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Improved Ranking Algorithm For Solution Of Fuzzy Assignment Problem Using Trapezoidal Fuzzy Numbers

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Abstract: The paper proposes an improved Ranking Algorithm to solve a Fuzzy Assignment problem, the assignment cost being taken as Generalized Trapezoidal Fuzzy Numbers. The new ranking algorithm reduces the number of computations required to transform the Generalized Trapezoidal fuzzy Numbers [GTFN] into crisp ones. Then Optimal Cost is obtained using Hungarian method [3] in minimum time.

The advantage of the method is illustrated using numerical examples to prove the efficiency of the proposed method for solving Generalized Fuzzy Assignment Problem (GFAP).

Keywords: Fuzzy Assignment Problem, Trapezoidal Fuzzy numbers, Hungarian method

1. Introduction

Fuzzy Assignment problem [FAP] is an effective tool in solving real life problems world- wide. An FAP plays a vital role in Engineering and Management studies. The FAP is efficient (in terms of cost, profit, time etc.,), if each facility (resource) is assigned to one and only one activity (job) by optimizing the given measure of efficiency. Sakawa et al.[4] dealt with the problems on production and work force assignment in a firm using interactive fuzzy programming for two level linear and linear fractional programming models. Chen [5] highlighted a fuzzy assignment model that considers all persons to have same skills. Long-Shen Huang and Li- pu Zhang [6] proposed a mathematical model for the fuzzy assignment problem and transformed the model as certain assignment problem with restriction of qualification. Chen Lian-Hsuan and Lu Hai-Wen [7] employed a procedure for resolving assignment problem with multiple inadequate inputs and outputs in crisp form for each possible assignment using linear programming model for optimum efficiency.

S. H. Chen [8] projected that in many cases it is not possible to restrict the membership function to the normal form and proposed the concept of gen- eralized fuzzy numbers. A. Kaufmann and M. M. Gupta [9] dealt with the transformation of generalized fuzzy numbers into normal fuzzy numbers through normalization process. In literature [10, 11, 19], many papers have applied gen- eralized fuzzy numbers for solving real life problems. Amarpreet Kaur and Amit Kumar [1] defined a new ranking method on generalized trapezoidal fuzzy numbers and also they applied the same technique in solving generalized fuzzy assignment problem which is beneficial over the existing fuzzy ranking methods.

The new proposed ranking algorithm reduces the number of computations required to transform the GTFN into crisp ones in comparison to the algorithm used in [2]

The paper is organized as follows: Section - 2 presents basic definitions. In section - 3, the fuzzy assignment problem and its mathematical formulation are presented. Section - 4 briefs the proposed algorithm. Section - 5 provides a numerical example of the proposed approach and Section - 6 ends with concluding remarks.

2. Preliminaries

2.1 Fuzzy Set

A Fuzzy set $\tilde{A}$ characterized by a membership function mapping elements of a domain, space, or universe of discourse X to the unit interval [0,1](i.e) $\tilde{A}=(a,b,c,d)$, $\tilde{A}=\{(x,\mu_{\tilde{A}}(x));x\in X\}$ here $\mu_{\tilde{A}}:X\rightarrow[0,1]$ is a mapping called the degree of membership function of the fuzzy set $\tilde{A}$ and $\mu_{\tilde{A}}(x)$ called the membership function value of $x \in X$ in the fuzzy set $\tilde{A}$.These membership grades are often represented by real numbers ranging from [0, 1].

2.2 Trapezoidal Fuzzy Number

A fuzzy number $\tilde{A}=(a,b,c,d)$ is said to be a trapezoidal fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x)=\begin{cases}\dfrac{(x-a)}{(b-a)}, a\le x\le b,\\ 1 \qquad b\le x\le c,\\ \dfrac{(x-d)}{(c-d)}, c\le x\le d.\end{cases}$$

2.3 Generalized Trapezoidal Fuzzy Numbers

A generalized trapezoidal fuzzy number $\tilde{A}=(a,b,c,d;\omega)$) is said to be a generalized fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \omega \dfrac{(x-a)}{(b-a)}, a \le x \le b, \\ \omega \qquad b \le x \le c, \\ \omega \dfrac{(x-d)}{(c-d)}, c \le x \le d. \end{cases}$$

3. Fuzzy Assignment Problem(FAP)

The generalized fuzzy assignment problem can be represented in the form of n x n fuzzy cost matrix $\left[\tilde{C}_{ij}\right]$ as given below,

$$\tilde{C}_{ij} = \begin{bmatrix} \tilde{C}_{11} & \tilde{C}_{12} & .. & \tilde{C}_{1j} & .. & \tilde{C}_{1n} \\ \tilde{C}_{21} & \tilde{C}_{22} & .. & \tilde{C}_{2j} & .. & \tilde{C}_{2n} \\ .. & .. & .. & .. & .. & .. \\ \tilde{C}_{i1} & \tilde{C}_{i2} & .. & \tilde{C}_{ij} & .. & \tilde{C}_{in} \\ .. & .. & .. & .. & .. & .. \\ \tilde{C}_{n1} & \tilde{C}_{n2} & .. & \tilde{C}_{nj} & .. & \tilde{C}_{nn} \end{bmatrix}$$

The costs or time $\left[\tilde{C}_{ij}\right]$ are generalized trapezoidal numbers $\tilde{C}_{ij} = \left[C_{ij}^{(1)}, C_{ij}^{(2)}, C_{ij}^{(3)}, C_{ij}^{(4)}; \omega_{ij}\right]$. The objective is to find a optimal way of assigning the j^{th} job to the i^{th} resource, (i.e) assigning all jobs to available resources, by minimizing the total cost with minimum time.

3.1. Mathematical Formulation of Generalized Fuzzy Assignment Problem(GFAP)

The generalized fuzzy assignment problem can be mathematically stated as

Minimize $\tilde{Z} = \sum_{i=1}^{n}\sum_{j=1}^{n} \tilde{C}_{ij},\ i = 1, 2, \ldots, n.$

subject to,

$$x_{ij} = \begin{cases} 1, & if\ i^{th} resource\ is\ assigned\ to\ j^{th}\ job, \\ 0, & \text{otherwise} \end{cases}$$

$\sum_{i=1}^{n} x_{ij} = 1$ (ie.) one job is done by i^{th} resource,(i=1,2,…n) and $\sum_{j=1}^{n} x_{ij} = 1$ (i.e) only one resource should be assigned to j^{th}job, j=1,2,...,n), where x_{ij}specifies that j^{th}

job is assigned to the i^{th}resource.

4 Proposed Method(Algorithm):

Step 1: Test whether the given GFAP is balanced or not.

a. If it is a balanced one (i.e., the number of resource are equal to the number of jobs) then go to step 3.

b. If it is an unbalanced one (i.e., the number of resources are not equal to the number of jobs) then go to step 2.

Step 2: Introduce dummy rows and / or dummy columns with zero fuzzy costs so as to form a balanced one.

Step 3: Find the rank of each cell $C_{ij}$of the chosen fuzzy cost matrix by using the ranking function: if $\widetilde{A}$=(a,b,c,d; ω) then $R(\tilde{A}) = \omega[(a + b + c - d)/2]$

Step 4: Proceed by the Hungarian method to solve fuzzy cost table to get optimal fuzzy assignment.

Step 5: Add the optimal fuzzy assignment using fuzzy addition defined as if $\widetilde{A}$ =(a,b,c,d; ω_1) and $\widetilde{B}$ =(e,f,g,h; ω_2) then $\widetilde{A} + \widetilde{B}$ =(a+e,b+f,c+g,d+h;min(ω_1, ω_2))

5 Numerical Examples:

Problem 1:

Consider a fuzzy assignment problem with four resources A, B, C, D and four jobs I, II, III, IV. Fuzzy cost considered here to be the generalized trapezoidal number. The problem is to find the optimal assignment in an efficient way.

	I	II	III	IV
A	(4,6,8,10;0.1)	(20,24,26,28;0.2)	(14,16,17,19;0.3)	(6,8,11,13;0.1
B	(8,11,13,15;0.2)	(24,26,28,30;0.3	(0,2,4,6;0.1)	(23,25,26,28;0.2)
C	(30,34,38,40;0.1)	(15,17,19,20;0.2)	(14,16,18,20;0.4)	(12,14,15,18;0.3)
D	(14,16,19,20;0.2)	(22,24,26,28,0.6)	(20,22,24,26;0.4)	(6,8,10,12;0.2)

Table 5.1:

Solution: The given generalized trapezoidal fuzzy cost table is balanced one. Using step 4, the rank of generalized trapezoidal fuzzy cost table is:

	I	II	III	IV
A	0.4	4.2	4.2	0.6
B	1.7	7.2	0	4.6
C	3.1	3.1	5.6	3.45
D	2.9	13.2	8	1.2

Proceeding by Hungarian method, the optimal allocations are: Therefore

	I	II	III	IV
A	0	3.8	3.8	0.2
B	1.7	7.2	0	4.6
C	0	0	2.5	0.35
D	1.7	12	6.8	0

The **assignment is A→I, B→III, C→II and D→IV.**

Hence (4,6,8,10;0.1), (0,2,4,6;0.1), (15,17,19,20;0.2), and (6,8,10,12;0.2) arethe generalized fuzzy trapezoidal assignments.

By fuzzy addition,

The Minimum cost is :(4+0+15+6,6+2+17+8,8+4+19+10,10+6+20+12; Min (0.1, 0.2)).

Minimum cost = (25, 33, 41, 48; 0.1).

The number of iterations taken to arrive at the solution is 2

Problem 2:

Consider a fuzzy assignment problem with four resources A, B, C, D and four jobs I, II, III, IV. Fuzzy cost considered here to be the generalized trapezoidal number. The problem is to find the optimal assignment in an efficient way.

	I	II	III	IV
A	(3,5,6,8;0.6)	(5,8,11,13;0.7)	(8,10,11,15;0.5)	(5,8,10,12;0.5)
B	(7,9,10,12;0.7)	(3,5,6,8;0.4)	(6,8,10,12;0.7)	(5,8,10,12;0.8)
C	(2,4,5,7;0.6)	(5,7,10,12;0.7)	(8,11,13,15;0.6)	(4,6,7,10;0.8)
D	(6,8,10,12;0.8)	(2,5,6,8;0.7)	(5,7,10,14;0.6)	(2,4,5,7;0.7)

Solution:

The given generalized trapezoidal fuzzy cost table is balanced one. Using step 4, the rank of generalized trapezoidal fuzzy cost table is:

	I	II	III	IV
A	1.8	3.85	3.5	2.75
B	4.9	1.2	4.2	4.4
C	1.2	3.5	5.1	2.8
D	4.8	1.75	2.4	1.4

Proceeding by Hungarian method, the optimal allocations are: Therefore

	I	II	III	IV
A	0	2.05	0	0
B	3.7	0	1.3	2.2

				5
C	0	2.3	2.2	0.6 5
D	4.3 5	1.3	0.2 5	0

the assignment is A→III, B→II, C→I and D→IV.

Hence (8,10,11,15;0.5), (3,5,6,8;0.4), (2,4,5,7;0.6), and (2,4,5,7;0.7) are the generalized fuzzy trapezoidal assignments.
By fuzzy addition the minimum cost:(8+3+2+2,10+5+4+4,11+6+5+5,15+8+7+7;
Min (0.5, 0.4, 0.6, 0.7)).
Minimum cost = (15, 23, 27, 37; 0.4).
The number of iterations taken to arrive the solution is 5

Problem 3:
Consider a fuzzy assignment problem with four resources A, B, C, D and four jobs I, II, III, IV. Fuzzy cost considered here to be the generalized trapezoidal number. The problem is to find the optimal assignment in an efficient way.

	I	II	III	IV
A	(2,4,7,10;0.1)	(4,6,12,14;0.2)	(7,9,13,17;0.3)	(4,7,11,13;0.3)
B	(6,8,11,13;0.1)	(1,4,7,10;0.3)	(5,7,11,13;0.1)	(4,6,11,13;0.1)
C	(1,3,6,8;0.1)	(4,6,11,14;0.1)	(7,9,14,16;0.2)	(2,5,8,11;0.1)
D	(5,7,11,13;0.1)	(1,3,7,9;0.1)	(4,6,12,15;0.2)	(1,3,6,8;0.1)

Solution: The given generalized trapezoidal fuzzy cost table is balanced one. Using step 4, the rank of generalized trapezoidal fuzzy cost table is:

	I	II	III	IV
A	0.1 5	0.8	1.8	1.3 5
B	0.6	0.3	0.5	0.4
C	0.1	0.3	1.4	0.2

		5		
D	0.5	0.1	0.7	0.1

Proceeding by Hungarian method, the optimal allocations are: Therefore

	I	II	III	IV
A	0	0.55	1.35	1.1
B	0.4	0	0	0.1
C	0	0.15	1	0
D	0.5	0	0.4	0

the assignment is A→I, B→III, C→IV and D→II.

Hence (2,4,7,10;0.1), (5,7,11,13;0.1), (2,5,8,11;0.1)and (1,3,7,9;0.1)are the generalized fuzzy trapezoidal assignments.

By fuzzy addition

The minimum cost:(2+5+2+1,4+7+5+3,7+11+8+7,10+13+11+9;Min (0.1)).

Minimum cost = (10, 19, 33, 43; 0.1).

The number of iterations taken to arrive the solution is 4.

The table below compares the number of iterations required to achieve optimal assignment using existing method[2] and the proposed method.

NUMERICAL PROBLEM	NUMBER OF ITERATIONS	
	EXISTING METHOD[2]	PROPOSED METHOD
1	3	2
2	7	5
3	9	4

6 CONCLUSION

Hence the proposed Ranking Algorithm reduces the computational efforts significantly to get the final optimal assignment. Hence it creates a scope to effectively solve and analyze many complex real life fuzzy assignment problems.

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