Ilkogretim Online - Elementary Education Online, 2021; Vol 20 (Issue 6): 1437-1445
http://ilkogretim-online.org
doi: 10.17051/ilkonline.2021.06.145

A Content Analysis And Validity For A Problem Situation Associated With The Antiderivative.

Wilson Gordillo, Faculty of Environment, Universidad Distrital Francisco José de Caldas, Bogotá, Colombia.

Miguel A. Ávila A., Faculty of Engineering, Universidad Distrital Francisco José de Caldas, Bogotá, Colombia.

Wilson J. Pinzon, Faculty of Environment, Universidad Distrital Francisco José de Caldas, Bogotá, Colombia.

ABSTRACT. We show a way to perform a content analysis for a problem situation about the antiderivative that allows to explore and characterize the primary mathematical objects through the tools proposed in the Ontosemiotic Approach to Knowledge (OSA), the results obtained from the problem situation and the solution to it provide knowledge to teachers regarding another way to perform content analysis through ontosemiotic analysis, helping also to characterize the understanding of mathematical object.

Keywords: Content analysis, antiderivative, validity, ontosemiotic approach, understanding.

I. INTRODUCTION

There are different approaches to understanding comprehension [1] [2] and there are two basic ways of understanding it as a mental process or as competence [3] [4]. The two points of view respond to epistemological conceptions that, at the very least, are divergent, if not clearly opposed. Cognitive approaches in the Didactics of Mathematics, at bottom, understand understanding as a mental process. The pragmatist positions of the Ontosemiotic Approach (OSA), on the other hand, lead to understand understanding basically as a competence and not so much as a mental process. That is, a subject is considered to understand a given mathematical object when he/she uses it competently in different practices.

This pragmatic way of understanding also implies conceiving it as knowledge and application of the rules that regulate a practice. It is, therefore, a point of view that seeks to elucidate the intelligibility of human actions by clarifying the thinking that informs them and situating it in the context of the social norms and forms of life within which they occur. It is necessary to clarify that, within OSA, the theoretical approach to which this study adheres, the term knowledge is used in the sense of a general epistemic-cognitive construct that includes understanding, competence and disposition

[3]. Disposition, or capacity, is related to the notion of personal mathematical and didactic object, i.e., that which makes practice possible. Competence is related to the mathematical practices of the subjects and to the activation, in these practices, of the appropriate cognitive ontosemiotic configuration, which should be suitably coupled to the epistemic ontosemiotic configuration of reference [4].

II. THEORETICAL AND METHODOLOGICAL NOTIONS

This paper has adopted the pragmatist positions provided by the theoretical framework known as the Ontosemiotic Approach (OA) to mathematical knowledge and instruction [5]. The OSA has introduced a typology of primary mathematical objects: situations/problems, languages, definitions, propositions, procedures and arguments. These primary mathematical objects are related to each other forming networks of intervening and emergent objects of the systems of practices, which in EOS are known as configurations. These configurations can be epistemic (networks of institutional objects) or cognitive (networks of personal objects).

Thus, for the performance of a mathematical practice and for the interpretation of its results as satisfactory, it is necessary to put certain knowledge into operation. If we consider, for example, the components of knowledge for the realization and evaluation of the practice that allows solving a problem-situation, we see the use of languages, verbal and symbolic. These languages are the ostensive part of a series of concepts, propositions and procedures that intervene in the elaboration of arguments to decide whether the simple actions that compose the practice, and it as a composite action, are satisfactory. Consequently, when an agent performs and evaluates a mathematical practice, it activates a conglomerate formed by situations/problems, languages, concepts, propositions, procedures and arguments, articulated in the configuration [6]. As shown in Figure 1.

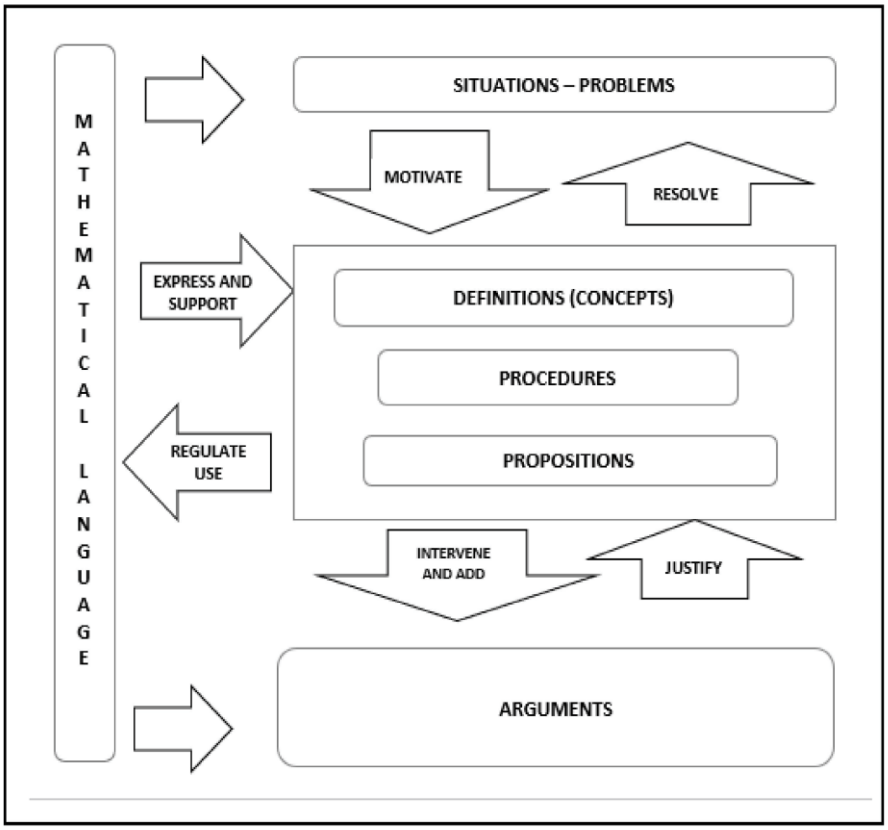

FIGURE 1: Configuration of primary mathematical objects. Source [6].

The definition of object as emerging from the systems of practices, and the typology of primary objects, respond to the need to be able to describe the systems of practices, in order to compare them with each other and make decisions in the design, development and evaluation of mathematical teaching and learning processes.

These primary mathematical objects that make up the configuration manifest themselves in various ways during mathematical activity: the language with which we refer to them, which in turn evoke concepts or definitions, which become operative through procedures and associated properties, which in turn manifest themselves during the solution of mathematical tasks. Moreover, each of the primary mathematical objects can be considered from different facets or dual dimensions [7] : personal - institutional; ostensive - non-ostensive; unitary - systemic; expression - content; extensive - intensive.

The emergence of the primary mathematical objects considered in the model of Figure 1, are associated, respectively, with the processes of problematization, communication, definition, algorithmizing, enunciation and argumentation.

Figure 2 shows the breakdown, and the interactions, of the primary mathematical objects, the dual facets from which they can be viewed, and the processes associated with them [8].

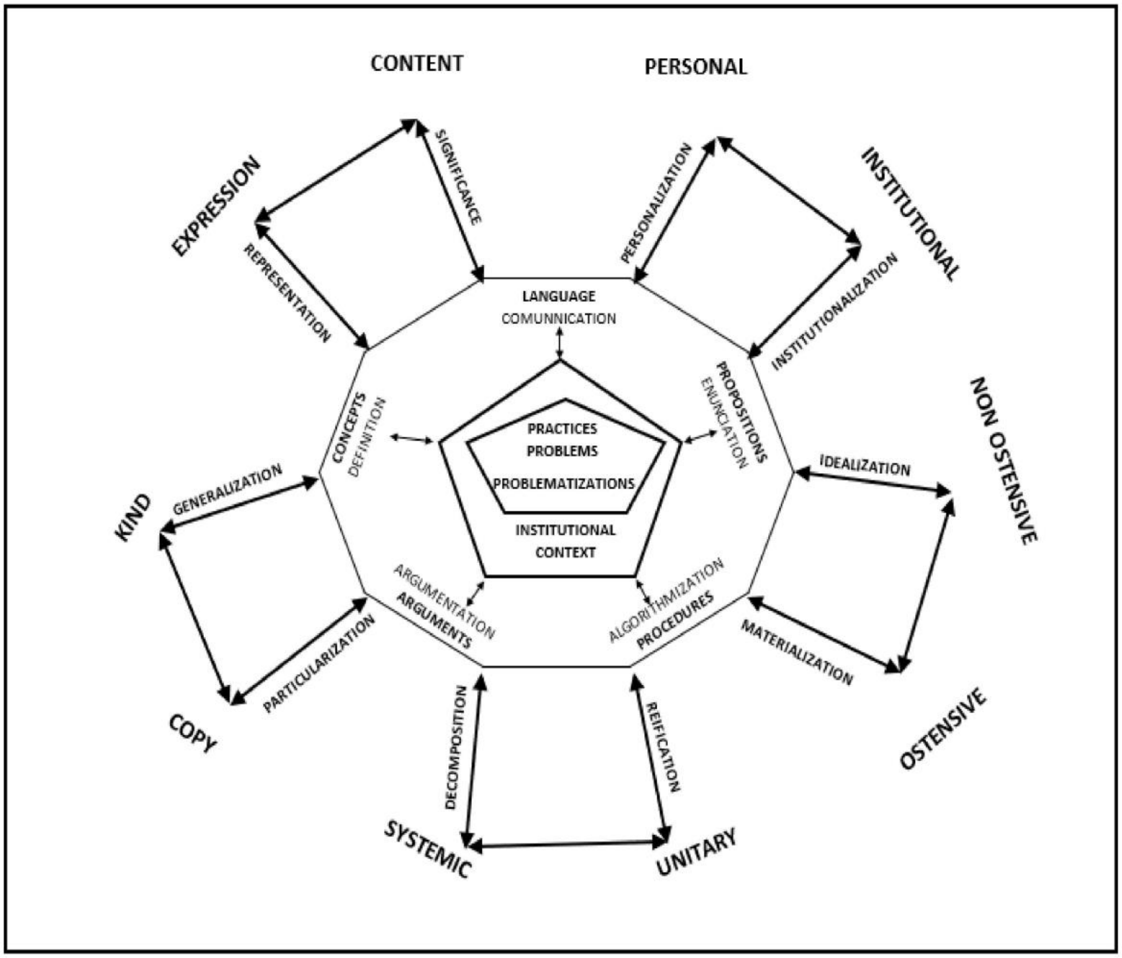

FIGURE 2: Configuration of primary mathematical objects. Source [9].

III. THE PROBLEM SITUATION

This problem situation explores mathematical knowledge in relation to other mathematical objects that are part of the curriculum [8], such as: the indefinite integral of a function or the fundamental theorem of calculus. Figure 3 shows the problem situation. The representations to be handled for the resolution of the problem situation are symbolic, graphical and tabular, activating the partial differential-summative meaning.

The processes of problematization, communication, definition, algorithmizing, enunciation and argumentation, respectively.

Figure 2 shows the breakdown, and the interactions, of the primary mathematical objects, the dual facets from which they can be viewed, and the processes associated with them [8].

For a given function $y = f(x)$, continuous in $\mathbb{R}$, the values of the following table are satisfied

x	$f(x)$
0	0
1	2
1,5	3
2	4
2,5	5

a) find a function for $f(x)$?
b) Can you find a second expression, different from the previous one for $f(x)$?
What would it be? justify the answer

FIGURE 3: Primitive function calculation, Source [9]

IV. PLAUSIBLE SOLUTION TO THE PROBLEM SITUATION

There are several solutions for each of the sections of this task, a possible solution of which is discussed below:

(a) Based on the data provided in the table 1, it is possible to find a pattern as follows:

Table 1: function and derivative table, source Authors.

x	f′(x)
0	$2(0) = 0$
1	$2(1) = 2$
1,5	$2(1{,}5) = 3$
2	$2(2) = 4$
2,5	$2(2{,}5) = 5$
.	.
.	.
.	.
x	$2(x) = 2x$

Therefore, given that $f'(x) = 2x$, and knowing that for a function $f(x) = x^n$ the derivative is given by $f'(x) = nx^{(n-1)}$, then an expression for f(x) would be: $f(x) = x^2$.

(b) Yes, you can; to find another expression for f(x) other than $f(x) = x^2$. If $f'(x) = 2x$ then $f(x) = \int 2x dx = x^2 + C$. Thus, f(x) can be any function of the family of functions

$f(x) = x^2 + C$ were, $C \in \mathbb{R}$.

V. CONTENT ANALYSIS AND VALIDITY

The designed problem situation is subjected to an ontosemiotic content analysis [8]. The content analysis and the possible difficulties in the resolution of each situation allow observing, describing and predicting the mathematical activity as a complex set of mathematical practices carried out by university professors when solving the problem situation, through the description of the processes, primary objects (linguistic elements, concepts/definitions, propositions/properties, procedures and arguments) and their meanings, immersed both in the approach and in the solution of the problem situation. The analysis of each problem situation is presented in a table 2 below.

Table 2: Content Analysis: Primitive Function Calculation, source Authors.

PRIMARY MATHEMATICAL OBJECTS	MEANINGS
Situation Problem	Calculation of the Primitive Function
Linguistic Elements	The statement of the problem situation denotes an indeterminate function, in this case, a function that fulfills certain conditions regulated by a table of values. The table of values. It corresponds to the tabular representation of an unknown function of which you are presented with five images for the values of the variable x given in the table, which provides ordered pairs of the type (x, f (x)). Such ordered pairs can be taken to the Cartesian plane to give a graphical representation of the given pairs. The questions derived from the statement refer to a procedure to find a function, which when derived meets the conditions and values of the table
Concepts	Function of unknown real variable. Function f(x) to be determined from its derivative function partially defined by five points. Ordered pairs. Originals and images of the given derivative function. Derivative function of a real variable. Partially defined by five points whose coordinates are ex-

	pressed in tabular form.
Propositions	Derivative rules. Specifically, "the derivative of a constant function is equal to zero", "the derivative of a sum is the sum of the derivatives" which allows us to determine that the function sought is any of the family functions $f(x) = x^2 + C$ were, $C \in \mathbb{R}$.
Procedures	Calculation of the antiderivative of $y = 2x$, this is done either by means of the derivative rules (derivative of the potential function), or by means of the integration rules. This procedure gives rise to the answer of both sections of the task. Trial and error. Trying possible correspondence rules between the values of x and those of $f'(x)$, from the values given in the table. This procedure is of a numerical-technical nature, focused on the search for a pattern that allows to establish the correspondence rule that allows to define the derivative function.
Arguments	The algebraic expression of the derivative function is $y = 2x$ because by evaluating the given points tubularly, $(x, f'(x))$; it is empirically deduced that it is the derivative function. The function sought is $y = x^2$ because the derivative of this function is $y' = 2x$. It establishes the validity for the function $f(x)$ taking into account the rule for deriving the potential function.

CONCLUSIONS AND RECOMMENDATIONS

This paper presented an analysis and content validity for a problem situation on the antiderivative, this analysis allows us to characterize through the design of questionnaires that allows us the knowledge and mathematical practices on the antiderivative, from a pragmatist point of view as adopted by the EOS, includes and links the activities of understanding, competence and disposition, which are involved in the mathematical practices that are developed in order to solve a problem.

This pragmatic way of understanding knowledge has been considered in the problem situation, since it requires for its resolution the congruent mobilization of the various registers of representation for the antiderivative [9], as well as the diversity of partial meanings of this mathematical notion [10].

The ontosemiotic analysis (content), and the possible difficulties in solving the tasks,

carried out for each of them, allows us to observe, describe and predict the mathematical activity as a complex set of mathematical practices carried out by university professors when solving the proposed tasks, around the mathematical object. Practices where it is possible to identify, the configuration of primary mathematical objects and processes; proposed by the theoretical framework of EOS, which has been called onto-semiotic analysis.
This onto-semiotic analysis used in the questionnaire, is foreseen as a powerful tool to be able to identify and analyze and have a good degree of content validity. Methodological theoretical tool of the EOS, which is validated and used in other questionnaire designs [10].

REFERENCES

[1] A. SFARD, "On the dual nature of mathematical conceptions: reflections on processes and objects as different sides of the same coin". Educational Studies in Mathematics, Vol. 22, no. 1, pp. 1-36, 1991. doi: 10.1007/BF00302715

[2] S. PIERE, and T. KIEREN, "Growth in mathematical understanding: How can we characterise it and how can we represent it? Educational Studies in Mathematics, Vol. 26 no.2-3, 165-190, 1994. doi: 10.1007/BF01273662

[3] L. PINO-FAN, J.D. GODINO, and V. FONT, "Didactic-mathematical knowledge on the teaching and learning of the derivative". Memories XIII School of Winter School in Educational Mathematics, pp. 206-213, 2010.

[4] L. PINO-FAN, J.D. GODINO and V. FONT, "Epistemic facet of didactic-mathematical knowledge about the derivative ". Education Mathematics Research, Vol. 13, no. 1, pp.141-178, 2011.

[5] J.D. Godino, C. Batanero, and V. Font, "The onto-semiotic approach to research in mathematics education", ZDM Mathematics Education, Vol. 39 no. 1, pp.127-135, 2007.

[6] V. FONT, and J.D. GODINO, "The notion of epistemic configuration as a tool for the analysis of mathematical texts: its use in teacher education". Educação Matemática Pesquisa, Vol.8 no.1, 67-98, 2006.

[7] J. D. GODINO, "An ontological and semiotic approach to mathematical cognition," Recherches en Didactique des Mathématiques, Vol. 22, no.2/3, pp.237-284, 2002.

[8] L. R. PINO-FAN, V. FONT, W. GORDILLO, V. LARIOS and A. BREDA, "Analysis of the Meanings of the Antiderivative Used by Students of the First Engineering Courses", International Journal of Science and Mathematics Education, vol. 16, no. 6, 2017, pp. 1091-1113. https://doi.org/10.1007/ s10763-017-9826-2

[9] L. PINO-FAN, " Evaluation of the epistemic facet of the mathematical didactic knowledge of future high school teachers about the derivative", Granada: University of Granada, 2014.

[10] W. GORDILLO and L. PINO-FAN, "A Proposal for Reconstructing the Holistic Meaning of the Antiderivative," Bolema, vol. 30, no. 55, pp. 535-558, 2016. https://doi.org/10.1590/1980-4415v30n55a12