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Using Bayesian Approach In Handling Mortality Ratios Deviations

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Abstract

Using Bayesian Approach in Handling Mortality Ratios Deviations was emerged as this research idea to meet the need of a scientific model handling the deviations resulting from using standard mortality tables that are inconsistent with the nature of the Egyptian context,where Bayesian is a high degree of precision statistical estimation approach depends on mixing information derived from previous experience with newly collected data, has a high degree of precision. The fundamental and influential difference between mortality rates derived from the experience outcomes in the Egyptian market and mortality rates derived from tables used in pricing of life insurance contracts. The results found the equation of the general trend line for safety margin rates is a sixth-degreepolynomial equation with a significant level of 99%. Equation used to calculate the value of mortality rates based on tabular values was estimated.

Keywords: Bayesian approach, mortality rates, life insurance contracts.

1. INTRODUCION

The cost of life insurance in society depends on three key elements, namely prevailing mortality ratios, applicable technical interest ratios and expenses ratios, as well as society burdens and prevailing economic and social conditions (Abid at el., (2005), Lauderdale at el. (2002), Benjamin&Pollard (1993), Robert (2015), Denison&at el. (2002). Given that prevailing mortality ratios in society are difficult to be estimated accurately due to the high cost of creating mortality tables in light of the need for expert and technician teams to be created in many developing societies, including the Egyptian society, therefore, life insurance companies working in the Egyptian insurance market rely on standard mortality tables derived from other communities experience not on real experience of the Egyptian context. This led to the lack of fair pricing in life insurance sector products in the Egyptian market. Moreover, we must consider previous experience when calculating the cost of these products, represented in distribution and volume of the insured claims during the previous period until we reach a proper system to price life insurance policies.The price rises of life insurance policies is a main reason for its low demand in Egyptian market (Reference). Moreover, using standard mortality tables not derived from the insured experience is one of the most important reasons behind the high cost of insurance protection, as life insurance companies rely on conservative mortality tables, i.e. the English tables AKA (A49/52ULTIMATE). This led to the fact that the mortality ratios derived from these tables are higher than the actual mortality ratios derived from experience of the insured with life insurance companies. This can be clarified through data in table (1) where there are clear deviations between expected estimated mortality claims in insurance companies operating in Egyptian market, which were calculated based on standard mortality ratios and the actual claims values derived from the experience of the insured with these companies. This deviation is an important reason for the high cost of insurance protection in insurance sectors.

Table (1) Analysis of excess mortality in life insurance sector in Egyptian market for the year ended on 30/06/2019

Company Name:	Actual value	Expected value	Excess	Deficit
Misr Insurance	209540000	273649000	64109000	-
Al-Sharq Insurance	33861518	49620723	15759205	-
Allianz	1546470	7956405	6409935	-
Mohandes Insurance Co.	5891876	7580814	1688938	-
N.S.G.P	217117	62568	-	154549
Al Tijari	210879	2019335	1794456	-
Delta Insurance	1554782	2742787	1188005	-
Pharaonic Insurance Company	7588368	9992953	2404585	-
Total	260411010	353610585	93199575	-

The research problem is represented in the lack of a scientific model to address the deviations resulting from using standard mortality tables that are inconsistent with the nature of the Egyptian context. These deviations are one of the most important factors affecting the high cost of life insurance in the Egyptian context, thus decreasing the demand for products of this sector. This research aims to propose a quantitative model to address deviations in mortality ratios in Egyptian life insurance market arising from the use of standard mortality tables inconsistent with such market, by providing a scientific method to determine the following variables:

- Safety margin to be added to or subtracted from the standard mortality ratios used in the Egyptian life insurance market, which are derived from standard tables used in English life insurance market, so that it can come close to the actual ratio and reach the most appropriate estimate of mortality in life insurance companies, thus reaching the fair cost of insurance protection service.
- Use of statistical method (Bayesian approach) to estimate the mortality loss ratio in Egyptian life insurance companies (mortality loss ratio) to be able to assess the technical performance of Egyptian life insurance companies based on this relatively recent concept in Egyptian insurance market.

There is lack of studies on addressing deviations in mortality rates arising from use of standard mortality tables not derived from experience of insured with insurance companies, in order to achieve the concept of justice in calculating cost of life insurance protection service.

2. LITERATURE REVIEW

The early work inAbid at el., (2005) addressed the actuarial methods used to settle crude mortality rate (CMRrelying on the population tables of the Kingdom of Saudi Arabia and aimed to develop a special formula to reflect the Saudi society experience in settling the mortality tables. The study concluded that equation is the most appropriate equation used in the settlement and concluded an estimate of the financial substitution tables depending on mortality rates derived(Heligman& Pollard (1980).While in Bravo (2010) addressed a proposed model for creating mortality tables to reflect the experience of small communities, relying on the statistical data of Portugal, used parametric mortality ratios settlement methods based on GM equations and results precision was tested. and concluded that the (GM (3,6)) function is the most appropriate function used in CMR settlement, reflecting the experience of small communities, subject of the study.

Later on,Contador(2012)aimed to create a life table for experience of the insured with insurance companies in Brazil. The study population included all insured records at 23 Brazilian insurance companies, which is considered statistically sufficient to create a reliable life table for pricing life insurance products. The study relied on equation in concluded CMR settlement(see Benjamin&Pollard (1993)). The advantage of relying on this equation is the interpretation of general trend of mortality curve within different ages, which was divided into three phases. The study concluded substitution tables for both males and females reflecting the insured experience in Brazil. Furthermore,Cutler&at el. (2006)relied on estimating mortality ratios in a specific community based on indicators that rely on community experience in application to the United Republic of Tanzania. This is done by comparing community-based mortality ratios estimates with mortality ratios derived from experience of health agencies along with measuring different mortality levels. The research team used the data available in civil records for living persons' numbers, as well as the mortality numbers with a focus on the cause of death. The research team relied on a sample consisting of two relatively large cities from the United Republic of Tanzania. Relative mortality ratios derived from civil records were used to estimate standard mortality ratios. The research adopted the international classification of diseases and related health problems, as well as dividing the community into five-year age groups. The research team concluded mortality tables based on health status, dividing them according to the health status of each individual separately.

Arias(2006) included an analysis of the most important quantitative indicators contained in the life tables of the United States of America in 2003. The study addressed the actuarial bases adopted to create this table, as well as the demographic divisions addressed. The study concluded that life expectancy is increasing annually by 0.2 for whites and 0.3 annually for blacks about females. For males, the increase rate was 0.2 for whites and 0.1 for blacks. Researchers attributed this result to improvement in health level. The study succeeded in creating life tables with built-in categories to be more realistic and simpler to use for life insurance companies, reflecting improvement in life expectancy results.

Finally, Butt & Haberman (2004) tested the possibility of using two models for mortality ratios settlement (GM and Gauss inverse) to reach the most appropriate models that lead to the highest precision in addition to reducing the linear correlation. The study concluded that GM models offer more successful results for crude data.

This research is based on main hypothesis that the standard mortality tables used in the Egyptian context is inconsistent with the actual reality of the market. This is more accurately translated in significant deviations between standard mortality ratios used in calculating the cost of insurance protection service and actual mortality ratios derived from the actual experience of the insured with life insurance companies, leading to higher costs of life insurance sector products. Therefore, this hypothesis can be formulated and be subject to testing and statistical analysis as follows:

- There is no statistically significant difference between standard mortality ratios used in Egyptian life insurance companies and actual mortality ratios derived from experience of insurance companies.
- There is no statistically significant difference between high costs of life insurance in Egyptian market and the deviation between both standard and actual mortality ratios.

3. RESEARCH APPROACH

The research adopts the Bayesian Approachfor analyzing data and estimating appropriate parameters and indicators to test the research hypothesis to estimate the safety margin variable required to address the deviations between standard mortality ratios and actual mortality ratios derived from actual experience of the insured with life insurance companies. The main idea of this statistical method is that there is additional information from the previous experience about the parameter on which the probable distribution of a particular phenomenon depends. This parameter takes different values and there are evidence that this parameter changes and that this change and additional information available to us as a result of previous experience can be represented by a probable distribution, i.e. this parameter becomes a random variable with a probable distribution (Abid at el., (2005), Lauderdale at el. (2002), Benjamin&Pollard (1993), Robert (2015), Denison at el. (2002).

4. RESEARCH METHOD

This research study is based on two complementary methods:

- Academic study method (theoretical): to cover the points that require a theoretical study in which the researcher relies on scientific references available scientific research, articles closely related to the research subject.
- Field study method (practical): depends on gathering published and unpublished detailed data on the study subject, which can be used to achieve the research objectives.

5. RESEARCH LIMITS

The research limits are: -

(1) Study period was five years observation data (2014/2015 to 2018/2019).
(2) Scope of research (place limits): Egyptian market, 18% of valid trade premiums, 17% of total valid insurance amounts and 24% of total insurance policies issued this year.
(3) The nature of data (objective limits): The research is limited to valid individual life insurance policies and the researcher relied on insurance policies in case of death only. This study relied on two types of published and unpublished data in general, which represent expected and actual claim amounts of all life insurance companies in the Egyptian market in details.

6. METHODS

There was a need of tool to enable those responsible for pricing life insurance policies to access the highest fairness degree in pricing. Accordingly, the researcher offers a model that uses the environmental method as a statistical tool that enables its users to estimate and predict with high precision. The model depends on combining the experience of insurance companies alone, which can be represented by available data during the past period and the standard tables used in pricing to reach closer mortality ratios of experience of the insured with insurance companies representing the community experience (see Hardy&Panjer(1996), Joseph& at el. (2015). Leonard& Hsu (1999), Fitzgerald (1996), Nelder&Verrall(1997).Hardy&Panjer (1996)). Below is a detailed explanation of the model: Let's consider $q_1, q_2, \dots, q_n$ to be the mortality ratios of different ages, which represents a random sample taken from a common distribution with a unknown value vector for the parameter θ, since the unknown parameter θ is treated as a random variable, knowing that $\theta^* = \theta$, therefore, q1 is independent of the probability density function that takes the shape of $f(q|\theta)$, which refers to the likelihood function of the variable q_i given the parameter θ. Usually in Bayesian statistics,(see Benjamin&Pollard (1993), Silcocks& at el. (2015), Verrall (1995),Mohamed (1997))density functions are indexed by their order, so that $f(q)$ and $f(\theta)$ may have different density functions. The possible values of $\theta^*$are obtained by using the density function $f(\theta)$ as well as the prior density function (for prior distribution). Bayes' theorem states that knowing that$\overline{q} = (q_1, q_2, \dots, q_n)$ the distribution function of the posterior probability density of the parameter θ is:

$$f(\theta|\overline{q}) = \frac{f(\overline{q}|\theta).\,f(\theta)}{f(\overline{q})}, \qquad (1)$$

where it does not depend on the value of parameter θ, therefore:

$$f(\theta|\overline{q}) = kf(\overline{q}|\theta).\,f(\theta), \qquad (2)$$

whereas k is a constant that depends on the value of parameter θ and arithmetic mean of parameter θ can be estimated as follows:

$$m(\theta) = E(\theta|\overline{q}) = \int q\, f(\overline{q}|\theta)dq. \qquad (3)$$

Accordingly, we can estimate the value of m(θ), which reduces the average value of mean-square error for posterior distribution after realizing the number of n of the ratios $\overline{q} = (q_1, q_2, \dots, q_n)$ as follows:

$$m(\overline{q}) = E(m(\theta)|\overline{q}) = \int m(\theta)\, f(\overline{q}|\theta)dq. \qquad (4)$$

Accordingly, we can estimate the distribution density function generated as follows:

$$f(y|\overline{q}) = \int f(y|\theta)\, f(\theta|\overline{q})d\theta. \qquad (5)$$

After explaining the general framework of the used model, we will show below the detailed used model based on the Bayesian model (Normal/Normal), which is based on a fundamental assumption that must be tested first before running the model, being that the probability density function for the prior distribution of data follows the normal distribution,(seeGary (2019), Contador(2012), Abid (2006), Andreev&Vaupel(2005)), and the probability density function for the posterior distribution of data also follows the normal distribution; accordingly, we can describe the detailed used model (see Beltrão& at el.(2005), Bongaarts& Feeney (2003)), as follows:

If we have data on the variable in question, which in this research represents the actual mortality rates during the study period, which can be referred to as: $q_1, q_2, \ldots, q_n$, where they represent values with normal distribution, which is the hypothesis on which the model is built, which will be tested and has an unknown mean value referred to as parameter θ and known degree of precision p (the term precision can be used as an alternative to variance), (the precision is the multiplicative inverse of the variance); accordingly, the probability density function of prior distribution (seeBongaarts& Feeney (2003)) becomes:

$$f(\overline{q}|\theta) = \prod_{i=1}^{n} f(q_i|\theta) = p\left(\frac{\frac{n}{2}}{2\pi}\right) e^{-\frac{p}{2}\Sigma(q_{i-\theta})^2} \tag{6}$$

The previous function represents the prior distribution of the parameter θ. For a small sample whose arithmetic mean is unknown, it is the same as a normal distribution with a known mean (m) and a degree of precision which can be represented by the following probability density function for the parameter θ (seeCarlin& at el. (1996)):

$$f(\theta) = \sqrt{\frac{Q}{2\pi}}.\, e^{-\frac{Q}{2}(\theta-m)^2}, \tag{7}$$

therefore, according to Bayes' theorem, the probability density function of the posterior distribution will be as follows:

$$f(\theta|\overline{q}) = k\, f(\overline{q}|\theta).\, p(\theta) = k.\, e^{-\frac{1}{2}(p\,\Sigma(q_{i-\theta})^2 + Q(\theta-m)^2} = k.\, e^{\left[\frac{-np+Q}{2}\right]\left[\frac{\theta-(n.p.q+Q.m)}{(np+Q)^2}\right]} \tag{8}$$

Since k is a constant depends on value of the parameter θ, hence the posterior distribution of the values of θ also has a normal distribution and probability function has the same normal distribution and the prior distribution also follows the normal distribution (see Mohamed (2008).). The posterior mean value will be as follows:

$$E(\theta^*|\overline{q}) = \frac{np\overline{q} + Qm}{np + Q}. \tag{9}$$

If the value of n is small compared to the degree of precision $(\frac{Q}{p})$, then the estimated value on a Bayesian basis will be more accurate than the conventional estimated value(see Leonard& Hsu (1999)).

Hence, the value of the predictive density function for product distribution can be estimated as follows:

$$f(y|\overline{q}) = k. \int e^{\frac{-p}{2}(y-\theta)^2}.\, e^{\left(\frac{-np+Q}{2}\right)(\theta-m(q))^2} d\theta. \tag{10}$$

Moreover, this indicates that product distribution is a normal distribution and has an estimated value for the variable that takes the following form(seeBeltrão& at el.(2005), Carlin at el. (1996), Mohamed (2008), Bravo (2010), Arias (2006)):

$$m(\overline{q}) = E(\theta^*|\overline{q}) \tag{11}$$

degree of precision is equal to:

$$\mathrm{Var}^{-1}((y|\bar{q}) = p.\frac{np + Q}{(n+1)p + Q}, \qquad (12)$$

using assumptions of the credibility theory, second degree moment will be as follows:

$$E(q_i|\theta) = m(\theta), \quad \mathrm{Var}(q_i|\theta) = v(\theta). \qquad (13)$$

Estimated value of mortality rates, $m(\theta)$, is approximated by the rates that represent a linear relationship to values as follows:

$$m(\theta) = \alpha_0 + \sum \alpha_j q_j,$$

whichreduces the mean square error value as follows:

$$E[m(\theta) - m(\theta)]^2 = \min \ ! \qquad (14)$$

Whereas, the coefficient of credibilityZ given for the mean of individual mortality ratesq can be calculated by the following equation:

$$Z = \frac{n \ . \ \mathrm{Var}[m(\theta^*)]}{n \, . \mathrm{Var}[m(\theta^*)] + E[\mathrm{Var}(q|\theta)]} \, . \qquad (15)$$

Based on aforementioned,the case of probability density function of the variable under study that has a normal distribution and has a previous distribution that follows the normal distribution, then credibility equation for it is equal to the posterior distribution of the arithmetic mean. This result is generalized to all families with a simple exponential distribution that have a mean (sufficient statistic) and that have a previous distribution is the same as the corresponding/ accompanying prior distribution and thus the mean square errors of credibility equation is(seeVerrall (1995), Mohamed (1997), Gary (2019)):

$$E\left(m(\bar{\theta}) - m(\theta)\right)^2 = \mathrm{Var}\left(E(q|\bar{\theta})\right) \qquad (16)$$

Frameworks of the Bayesian Method in general and the theory of credibility represented here can be generalized to all risks, for observations of homogeneous weights and for linear models (linear relationship) (G.L.M), and based on the above, the detailed steps of the proposed model can be formulated more precisely as follows:

1. If we have the actual mortality rates for individual ages during a previous period of experience, which can be symbolized as follows(Nedler&Verrall (1997), Hardy&Panjer (1998), Benjamin&Pollard (1993)):
q_{ij}:refers to actual mortality rates for age i in the year under study j (note: $i = 20, 21, 22, \dots, 60$) (as $j = 1, 2, 3, \dots, 5$).$\mathring{q}_{ij}$: refers to tabular (typical) mortality rates.

$$\bar{q}_i = \sum \left(\frac{\mathring{q}_{ij}}{\mathring{q}_i}\right) q_{ij} \text{with} \mathring{q}_i = \sum \mathring{q}_{ij} \qquad (17)$$

According to Buhlmann-Straub (1970) principles, the weighted mortality rate reflecting the theory of credibility for age i is

$$\tilde{\mu}_i = \mu + \alpha_i (\bar{q}_i - \mu).$$

The equation can be written more precisely:

$$\tilde{\mu}_i = \alpha_i \bar{q}_i + (1 - \alpha_i)\,\mu,$$

where μ is the average weight of experience (mean averages) and α i is the coefficient of credibility for age i(seeLauderdale & Kestenbaum (2002), Mohamed (1997)) where:

$$\alpha_i = \frac{q_i^{\circ}.\,b}{q_i^{\circ}.\,b + w}, \qquad (18)$$

where $b, w$is the variance between and within rates respectively (between and within the variance).

Buhlmann-Straub also presented a method for estimating the values of μ, b and w(seeBravo, J. M. (2010), Carlin& at el. (1996))as follows:

$$\tilde{\mu} = \sum \frac{\alpha_i}{\alpha}.\,\bar{q}_i \text{ with} \alpha = \sum \alpha_i,$$

$$\tilde{w} = \frac{1}{N}.\frac{1}{n-1}\sum\sum q_{ij}^{\circ}(q_{ij} - \bar{q}_i)^2,$$

$$\tilde{b} = c^{-1}\left[\sum \frac{q_i^{\circ}}{q^{\circ}}.\,(\bar{q}_i - \bar{q})^2 - (N-1)\left(\frac{\tilde{w}}{q^{\circ}}\right)\right] (19)$$

where: $q^{\circ} = \sum q_i^{\circ}$, $q = q^{\circ -1}\sum q_i^{\circ}\,\bar{q}_i$, $c = \sum \frac{q_i^{\circ}}{q^{\circ}}.\left[1 - \frac{q_i^{\circ}}{q^{\circ}}\right]$

A robust version for credibility analysis assumed that individual mean$\mu(\theta_i) = E[q_{ij} \mid \theta_i]$ which is the total of individual parts. Individual parts $\mu_0(\theta_i)$ are estimated based on the robust statistical amounts,$t_i$andμ_{qs} are estimated based on the values observed for qs, as an estimated robust value. A so-called M-estimator for valuet_i(seeSilcocks& at el. (2015)) can represented as follows:

$$t_i = \sum \left(\frac{q_{ij}^{\circ}}{q_i^{\circ}}\right) \min\left(q_{ij}, c_{ij} t_i\right) (20)$$

where$c_{ij} = 1 + \left(\frac{\bar{q}_i^{\circ}}{q_{ij}^{\circ}}\right)^{\frac{1}{2}}$, $\quad \bar{v} = \frac{1}{n.N}\sum\sum v_{ij}$

$$\bar{\mu}_i = \bar{\mu}_t + \alpha_i\left(t_i - \bar{\mu}_t\right) + \bar{\mu}_{qs} \qquad (21)$$

where$\bar{\mu}_{qs} = \frac{1}{v}\sum\sum q_{ij}^{\circ}.\,qs_{ij}$, $\bar{\mu}_t = \sum\left(\frac{\alpha_i}{\alpha}\right) t_i$, $\quad \alpha = \sum \alpha_i.$

The Buhlmann-Straub input of credibility can be used to find a suitable estimate of mortality loss rate for life insurance companies, compared to standard (typical) mortality tables, to assess business results of these companies. It is a method to calculate margin to be added to mortality rate to match reverse selection phenomenon or selection against interests of the company, then credibility theory can be applied to variance of mortality loss rate, thereby offering a method to calculate the amount of (capital) surplus required to cover annual fluctuations in the experience of mortality rates. Common practice of life insurance in most countries proved that life insurance companies lay down policies to calculate reserves, taking into account some safety margins in addition to developing some metrics to calculate excess, which is placed as a buffer against unexpected results.

Accordingly, it is necessary to accurately determine the following amounts (see Lauderdale & Kestenbaum (2002)):

1. The "best estimate" of the future death experience (future here depends on the best future estimate) of the insurance company, assuming that there are no significant or fundamental changes in the nature of the company's business.
2. Safety margin: To be added or subtracted (as appropriate) to the best estimate of mortality rate in order to provide a backup rule that reflects the uncertainty of amount of the best estimate of mortality rate, knowing that the margin in addition to the best estimate shall be greater than the real mortality rate which has a high probability.
3. A measure of adverse fluctuations, which is translated in the form of the capital that the enterprise must maintain in addition to the reserves, as a buffer for the company against adverse fluctuations in the mortality experience in the company.

Used assumptions and symbols

Q_{ij}: is a random variable that expresses the claims amounts for each document (for age) i in year j (realmortality rate for age i in year j).

$\hat{q}_{ij}$: - The expected value of claims for the document (for age) i in year j is based on tabular mortality rates, and this is considered a measure of risk, which expresses the effect of enterprise size on risk of anticipated claims, which was calculated with reference to the standard tables (expected mortality rate for age i in year j).

$$X_{ij}:\ (X_{ij} = \frac{q_{ij}}{\hat{q}_{ij}}) \qquad (22)$$

refers to mortality loss rate (which is the ratio between real to expected claims) for document iin year j, which will be the focus of the study.

Let us assume the following assumptions for the random variable X_{ij}:

First assumption: distribution of variable X_{ij}depends on a fixed but unknown risk parameter (or parameter vector), θ_i and on risk magnitude$\hat{q}_{ij}$.

Second assumption: The random variable X_{ij} is a conditional variable in the fixed value θ_i which are independent random variables (X_{ij}) and have a mean and variance equal to:

$$E[X_{ij}|\theta_i] = \mu(\theta_i) \quad \text{and} \quad V[X_{ij}|\theta_i] = \frac{\sigma^2(\theta_i)}{\hat{q}_{ij}}. \tag{23}$$

Fourth central moment:

$$\mu^4\, [X_{ij} \mid \theta_i] = \frac{3}{P_{ij}^2}\sigma^4(\theta_i). \tag{24}$$

Third assumption: Risk parameterθ_i can be treated as a random variable distributed in a symmetric and independent distribution and drawn from a common distribution.
Fourth assumption: Pairs $(\theta_i, X_{ij})$and $(\theta_k, X_{ki})$are independent random variables.

The two equations for the second assumption are the traditional assumptions of Buhlmann-Straub's theory of credibility. The second assumption is that $P_{ij}V[X_{ij} \mid \theta_i]$is independent with respect to j and is equal to the assumption that claims variance for a given θ_i, then $V[X_{ij} \mid \theta_i]$, changes with respect to the size of the risk P_{ij}. This will be true only when ages and gender are the same for the different years of study, and if insurance amounts are distributed evenly during the different years of study. If this condition is not met, then risk size will be equal to square of expected tabular claims value divided by claims variance value, since if there is a K_{ij}contract in insurance company and it has aS_K refers to sum of insurance amounts for the number of k contracts for the portfolio and has an appropriate tabularmortality rate, it takes the form q_x^{ij} and risk size will be V_{ij}(Hardy&Panjer, (1998),Beltrão& at el.(2005)):

$$V_{ij} = \frac{P_{ij}^2}{\sum_{k=1}\left(S_k^{ij}\right)^2 q_x^{ij}\left(1 - q_x^{ij}\right)} \tag{25}$$

The third equation for the second assumption of $\mu^4\, [X_{ij} \mid \theta_i]$, representing the kurtosis coefficient of individual risk, which is equal to 3.0, as the kurtosis coefficient related to normal distribution. The fourth assumption indicates that company's successive years outcomes are independent of each other.

The main purpose is to find the best estimate for the amount $E[X_{in+1}|\theta_i] = \mu(\theta_i)$which is an unknown value, given a set of unknown values for the document i, $X_i = X_{i1},, \dots \dots, X_{in}$. We will refer to this estimate $\bar{\mu}_i$for document i wherei $= 1,2, \dots, N$ and $P_i = \sum_{j=1} P_{ij}$, as $P_i$represents the total expected claims for document i during the n years of study. Buhlmann-Straub's credibility estimate $\bar{\mu}_i$ for $\mu(\theta_i)$ is an estimator which is a linear equation (Linear) in data $X_{i1},, \dots \dots, X_{in}$ which reduces the square of losses errors as a whole where each value has a linear estimate $\bar{\mu}_i(\theta_i)$(seeRobert P. C. (2015), Contador (2012)):

$$E[\bar{\mu}_i - \mu(\theta_i)^2] \le E[\bar{\mu}_i(\theta_i) - \mu(\theta_i)^2] \tag{26}$$

The solution to this which is considered the optimal linear estimate of the amount$E[\mu(\theta_i)|X_i]$ takes the following form:

$$\bar{\mu}_i = Z_i\bar{X}_i + (1 - Z_i)E[\mu(\theta_i)] \quad (27)$$

where Z_i is credibility parameter of the document i and can be reached from the following equation:

$$Z_i = \frac{P_i}{P_i + \varphi} \text{ and } \varphi = \frac{E[\sigma^2(\theta_i)]}{V[\sigma(\theta_i)]}. \quad (28)$$

As risk parameters, $\{\theta_i\}_{i=1}$ are assumed to be symmetrically distributed, torques of parameter θ_i equations are symmetrically distributed across all documents.

Death margin for irregular risks: We can measure the precision of the estimated value $\bar{\mu}_i$ using the expected square losses function.(Beltrão at el., (2005), Bravo (2010)):

$$E\left[\left(\mu(\theta_i) - \bar{\mu}_i\right)^2\right] = (1 - Z_i)V\mu(\theta_i)]. \quad (29)$$

The required margin for irregular risks can be shown as an addition to best estimate mortality (PAD) in order to address MIS-ESTIMATION risk for expected deaths. The used margin depends on the required probability (p, say) that enables the company to make mortality loss rate higher than actual mortality loss rate, hence we wish to find the margin $m_p(i)$ for document i in the form:

$$\Pr\left[\mu(\theta_i) < \bar{\mu}_i + m_p(i)\right] \approx P. \quad (30)$$

If we assume that the posterior distribution of $\mu(\theta_i)$ is approximately following a normal distribution, then we will use the symbol $\Phi(z)$ to refer to equation of standard normal distribution:

$$\Pr\ C < \bar{\mu}_i + \Phi^{-1}(p)\sqrt{v_i}] \approx P,\ \ m_p(i) = \Phi^{-1}(p)\sqrt{v_i}. \quad (31)$$

In this model, we will use the correct (but unknown) random fluctuation value of the mortality loss rate for document i in the future year r given the expected claimsP_{ir}which is $[\sigma^2(\theta_i)/P_{ir}]$ .Credibility estimate of the value $\sigma^2(\theta_i)$ is the closest linear equation (in S_i^2) for the posterior mean.

$$E[\sigma^2(\theta_i)] = E[P_{ij}V[X_{ij}|\theta_i]|\ X_i] \quad (32)$$

The estimated value will be as follows:

$$\tilde{\sigma}_i^2 = C_i S_i^2 + (1 - C_i) E[\sigma^2(\theta_i)], \tag{33}$$

whereas:

$$S_i^2 = \frac{1}{n_i - 1} \sum p_{ij}(X_{ij} - \bar{X}_i)^2, \qquad C_i = \frac{1}{1 + \left(\frac{2}{n_i - 1}\right)\varphi}, \qquad \varphi = \frac{E[\sigma^4(\theta_i)]}{V[\sigma^2(\theta_i)]}. \tag{34}$$

This estimate relates to the value of $P_{ij}E[V[X_{ij}|\theta_i]|\, X_i]$and all the coefficients $[\sigma^2(\theta_i)], E[\sigma^4(\theta_i)]$, and$V[\sigma^2(\theta_i)]$are independent coefficients for each document, and $\{\theta_i\}$ is assumed to be symmetrically distributed, as the only dependent variable for document in Ci is ni which is the number of study years.Diversifiable risk is the risk of financial distress resulting from random fluctuations in claims experience. The capital needed to cover this risk is held along with the firm's reserves. Thus, the significant fluctuations that shall be considered are not the variance in total claims but the variance in death strain. This is defined as the increase in insurance amounts over the reserves at the end of the year, and amount at risk for document i in year jis expressed with the symbol D_{ij}, and a_{ij}^k is the net amount at risk (NAAR) for the number of K documents and in year J. The net amount at risk of the individual document is insurance sum less the reserve required to be kept at the end of the year if document holder is still alive (which is known as the amount at risk), and IK(an indicator variable for that contract) as IK = 0 if the person is still alive and it is equal to 1 if the death occurred during the year, and accordingly, (see Contador, (2012), Andreev&Vaupel(2005), Bravo (2010)):

$$D_{ij} = \sum a_{ij}^k I_k. \tag{35}$$

To reach the variance value of amounts at risk, we will need an estimate of the value of $\Pr[I_k = 1]$, and we will assume givenX_{ij} that this probability will be obtained from the relation$q_k^{ij}X_{ij}$ as$q_k^{ij}$is the tabular mortality rate of individual document, and therefore estimation of change value in amount at risk will be given by the following relation:

$$\begin{aligned} E[V[D_{ij}|\theta_i]] &= E[E\left[V\left[[D_{ij}|X_{ij}]\middle|\theta_i\right] + V[E[D_{ij}|X_{ij}]|\theta_i]\right] \\ &= E\left[E\left[\Sigma\left(a_{ij}^k\right)^{2q^{ij}X_{ij}}(1 - q^{ij})\middle|\theta_i\right] + V[\Sigma a_{ij}^k q^{ij} X_{ij}|\theta_i]\right] \\ &= E\left[\Sigma\left(a_{ij}^k\right)^{2q^{ij}} \mu(\theta_i) - \Sigma\left(a_{ij}^k\right)^2 \left(q^{ij}\right)^2 \left(\mu(\theta_i)^2 + \left(\frac{\sigma^2(\theta_i)}{P_{ij}}\right)\right) + \left(\Sigma a_{ij}^k q^{ij}\right)^2 \left(\frac{\sigma^2(\theta_i)}{P_{ij}}\right)\right] \end{aligned} \tag{36}$$

We will substitute the estimated value of credibility$\mu(\theta_i)$ and $\sigma^2(\theta_i)$ to provide an estimated value of the variance of the amounts at risk:

$$\left(\sum \left(a_{ij}^k\right)^2 q^{ij}\right)\bar{\mu}_i - \sum \left(a_{ij}^k\right)^2 \left(q^{ij}\right)^2 \left(\frac{\bar{\sigma}_i^{\,2}}{P_{ij}} + \bar{\mu}_i^{\,2}\right) + \left(\sum a_{ij}^k q^{ij}\right)^2 \left(\frac{\bar{\sigma}_i^{\,2}}{P_{ij}}\right) \tag{37}$$

The model parameters are common parameters for all documents due to the assumption that θ_i is symmetrically distributed, the estimation of the value of $E[\mu(\theta_i)]$ and we will assume:

$$\bar{\mu} = \frac{\sum Z_i \overline{X}_i^{\,2}}{\sum Z_i}, \qquad (38)$$

estimation the value of $E[\sigma^2(\theta_i)]$ and we will use $\tilde{\sigma}^2$ where:

$$\tilde{\sigma}^2 = \frac{\sum C_i S_i^{\,2}}{\sum C_i}, \qquad (39)$$

also, the estimation of the value of $V[\mu(\theta_i)]$ and we will use $\tilde{v}$ where:

$$\tilde{v} = \frac{PW - \tilde{\sigma}^2}{\prod p}. \qquad (40)$$

The researcher relies on the Bayesian approach as a statistical method for estimating the loss ratio value resulting from death risk. Available data gets close to normal distribution as a probability distribution describing the general shape of probability distribution function for the variables under study, which has been concluded and proven at the beginning of this research. Thus, the researcher will rely on (Normal / Normal) model, i.e the model will be run under the assumption that previous distribution of probability density function will be represented by a probability distribution function close to the normal distribution. Moreover, posterior distribution function will be represented by a normal distribution function. Various stages related to the model operation are as follows (see Hardy&Panjer (1998), Mohamed (1997), Carlin& at el. (1996), Mohamed (2008)):

(1) Model input:

The proposed model is based on tabular mortality rates derived from standard table A67 / 70 ULT as well as actual mortality rates derived from experience of an Egyptian life insurance company for a study period starting from 2014/2015 to 2018/2019 and for ages 20 to 65.

(2) Model operation stages:

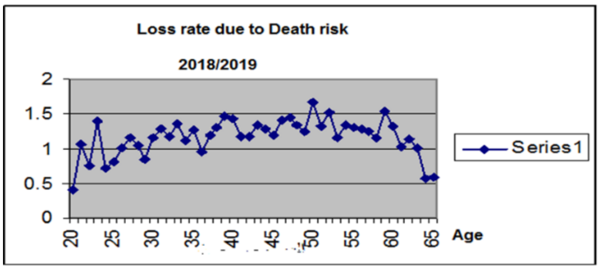
Operation stages of the proposed model for calculating loss rate resulting from death risk are five stages. First stage, model variables calculation, is centered on one variable, i.e. loss rate resulting from death risk. The last stage is the proposed model quality testing. In addition, various stages can be reviewed as follows: -

First: Calculation of loss rate resulting from death risk: Loss rate resulting from the death risk represents the result of dividing actual mortality rate and table mortality rate. The loss rate can be calculated by the following equation: -

$$X_{ij} = \frac{q_{ij}}{\mathring{q}_{ij}}$$

Results & Discussions

Figure No. (1) in the research appendices shows results of calculating loss rates according to the previous equation through available data. The general trend curve represented by loss rate resulting from death risk during the year 2018/2019 can be represented by the following chart:



When analyzing such chart related to loss rate, the researcher will focus on comparing these findings with the optimal rate (i.e. 1). Based on the previous chart, we find that loss rate generally takes the direction of increase over the optimal rate, which can be explained by the fact that actual mortality rates are higher than tabular rates. This increase was concentrated in the period between ages from 32 to 61, as increase in actual mortality ratescompared totubular rates lead to material damage to life insurance companies. Possibility of their exposure to material losses are reflected in inability to pay their obligations as actual claims are beyond the expected ones.

Second: Equations used to reach values estimated for loss rate resulting from death risk by using the credibility model:

1) Calculate value of tabular mortality rateexpressed for each age separately: -$\overline{q_i^{\circ}} = \sum q_{ij}^{\circ}$

2) Weighted average value of loss rate resulting from death risk:

$$\overline{X}_i = \sum \frac{q_{ij}^{\circ} . X_{ij}}{\overline{q^{\circ}}_i} \qquad (41)$$

3) Variances value $\sum q_{ij}^{\circ} . (X_{ij} - \overline{X}_i)^2$ and$\sum q_{ij}^{\circ} . (X_{ij} - \overline{X})^2$:

Calculated values of variance represent $\sum q_{ij}^{\circ} . (X_{ij} - \overline{X})^2$noting that before calculating the variance value $\sum q_{ij}^{\circ} . (X_{ij} - \overline{X})^2$value $\overline{q^{\circ}}$and value Xshall be calculated since $\overline{X}$ and $\overline{X}_i$are weighted values of valueX_{ij} through the following equations:

$$\overline{q^{\circ}} = \sum q_i^{\circ}, \quad \overline{X} = \sum \frac{\overline{q_i^{\circ}} . \overline{X}}{\overline{q^{\circ}}} = \qquad (42)$$

through available data collected from the previous tables,$\overline{q^{\circ}}$ and $\overline{X}$ values were calculated as follows:

$\overline{q^{\circ}}$ = 1.19811665 , $\overline{X}$ = 1.984270113.

Calculating the value of$q^{\circ *}$:

$$q^{\circ *} = (Nn - 1) \sum \overline{q_i^{\circ}} \left(1 - \frac{\overline{q_i^{\circ}}}{\overline{q^{\circ}}}\right), \qquad (43)$$

the value of $q^{\circ *}$was calculated as per the model and equals 0.004951871.

Based on the above, the indication of each used equation, as well as its estimated value, can be clarified through the following table:

Table (2): Indication of each estimator.

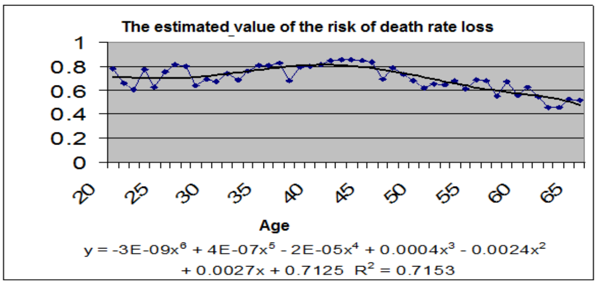
Estimator	Parameter
$\overline{X}$ = 1.98427011	$E[m(\theta)]$

$\dfrac{\sum[\sum \mathring{q}_{ij}(X_{ij} - \overline{X}_\imath)^2]}{N(n-1)} = 0.011568561$	$E[S^2(\theta)]$
$\dfrac{\sum[\sum \mathring{q}_{ij}(X_{ij} - \overline{X}_\imath)^2] - E[S^2(\theta)]}{N(n-1)} = 5.560351694$	$V[m(\theta)]$

Estimator of credibility parameter:
Credibility parameter for the values of loss rates resulting from death risk for each age can be separately estimated as follows:

$$Z = \frac{\overline{\mathring{q}_\imath}}{\overline{\mathring{q}_\imath} + \frac{E[S^2(\theta)]}{V[m(\theta)]}} \qquad (44)$$

Estimating value of loss rate resulting from death risk: Equation of estimating loss rate relies on credibility equation as follows: - $\widehat{X} = Z\overline{X}_\imath + (1-Z)\overline{X}$
The following chart shows the general trend represented by estimated values of loss rate resulting from death risk:



It is clear from the previous chart that estimated values of loss rates represent rates less than 1, which reflects good quality of model used in the estimation. In addition, all values are close to 85% as the rate, if more than 1, indicatesthat actual mortality rates are greater than tabular rates which lead to material losses for life insurance companies. Insurance price was calculated on the basis of tabular rate and increased actual rate will lead to insufficient calculated provisions to cover claims.
Operation outcomes: Based on the above, we conclude the great importance represented by loss rates resulting from the death risk, as they represent an indicator for insurance companies on quality of used tabular rates. In addition, such rates provide a measure of the estimated value on safety margin required to be added to address the deviation between actual and tabular mortality rates of. This is confirmed as loss rates estimated through the proposed model, enables us to adjust tabular mortality rate during the coming periods based on previous outcomes.

7. Conclusion

Bayesian approach is method of statistical estimation that depends on mixing information derived from previous experience with newly collected data, which leads to an estimate of a high degree of precision. There is a fundamental and influential difference between mortality rates derived from the experience outcomes in the Egyptian market and mortality ratesas it in tables used in pricing of life insurance contracts. The ideal equation representing the general trend line for safety margin rates is a polynomial equation of the sixth degree with a significant level of 99.16%, and that equation can be determined as follows:
$M_G = 5E(-11)X^6 - 7E(-9)X^5 + 3E(-7)X^4 - 7E(-6)X^3 + 7E(-5)X^2 -$ $0.0002X - 5E(-5)$.Equation used to estimate value of mortality rates based on tabular values is as follow$q_x = 1.276666635\ q_x^s - 5.65395\ E(-06)$ .Loss rates resulting from death risk

represent an indicator for insurance companies on quality of the used tabular rates. In addition, such rates provide a measure of estimated value related to safety margin to be added or subtracted to address the deviation between actual and tabular mortality rates.A mortality table should be promptly prepared based on data of Egyptian insurance companies, and this proposal shall be. The researcher believes that interest rate should be periodically adjusted in line with interest rates granted in the other competitive savingsfunds.

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