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Some Analysis Tools To Identify Difficulties In The Teaching And Learning Of The Integral Concept

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ABSTRACT. This article is the result of a research process where the instruments that allow a macroscopic and microscopic analysis of calculus textbooks are presented, in order to identify some difficulties that may arise in the teaching and learning process of the integral concept. The instruments make explicit the criteria to be evaluated, in order to facilitate the analysis and the curricular and methodological decision making.

Keywords: Teaching, learning, integral, calculus, difficulties.

I. INTRODUCTION

Learning and teaching the integral is a particularly valuable topic, as integration serves as the basis for many real-world applications and subsequent coursework [1] [2], appearing in a variety of contexts within physics and engineering [3] [4] [5]. The identification of students' difficulties with the concept of integral has been documented over the years in several studies [6] [7] [8] [9] [10] [11].

This paper presents four instruments designed to identify the difficulties present in the teaching and learning of the integral concept in a calculus text. The approach is based on two levels of analysis, one macroscopic and the other microscopic.

II. MACROSCOPIC TEXTBOOK ANALYSIS

The macroscopic analysis of a calculus textbook seeks to characterize the institutional meaning of the object to be taught - the integral - by taking into account the number of pages, the number of units, their distribution by chapters, sections and units, the structure of the unit, the location of the activities, the use of computer resources and their greater or lesser approximation to the constructivist model.

In order to achieve an orderly and effective analysis, concept maps should be constructed through which it is possible to detect conceptual errors and at the same time to visualize the evolution of the concept over time. This same instrument makes it possible to determine the level of prior knowledge of the author of a text, which interacts with the knowledge presented in formal instruction, often giving rise to a diverse set of learning that is not always desired.

III. MICROSCOPIC TEXTBOOK ANALYSIS

In the micro analysis, which deals with the description and interpretation of the author's ways of knowing about the concept of the integral as a mathematical object and as an object of teaching and learning, four instruments were designed that allow a detailed look at aspects such as: the integral as a mathematical object [12] [13], as an object of teaching and learning, the design and conformation of the exercises and their classification according to their typology.The first instruments that are proposed, inquire about the integral as a mathematical object; for this purpose, the following are taken into account:

(a) Syntactically, the symbology, the definition, the way the concept is introduced and the techniques used to define the integral.

Table 1: Syntactic aspects of the integral as a mathematical object present in the text, source Authors.

<table>
<tr><td colspan="5"></td><td>Yes</td><td>No</td></tr>
<tr><td rowspan="21">FORM (SYNTAX)</td><td rowspan="4">What Symbols are used?</td><td colspan="3">Δx, Δy</td><td></td><td></td></tr>
<tr><td colspan="3">dy, dx</td><td></td><td></td></tr>
<tr><td colspan="3">$\sum$</td><td></td><td></td></tr>
<tr><td colspan="3">$\int$</td><td></td><td></td></tr>
<tr><td rowspan="7">What does $\int_a^b f(x)\,dx$ or $\int_c^d f(y)\,dy$ mean ?</td><td colspan="3">Perimeter and area</td><td></td><td></td></tr>
<tr><td colspan="3">Area under the curve</td><td></td><td></td></tr>
<tr><td colspan="3">Coincidence of functions</td><td></td><td></td></tr>
<tr><td colspan="3">Multiplicationbased summation (MBS)</td><td></td><td></td></tr>
<tr><td colspan="3">Indivisible</td><td></td><td></td></tr>
<tr><td colspan="3">Fragmentation method</td><td></td><td></td></tr>
<tr><td colspan="3">Other</td><td></td><td></td></tr>
<tr><td rowspan="5">How do you introduce the concept?</td><td rowspan="2">Following the historical genesis of the concept</td><td colspan="2">1</td><td></td><td></td></tr>
<tr><td colspan="2">2</td><td></td><td></td></tr>
<tr><td rowspan="3">Following the historical genesis of the concept</td><td colspan="2">1</td><td></td><td></td></tr>
<tr><td rowspan="2">2</td><td>a</td><td></td><td></td></tr>
<tr><td>b</td><td></td><td></td></tr>
<tr><td rowspan="5">What techniques are covered?</td><td colspan="3">Direct technique: definition of the derivative</td><td></td><td></td></tr>
<tr><td colspan="3">Indirect derivative technique: derivation rules</td><td></td><td></td></tr>
<tr><td colspan="3">Numerical approximation technique</td><td></td><td></td></tr>
<tr><td colspan="3">Graphical approximation technique</td><td></td><td></td></tr>
<tr><td colspan="3">Transformation</td><td></td><td></td></tr>
</table>

	How do you handle the concept of area?	Approximation		
		Variation		
		Transformation		
	What is the working meaning of area?	Bounded surface		
		Infinite sum		
		Surface under a curve		
		Sum of integrands		
	How do you handle the transition from the derivative to the integral?			
	What geometric meaning is being worked out?	Area under curve		
		Perimeter and area		
		Multiplicatively based summation (MBS)		
		Indivisibles		
		Chopping up method		

(b) In terms of semantics, the approach used, the situations through which the concept is introduced, the handling given to the concept of area and its meaning, the different geometric meanings attributed to it, etc.

Table 2: Semantic aspects of the integral as a mathematical object present in the text, source Authors.

			Yes	No
CONTENT (SEMANTIC)	What approach do you use?	Algebraic		
		Numerical		
		Formal		
		Infinitesimalist		
		Local affine approximation		
		Geometric		
		Variational		
		Computational		
	With what situation do you introduce the concept?	Approximation		
		Variation		
		Transformation		
	How do you handle the concept of area?	Approximation		
		Variation		
		Transformation		
	What is the working meaning of area?	Bounded Surface		
		Infinite sum		
		Surface under a curve		
		Sum of integrands		
	How do you handle the transition from the derivative to the integral?			
	What geometric mean-	Area under curve		

	ing is being worked out?	Perimeter and area		
		Multiplicatively based summation (MBS)		
		Indivisibles		
		Chopping up method		

(c) Regarding the definition, the characteristic elements (conventionality, minimality), the type of definition used (area, antiderivative, summation, etc.)

Table 3: Aspects of the definition of the integral as a mathematical object present in the text, source Authors.

				Yes	No
DEFINITION	What is the characteristic of the definition?	Conventionality	Esthetics		
			Operations		
			Didactics		
			Hierarchical		
			Partitional		
		Minimality	Nominal		
			Sufficient condition		
			Necessary condition		
			p if and only if q		
	What definition do you use?	Area			
		Antiderivative			
		Summation			
		Sum Integrating			
		Average			

(d) In relation to representation, phenomenology and context.

Table 4: Aspects of the representation of the integral as a mathematical object present in the text, source Authors.

			Yes	No
REPRESENTATION	Phenomenology	Mathematics		
		Physics		
		Other		
	Context	Algebraic		
		Numerical		
		Verbal		
		Graphical		

The following proposed instruments seek to analyze aspects of the integral as an object of teaching and learning present in the text.

(a) Through a look at the form-content of the object (syntax/semantics).

Table 5: Aspects of the representation of the integral as a mathematical object present in the text, source Authors.

FORM / CONTENT SYNTAX/SEMANTICS	What is the sequence of the contents?		
	From what point is the concept introduced?	Definition	
		Sample Exercise	
		Examples	
		No examples	
		Actual situation	
		History of science	
	What type of activities are presented?		
	What type of exercises and/or problems are presented?	Approximation	
		Variation	
		Transformation	
	What cognitive processes emerge from the mathematical activity?	Internalization of actions into processes	
		Encapsulation of processes into objects	
		De-encapsulation of objects into processes	
		Generalization	
		Generalization of schemas	
	What perspectives of the concept are encouraged in the activities?	Action	
		Process	
		Object	
		Schema	
	What is the use of the history of science?	To introduce the concept	
		Handling historical problems	
		Phenomenological aspects that gave rise to the construction of the concept.	

(b) On the translations and relations between the representations that the text makes of the object.

Table 6: Aspects of translations and relations between the representations of the integral as an object of teaching and learning present in the text, source Authors.

TRANSL ATIONS AND RELATI	What role is given to translations and relations between representations in the task ap-	

	proach?	
	What translations between semiotic representations are used?	
	What relations between semiotic representations are used?	

The exercises or problems proposed by the author are classified into three typologies of problems that are usually treated in differential calculus textbooks. These typologies allow to simplify the analysis of the exercises proposed in each section of the textbook, helping to establish comparative relations between them and thus, helps to show the absent and not evident mathematical topics that can become a didactic obstacle of the integral concept, these typologies are:

- Of approximation

- Of variation

- Of transformation

The analysis of each of the above groups is done through the following instrument, which consists of seven general categories called: phenomenology, statement, representation, techniques, technological resources, translations and relations ∫f(x) to F (x) and cognitive processes; these in turn are divided into subcategories that seek to facilitate a more thorough analysis, the instrument is presented below.

Table 7: Aspects of translations and relations between the representations of the integral as an object of teaching and learning present in the text, source Authors.

			Yes	No
Phenomenology	Pure	Mathematics		
		Physics		
		Other		
	Historical	Mathematics		
		Physics		
		Other		
Statement	Verbal			
	Non-verbal			
Context	Algebraic			
	Numeric			
	Verbal			
	Geometric			
	Graphical			
Technique or procedure	Area under curve			
	Perimeter and area			
	Multiplication-based summation (MBS)			
	Indivisible			
Resources (technology)				

Translations and relations$\int$f(x) to F (x)	
Cognitive processes	

The first category called phenomenology uses the classification proposed by Puig [14], i.e. it differentiates between pure phenomenology, understood as the activities proposed by the author with mathematics in its current state and its current use, and historical phenomenology, understood as the activities proposed by the author based on the phenomena that gave rise to the mathematical concept in question and how these extend to other phenomena in other sciences.

The second category, the statement visualizes the tasks proposed by the author, facilitating the observation of the common characteristics among the problem situations, and helps to observe the frequency with which certain types of problems appear in the exercises proposed by the author and, in turn, the similarities and differences among the examples proposed.

The third category, identified as context, seeks the classification of the mathematical problems or exercises proposed by the author in the text according to their verbal or descriptive, algebraic, numerical and geometric environment.

The fourth category taken into account is the technique or procedure used to classify the exercises or problems depending on the way to integrate functions, that is, the different proposals for the calculation of the integral (analyticity, prediction and trend behavior of the function) are examined.

In the fifth category or resources, attention is focused on the author's use of new technologies as an enriching element in the teaching of mathematical concepts. The use of new technologies in textbooks brings with it a new range of possibilities for the teaching-learning of mathematics. The main reason is that these instruments can show, in a dynamic way, concepts that are very difficult to teach in the traditional way.

The sixth category, or translations and relations between representations, specifies the understanding of the integral concept.

The last category, cognitive processes, is based on the didactic decomposition of the integral concept, taking up the categories for the analysis of the tasks that deal with the translation between representations from $\int$f(x) to F (x), and vice versa.

IV. CONCLUSIONS

A macroscopic analysis makes it possible to determine the structure of the text, taking into account its organization and other general aspects.

The microscopic analysis attempts to establish in detail and with the help of the instruments proposed in this article, the nature and characteristics that from the author's perspective are attributed to the mathematical objects, to the teaching and learning objects and to the exercises contained in the chapter under study.

Both the macroscopic analysis and the microscopic analysis jointly seek to determine the existence of evidence in relation to elements that can be considered generators of difficulties in the teaching and learning of the integral.

The didactic choices established in a situation of teaching a mathematical concept can generate a difficulty, centered on two aspects of the construction of knowledge: one related to the use of the language of calculus, and the other to the contexts of exemplification and experimentation for the construction of knowledge.

The integral concept has difficulties that start from its meaning and are intrinsically related to the concept itself.

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