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Refined Advanced Surrogate Assisted Multi-Objective Optimization Algorithm- RASAMO

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ABSTRACT- Multi- objective optimization in structural applications is generally performed with the help of complex computer codes such as Finite Element Analysis (FEA) which are computationally very expensive. Surrogate models or meta- models are comparatively economical and very useful to optimize design solutions. In the earlier studies, authors have developed Advanced Surrogate Assisted Multi- objective Optimization Algorithm (ASAMO) by creating and selecting best single and mixture surrogate models for each offspring solution by Dempster- Shafer theory (DST). For this purpose MATSuMoTo, the MATLAB based tool box is modified for multi- objective optimization problems. In the present study, a Refined Advanced Surrogate Assisted Multi- objective Optimization Algorithm (RASAMO) is presented in which the quality of Pareto- Front of ASAMO algorithm is improved by adopting a Target Value Strategy. The effectiveness of Target Value Strategy is improved: (i) by adding multiple points per optimization iteration, and (ii) by developing most efficient surrogate models. RASAMO is applied to multi- objective machine tool spindle design problem. RASAMO resulted into 1.5% improvement in NHV value and 8.5% for the spread value for less number of function evaluations as compared to ASAMO. RASAMO is very easy to apply on benchmark and engineering applications.

Keywords: Multi- Objective Optimization, MATSuMoTo, Surrogate Models, FEA, RASAMO

I. INTRODUCTION

Multi-objective optimization problems (MOOP) have more than one objective functions and all of them are required to be simultaneously minimized. The actual engineering optimization problems fall under this category of optimization [1]. These problems have several design variables and subjected to a large number of constraints. Solution of these MOOPs usually involves multiple optimization iterations. Pareto-Front solutions are the set of optimized solutions where it is not possible to further minimize one objective function without compromising on the value of other objective functions.

The main objective of any MOOP solving algorithm is to find out this Pareto- Front solution. To solve the complex engineering problems such as crash simulations usually involve the use of Finite Element Analysis (FEA) which are very time consuming even with today's advanced computers. For FEA based multi- objective complex engineering problems requires numerous number of these computer simulations. Surrogate models or meta- models are useful in approximating these simulations and are very less time consuming to solve. In the present study, A new algorithm RASAMO is proposed in which the quality of Pareto- Front of ASAMO algorithm is improved by adopting a Target Value strategy. The target value strategy is a non- evolutionary algorithm (SOCEMO algorithm) [3], which adds new points in to objective space and thus, results into well diversified equally spaced Pareto- Front solution. The effectiveness of original target value algorithm is increased by adding multiple points per optimization iteration and by developing most efficient surrogate models. The developed algorithm is applied to multi-objective machine tool spindle design problem.

The brief outline of the paper is as follows. Section 2 provides the brief literature review carried out for the history of MOOP solving algorithms and surrogate models. In section 3, the details of MOOP and surrogate model based MOOP problem definition are provided. RASAMO algorithm with suggested improvements in ASAMO algorithm by target value strategy is presented in section

5. The developed concepts are applied on the real engineering optimization problem of machine tool spindle design. This is a multi- objective optimization problem with non- linear constraints. Key results

and their discussion are detailed in Section 6. Important conclusions and future scope of this study are summarized in Section 7.

II. LITERATURE REVIEW

On the basis of type of solution methods, MOOP solving algorithms are of evolutionary non-evolutionary types.

In evolutionary category, NSGA-II (Deb [4]) is the most prominent algorithm. Since NSGA-II algorithm is population based, it requires many iterations to get the final solutions. Surrogate model based evolutionary algorithms developed in this field. Different types of surrogate models are developed for this purpose. The combination of NSGA-II with Artificial Neural network was suggested by Nain et al. [5].

SMES-RBF algorithm developed by Datta et al. [6] is developed for constrained MOOP solution. The effect of various types of surrogate models on the quality of non- dominated solutions is studied by Kunakote et al. [7] in SPEA-II algorithm. Pareto- efficient global optimization – ParEGO, a Kriging surrogate model based algorithm was introduced by Knowles, J. [8]. SAMO- a surrogate assisted evolutionary algorithm is developed for constrained MOOP solution (Bhattacharjee et al. [9]). ASAMO- an advanced Surrogate model assisted multi- objective optimization (ASAMO) problem solving algorithm is developed by authors [2]. MATSuMoTo [10], a MATLAB based tool box is used to develop the advanced surrogate models ([11], [12]). Mixture surrogate models are created by using Dempster-Shafer Theory [13].

In non- evolutionary category, Müller [3] has developed SOCEMO algorithm which is a RBF surrogate model based optimization method for solving computationally expensive unconstrained multi-objective problems. In this study, target value strategy [3] is developed for ASAMO algorithm to improve the quality of final non- dominated solutions. Sugumaran Narayanan [14] has used statistical models for unified explanation of occurrence of Civil Wars. William Lau [15] has used regression models for predicting Hong Kong petroleum stock pricing model.

III. SURROGATE MODELS BASED MULTI- OBJECTIVE OPTIMIZATION

In general, a MOOP is defined [16] by the following expressions:

$$\min_{x} \; [f_1(x),\ f_2(x),\ ,\ \dots\dots,\ f_m(x)]^{\mathrm{T}} \tag{1}$$

subjected to

$$g_j(x) \leq 0;\ ,\ j=1, 2, \dots, k$$

$$x_j^l \leq\ x_j \leq\ x_j^u,\ ,\ j = 1, 2, \dots, d$$

In the above expression, $f_i(\mathrm{x})$ represent M objective functions subjected to I constraints $g_i(x)$ with V number of design variables $\mathrm{x_k}$. The solutions of these problems involve many iterations which are based on complex computer codes such as Finite Element Analysis (FEA) and thus require a huge amount of computational efforts. Surrogate models are the approximations of these solutions and are found to be very useful in such situations as they are computationally very less expensive. In the surrogate-based multi-objective problem solution approach, non- dominated (Pareto- front) solutions are obtained by approximation of these functions by equivalent surrogate models ($\mathrm{s_i}$ and $\hat{\mathrm{g}}_\mathrm{j}$).

The problem is formulated as follows:

$$min\ [s_1(x),\ s_2(x),\ ,\dots,s_m(x)]^T \tag{2}$$

subjected to $\hat{g}_j(x) \le 0$, j=1, 2,....,k

$$x_j^l \le x_j \le x_j^u, j = 1, 2, \dots, d$$

For the creation and choice of the finest metamodel, Matlab [17] based MATSuMoTo is used, which utilizes DS theory to combine the effect of four surrogate model characteristics.

Advanced surrogate models are created by creating and selecting best single and mixture surrogate models for each offspring solutions by Dempster- Shafer theory (DST).Table 1 shows the summary of surrogate model types.

Table 1: Summary of surrogate model types [10]

Single Surrogate	
"RBFcub"	"Cubic radial basis function interpolant."
"RBFlin"	"Linear radial basis function interpolant."
"POLYquad"	"Full quadratic regression polynomial."
"POLYcub"	"Full cubic regression polynomial."
Mixture Surrogate:	
"MIX RcPc"	"Ensemble of RBFcub and POLYcub"
"MIX RcPq"	"Ensemble of RBFcub and POLYquad"

ASAMO algorithm has evolutionary base and does not use any mathematical operators for the optimizations so it is difficult to ensure the convergence [18]. Various strategies are adopted to improve the convergence property of non- dominated solutions.

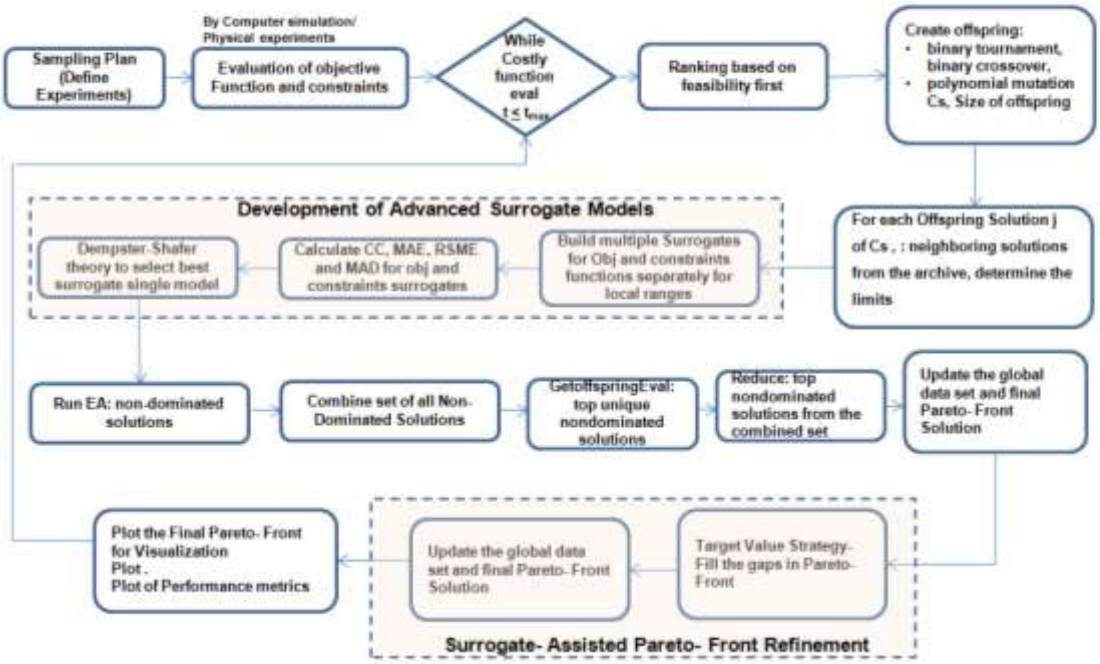

Fig. 1. Refined Advanced Surrogate Assisted Multi- objective Optimization Algorithm (RASAMO)

IV. REFINED ADVANCED SURROGATE ASSISTED MULTI- OBJECTIVE OPTIMIZATION ALGORITHM (RASAMO)

In the related work by authors, an advanced surrogate assisted multi- objective (ASAMO) algorithm [5] is developed to solve constrained MOOP. In this study, authors refined this algorithm and presented the effectiveness of RASAMO algorithm for the constrained MOOP solution. This study showed the effect of various surrogate models on the quality of final Pareto- Font solutions. In the present article, the strategies of the improvement of the quality of non- dominated solutions are discussed.

The Target value strategy is developed by Muller et al. [3] for their non- evolutionary MOOP solving algorithm (SOCEMO). This algorithm uses RBF surrogate model and is capable to solve un-constrained MOOPs. In each optimization cycle, only single point is added in the objective space. The following section describes about the target value in brief. Further details of this strategy are provided in SOCEMO algorithm [3].

The target value strategy is a two-step process of determining feasible solutions. The algorithm starts with the determination of lower and upper bounds of each objective functions. In the objective space, any one of the objective function is assumed as independent variable and a linear RBF function is created by treating others as dependent variable. The choice of independent objective function is not important. Next step is the determination of points in the gap of the Pareto- Front by following Max_Min distance approach [3] with respect to all the existing points. This step is illustrated in Figure 2 by addition of two target values in the gap of Pareto- Front solution of ASAMO algorithm.

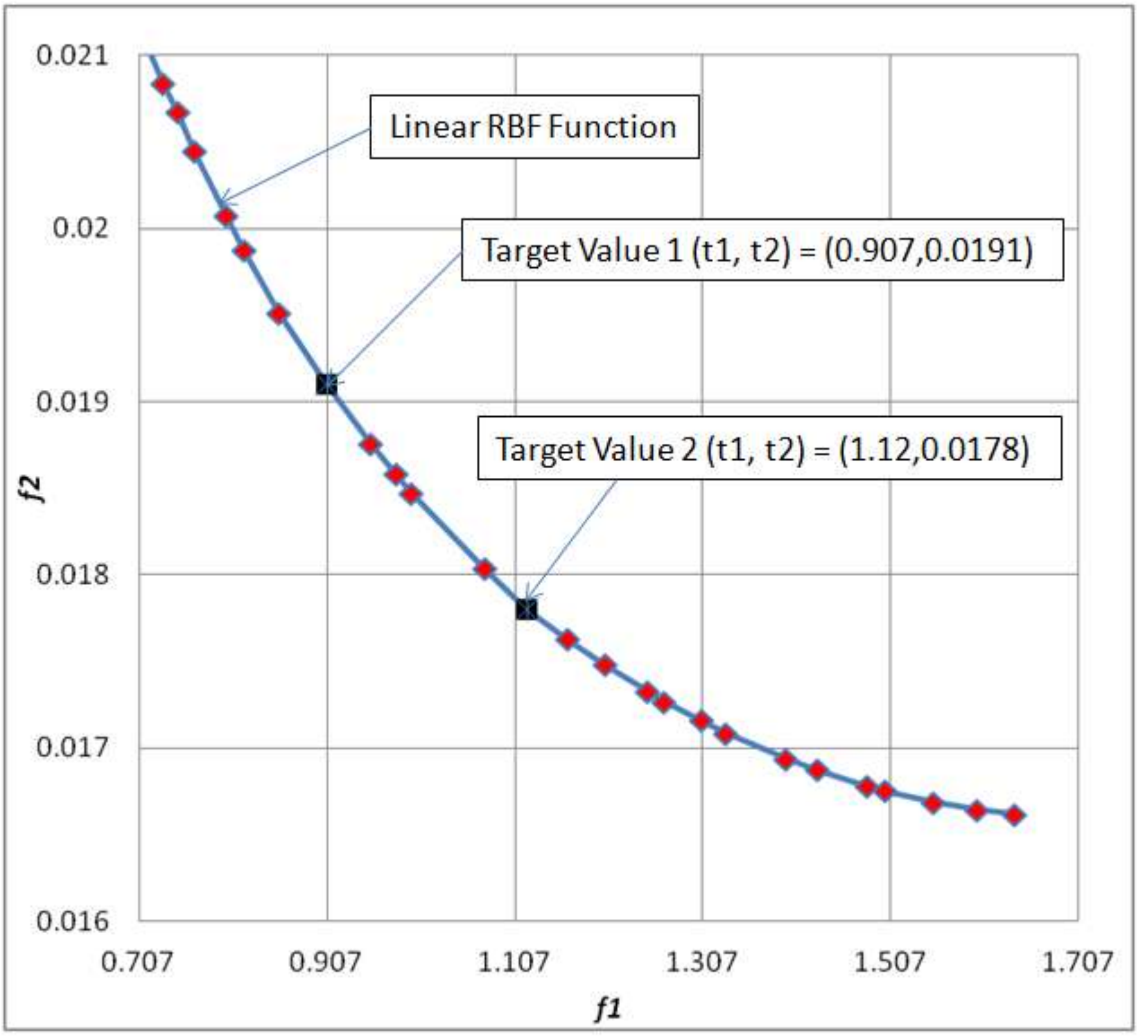

Fig. 2 : Illustration of finding multiple target values in the objective space for RASAMO algorithm

In the next step, for each of the target value, corresponding point in the decision space is determined by solving following multi- objective surrogate model based optimization problem [3].

$$min\ [|s_1(x) - t_1|,\ |s_2(x) - t_2|, \ldots \ldots, |s_m(x) - t_m|]^T \qquad (3)$$

subjected to $\hat{g}_j(x) \leq 0$, j=1, 2,....,k

$$x_j^l \leq x_j \leq x_j^u, j = 1, 2, \ldots, d$$

In this MOOP definition, s_i is the i^{th} global surrogate model for i^{th} objective function and t_i is the target value for this objective function. $\hat{g}_j$ is the surrogate model for the individual constraint functions. This MOOP is solved by genetic algorithm to get the point in the decision space. If the points in the decision space are feasible then they are added to global data set. The target value strategy is modified to make it suitable for ASAMO algorithm. Following changes are made for this adaptation. For faster convergence, the target value strategy is modified for multiple target values as oppose to addition of single target value for each optimization cycle.

As RBF surrogate model may not be suitable to approximate all the objective and constraint functions, single best and best mixture surrogate models are developed for target value strategy. This is in addition to RBF surrogate models.

The performance of any MOOP solving algorithm is quantitatively measured by performance metrics of the algorithm. Generally the performance metrics should be able to measure the convergence property and diversity of non- dominated solutions. Two parameters should be considered to measure the performance of any MOOP solving algorithms.

1. Normalized Hyper Volume (NHV) is considered as one of the performance metric which simultaneously measures the convergence property and diversity of the final non- dominated solution[19]. Higher value of NHV parameter is preferred.

2. Spread is another performance metric which measures the diversity of the solution by measuring the spread of the Pareto-Front solution [20]. Lower value of Spread is preferred.

V. METHOD OF ANALYSIS (NUMERICAL EXPERIMENTS)

For testing the efficiency of the improvement in the advanced surrogate based multi- objective optimization algorithm by target value strategy, optimization of machine tool spindle design [16] is chosen which has 4 design variables. These design variables are combination of continuous and discrete design variables. Figure 2 displays the schematic diagram of machine tool spindle design [16].

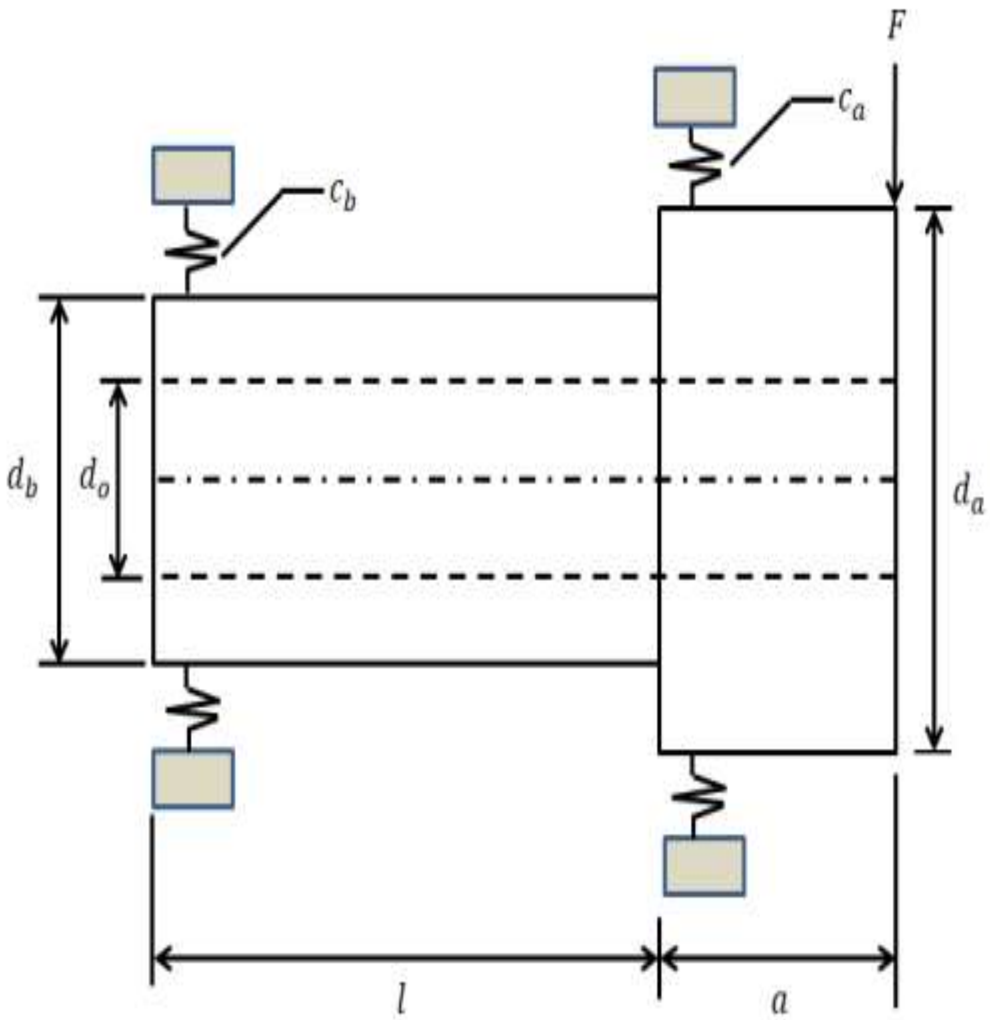

Fig. 3: Details of machine tool spindle with design variables $\mathbf{x} = \{\mathbf{l}, \mathbf{d_o}, \mathbf{d_a}, \mathbf{d_b}\}$.

This problem is a MOOP with 2 objective functions, 3 constraints and 4 design variables. In mathematical terms, the problem is defined as follows

$$min\ [f_1(x),\ f_2(x)]^T \quad (4)$$

subjected to $g_i((x) \leq 0$, j=1, 2,….,3

$$x_j^l \leq x_j \leq x_j^u, j = 1, 2, \ldots, 4$$

This is a typical example of multi- objective optimization with minimization of volume $f_1(x)$ and deflection $f_2(x)$ [11].

$f_1(x)$ is defined by following expression–

$$f_1(x) = \frac{\pi}{4}\left[a(d_a^2 - d_0^2) + l\left(d_b^2 - d_o^2\right)\right] \quad (5)$$

$f_2(x)$ is defined by following expression

$$f_2(x) = \frac{Fa^3}{3EI_a}\left(1 + \frac{l\,I_a}{a\,I_b}\right) + \frac{F}{c_a}\left[\left(1 + \frac{a}{l}\right)^2 + \frac{c_a a^2}{c_b l^2}\right)\right] \quad (6)$$

Here, $I_a = 0.049(d_a^4 - d_o^4)$; $I_b = 0.049\left(d_b^4 - d_o^4\right)$

Bearing stiffness, $c_a = 35400|\delta_{ra}|^{1/9} d_a^{10/9}$; $c_b = 35400|\delta_{ba}|^{1/9} d_b^{10/9}$

$g_1(x)$; $g_2(x)$ are the design proportionality constraints

$$g_1(x) = p_1 d_o - d_b \leq 0 \quad (7)$$

$$g_2(x) = p_2 d_b - d_a \leq 0 \quad (8)$$

$g_3(x)$ is spindle nose radial runout constraint

$$g_3(x) = \left|\Delta_a + (\Delta_a - \Delta_b)\frac{a}{l}\right| - \Delta \leq 0 \quad (9)$$

Bound Constraints: $l_k \leq l \leq l_g$; $d_{a2} \leq d_a \leq d_{a1}$; (10)

$d_{b2} \leq d_b \leq d_{b1}$; $d_{om} - d_o \leq 0$

Target value strategy is developed for ASAMO algorithm and the effectiveness of this strategy is studied on this engineering problem.

VI. Results and Discussion

For RASAMO, NSAGA- II is the base evolutionary algorithm [2]. The crossover distribution index (etc) is set to 20. Etam (Mutation distribution index/ mutation constant) is set at 20. Mutation probability (pm) is set at 0.4. Number of start point is set at 30. The population size is 30. Number of generations is set to 50. SHLD is used for initial design. The maximum number of costly function evaluations is 400. For target value strategy, 3 points per optimization cycle are added on the non-dominated solutions to ensure faster convergence.

RASAMO algorithm utilizes target value strategy. As shown in table 1, various types of single and mixture surrogate models are created locally for each offspring solution. The results of the target value strategy are compared for RBF, single best and best mixture surrogate models. Table 2 provides the Comparative study of quality of Pareto- Front for all surrogate models with target value strategy

Table 2: Comparative study of quality of Pareto- Front for all surrogate models with target value strategy

Type of Surrogate Model	Figure Numbers	Remarks
ASAMO with RBFlcube model	4	Un- equally spread, 3 disconnected regions, un equally spread regions
RASAMO with RBFcube model	5	Equally spread, 4 disconnected regions, equally spread,
ASAMO Singe best surrogate model	7	Equally spread 2 disconnected regions, un equally spread regions
RASAMO Singe best surrogate	8	Equally spread, 4 disconnected regions, equally spread,
Best mixture surrogate model	10	Un- equally spread, 3 disconnected regions, un equally spread regions
RASAMO with Best mixture surrogate	11	Equally spread, 4 disconnected regions, equally spread,

Figures 4 shows the Pareto- Front solution obtained from ASAMO algorithm with RBF cube surrogate model. From this figure it can observed that the non- dominated solutions are unequally spaced in the form of 3 disconnected regions. Figure 4 shows the above configuration of ASAMO algorithm with Target value strategy (RASAMO). This Pareto- Front clearly shows the improvement in spread of non- dominated solutions with 4 disconnected uniform spread solutions. Figure 6 compares the Pareto- Front of obtained by both the above configurations.

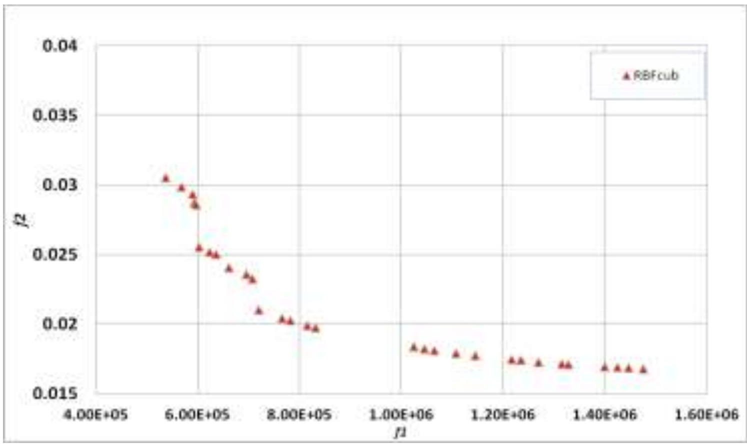

Fig. 4: Non dominated front for ASAMO algorithm for RBF cube surrogate model

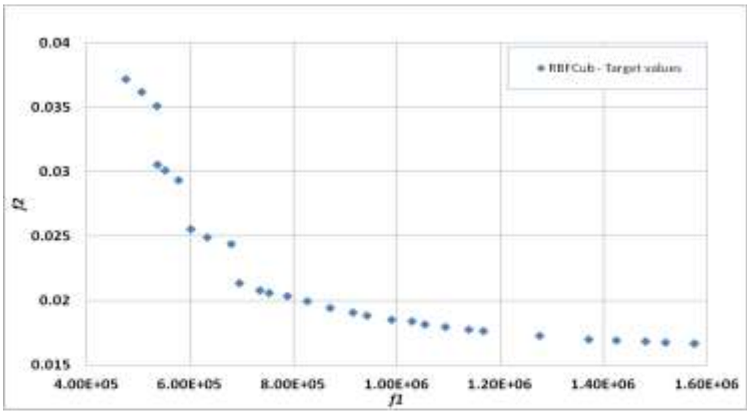

Fig. 5: Non dominated solution for RASAMO algorithm for RBF cube surrogate model with and without target value strategy

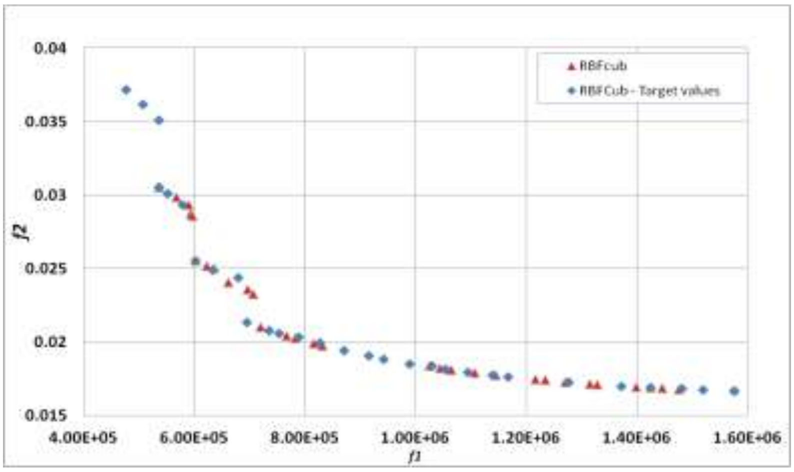

Fig. 6: comparison of Non dominated solution for RASAMO algorithm for RBF cube surrogate model with and without Target value strategy

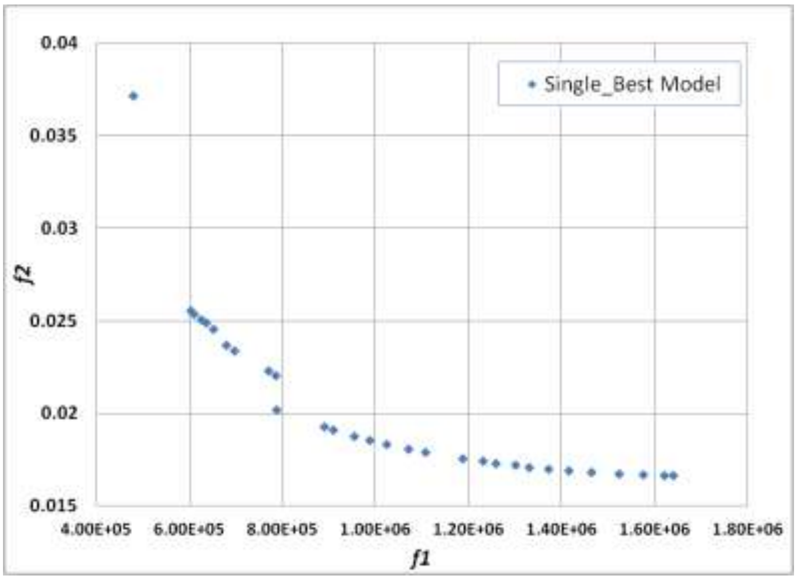

Fig. 7: Non dominated solution for ASAMO algorithm for Single best surrogate model

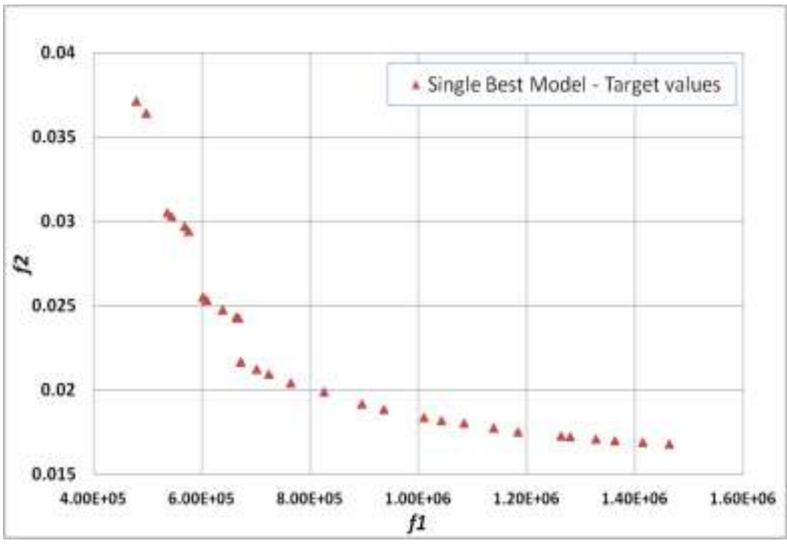

Fig. 8: Non dominated solution for RASAMO algorithm for Single best surrogate model with target value strategy

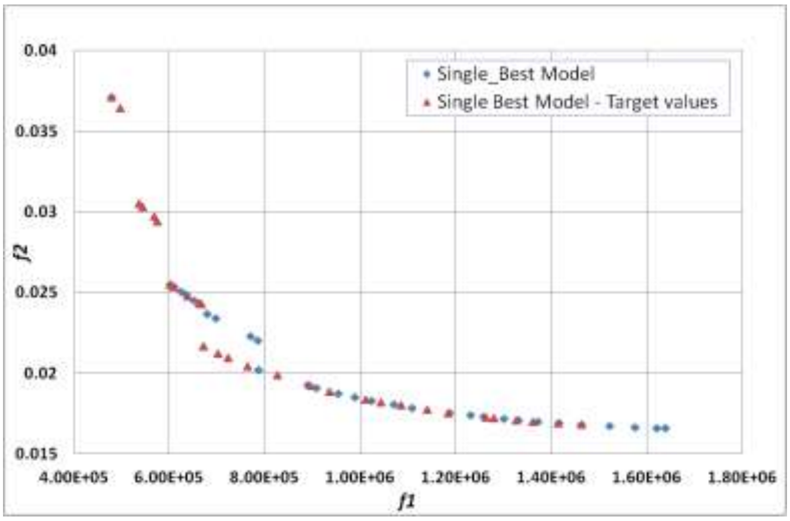

Fig. 9. Comparison of Non dominated solution for RASAMO algorithm for Single best surrogate model with and without Target value strategy

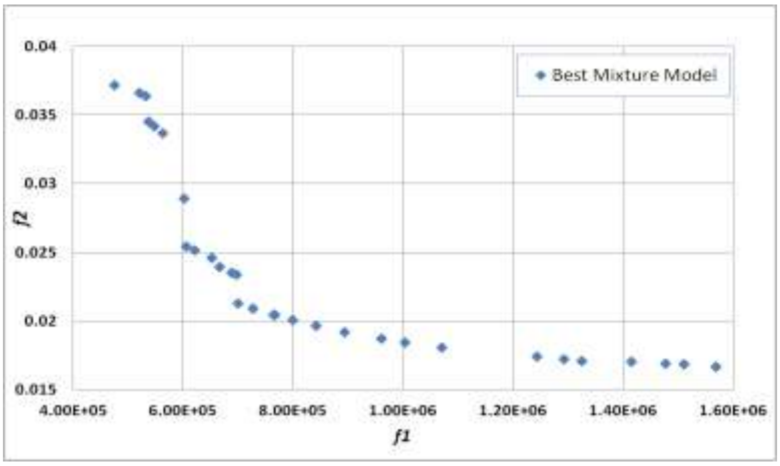

Fig. 10: Non dominated solution for ASAMO algorithm for Best mixture surrogate model

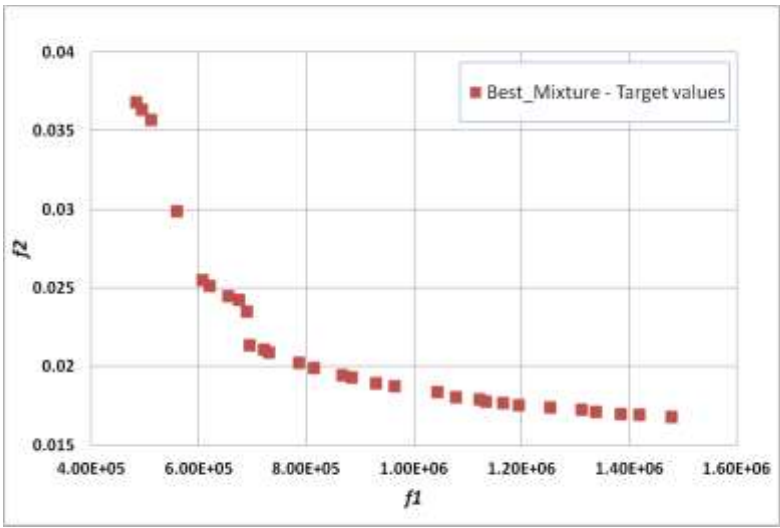

Fig. 11: Non dominated solution for RASAMO algorithm for Best mixture surrogate model with target value strategy

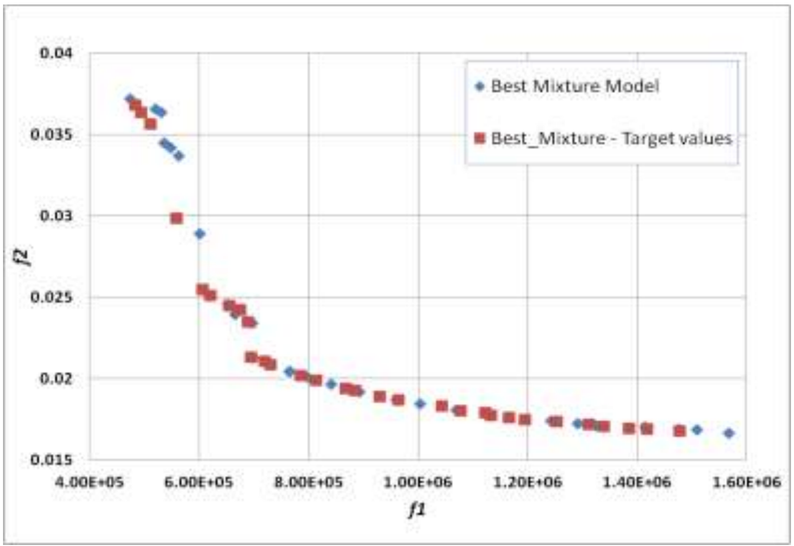

Fig. 12: comparison of Non dominated solution for RASAMO algorithm for Best mixture surrogate model with and without Target value strategy

Similar study is performed for the single best (Figures 7 to 9) and best mixture surrogate models (Figures 10 to 12). In all the cases, the RASAMO algorithm results in equally spaced diversified non- dominated solutions. Performance of RASAMO algorithm with single best surrogate model is found to be best amongst all types of surrogate models.

Table 3 compares the performance metrics (NHV and spread) at 400 number of costly function evaluations. Figures 13 and 14 show the NHV and spread performance parameters of RASAMO algorithms for various number of costly function evaluations. RBFcube surrogate models are developed for the ASAMO and RASAMO algorithms. These surrogate models are used for target value strategy also. As explained earlier, higher NHV value of non- dominate solutions is preferred for better convergence and diversity of the solutions. Algorithm with lower spread value of Pareto-front solution is preferred. The effect of target value strategy with RBFcube surrogate model results into 3% increase in NHV value and 12.6% decrease in spread value for 400 number of costly function evaluations. In cases where prior information regarding the best type of surrogate models for a type of problem is not available, strategy of selection of best surrogate model is found to be very useful.

Table 3: Performance metrics (NHV and spread) at 400 number of costly function evaluations.

Type of surrogate model	NHV value	Spread value
ASAMO- RBF Cube Model	0.410791	1.063897
RASAMO- RBF Cube Model-Target Value	0.42364	0.929900
% Improvement	**3.1%**	**-12.6%**
ASAMO- Single Best Model	0.416319	1.002770
RASAMO- Single Best Model- Target Value	0.422671	0.917437
% Improvement	**1.5%**	**-8.5%**
ASAMO- Best Mixture Model	0.417925	0.962065
RASAO- Best Model- Target Value	0.420041	1.025527
% Improvement	**0.5%**	**6.6%**

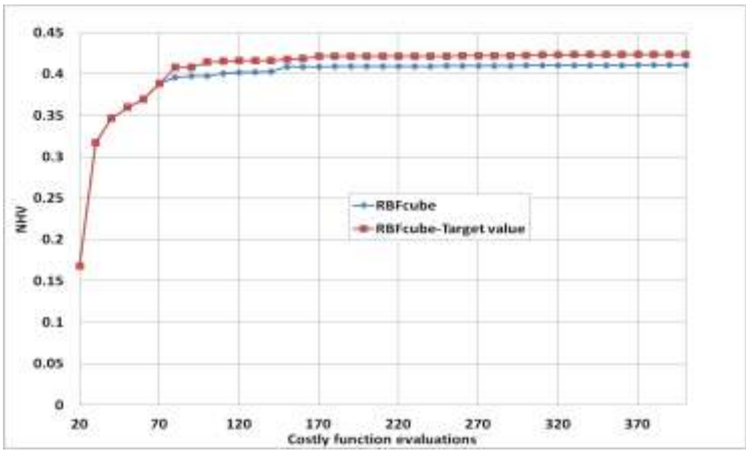

Fig. 13: Effect of target value strategy on NHV for RASAMO algorithm for RBFcube surrogate models

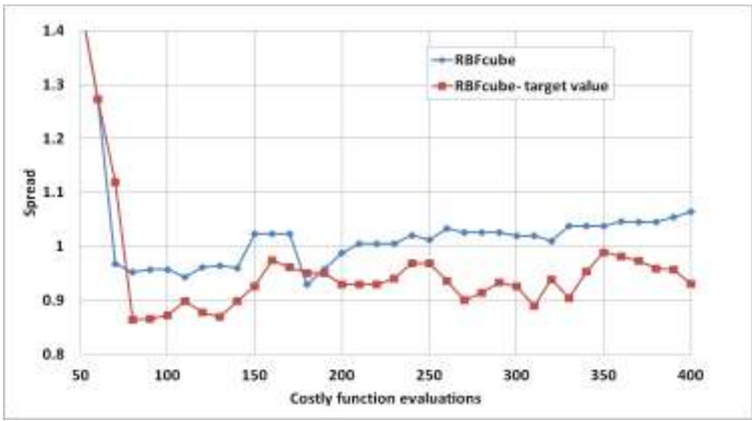

Fig. 14 : Effect of target value strategy on Spread for RASAMO algorithm for RBFcube surrogate models

Figures 15 to 16 show the performance comparison of target value strategy on the NHV and spread for RASAMO algorithm with single best surrogate models. From the study of these figures, it can be observed that there is a 1.5% improvement in NHV value and 8.5% reduction of spread value at 400 numbers of costly function evaluations. Figures 17 to 18 display the effect of target value strategy of RASAMO algorithm with best mixture surrogate models. For this configuration, there is only a marginal improvement in the NHV property for target value strategy. For best mixture surrogate models, already convergent solutions are obtained and there is no further scope of improvement for the target value strategy.

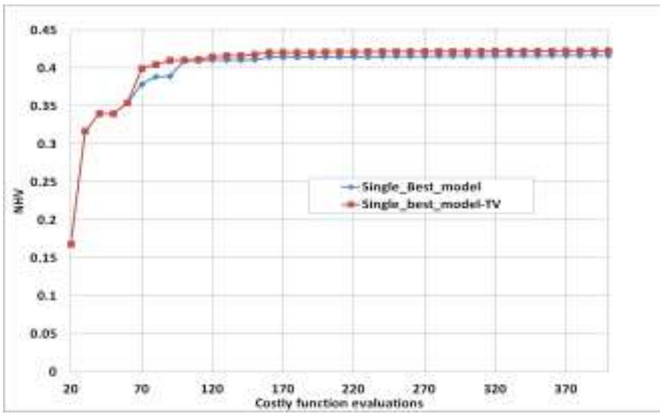

Fig. 15: Effect of target value strategy on NHV for RASAMO algorithm for single best surrogate models

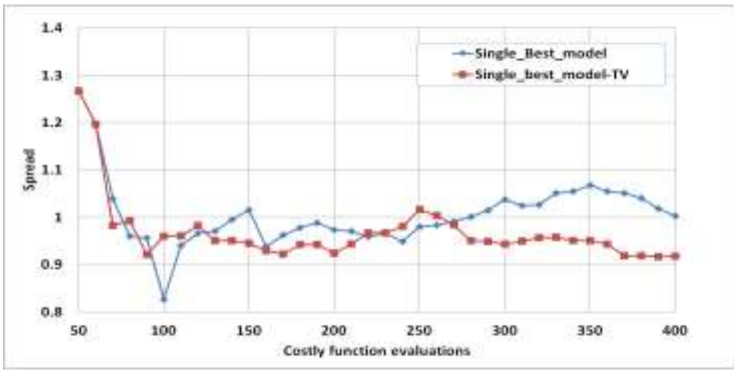

Fig. 16: Effect of target value strategy on Spread for RASAMO algorithm for single best surrogate models

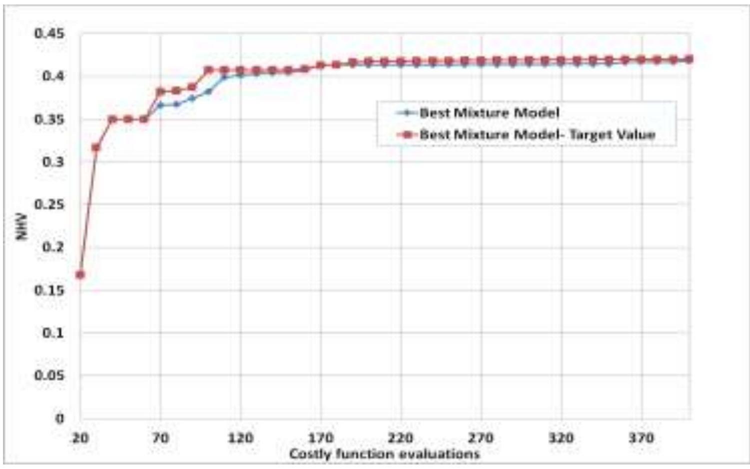

Fig. 17: Effect of target value strategy on NHV for RASAMO algorithm for best mixture surrogate models

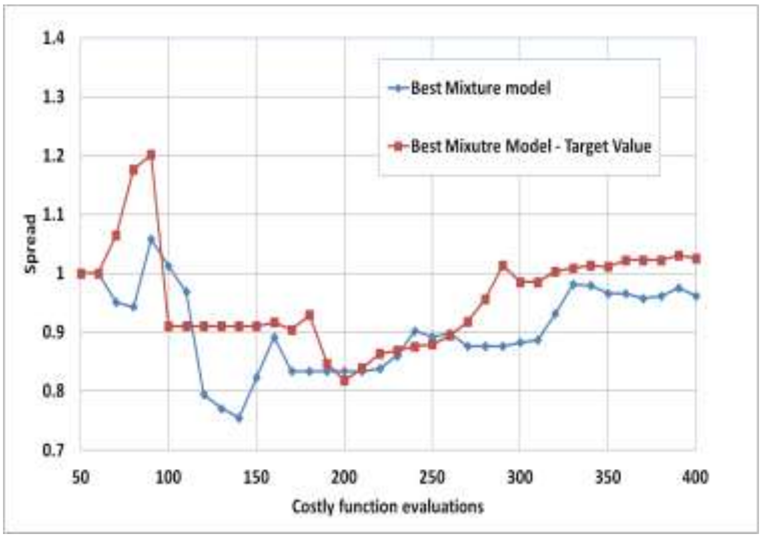

Fig. 18: Effect of target value strategy on Spread for RASAMO algorithm for best mixture surrogate models

VII. CONCLUSION AND SCOPE FOR FUTURE WORK

The comparison of various performance metrics for the ASAMO and RASAMO algorithms clearly shows the improvement in convergence and divergence property of the final Pareto- Front solutions for same number of costly function evaluations. This means that RASAMO is effective in finding the better solutions with less number of expensive function evaluations. The new algorithm is successfully applied on the complex engineering application problems with multiple objectives and several continuous and discrete variables. The nature of the constraints is also non- linear in nature. The suggested changes in the original target value strategy such as development of advanced surrogate models and addition of multiple points per optimization cycle has resulted into enhancement of overall efficiency of RASAMO algorithm to solve MOOP problems.

In cases where the prior knowledge of the suitability of surrogate model for a particular problem type is not available, the strategy of creation and selection of single best surrogate model has been found to be effective for RASAMO algorithm. From this study, it is evident that addition of multiple points per optimization cycle result into faster convergence. There are multiple other parameters affecting the performance of RASAMO algorithm. Their effect is required to be determined on the effectiveness of suggested improvements. In addition to target value strategies, there are numerous other techniques to improve the quality of non- dominated solutions of any MOOP solving algorithms.

Nomenclatures

$f_i(x)$	Objective Function
M	Number of objective functions
$g_i(x)$	Constraint functions, where i is the constraint number (J)
V	Number of design variables
X	Variable vector
$s_i(x)$	Surrogate model of objective function
$\hat{g}_j(x)$	Surrogate model of constraint function

Abbreviations

MOOP	Multi – Objective Optimization Problem
ASAMO	Advanced Surrogate Assisted Multi-Objective Algorithm
SOCEMO	Surrogate Optimization of Computationally Expensive Multi-objective Problems
DOE	Design of Experiments
DST	Dempster-Shafer Theory
MATSuMoTo	Matlab based Surrogate Model Toolbox
EMO	Evolutionary Multi Objective Algorithm
EA	Evolutionary Algorithm
NHV	Normalized Hyper Volume

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