories of analysis proposed, it can be established that in the interpretation of the graph, associated with the category of figuration, students intervene the graphs, extending the curve or adding axes or making marks, as evidenced in [10]. Such interventions seek to visualize and/or focus the interpretative gaze of the graph. In the category of justification, the students' written answers assume as an argument the visual perception, "at a glance" of the figure.

This seems to hinder the construction of mathematical written arguments, as opposed to the diversity of oral and gestural arguments during their activity. Regarding gestures, in agreement with [6], we identified gestures or movements of the students that contribute to the communication and justification to their classmates of the ideas they raise.

In particular, the inclination of a curve referred to as the slope that is associated to the inclination of the straight segment of their arm graphs 7, 8, 9 and 10, accompanied by the orality that associates the concept of slope. As evidenced in the selected episodes of the filmic record, the categories are interrelated in the construction of the responses to the activity.

The use of the intervention of the figures articulated in the orality of the explanation to their partner, shows how germinal arguments or ideas emerge to approach variational thinking, which cannot be translated into written mathematical formalization, but can constitute links for the teaching of concepts in calculus. On the part of the researchers, there is a starting point to generate and build didactic proposals that address the interrelation shown between figuration, gesture and formalism of mathematics for the appropriation of concepts.

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