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Modeling mathematics achievement using hierarchical linear models[1]

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Abstract: The purpose of this study was to investigate the effects of student-level and school-level variables on mathematics achievement in four countries including Finland, South Korea, Taiwan, and Turkey. Based on eighth grade students' responses to a student questionnaire and a mathematics test, and school principals' responses to a school questionnaire, the Trends in International Mathematics and Science Study (TIMSS) 2011 dataset was analyzed using Hierarchical Linear Models. The results revealed that students' having educational resources at home, their self-confidence in mathematics, and schools' emphasis on academic standards were the common variables that influenced mathematics achievement in all four countries. On the other hand, no significant effects of education levels of parents (except education levels of mothers in Turkey), fights or physical injuries to other students, and lack of resources for mathematics instruction on students' mathematics achievement were found in these countries. Implications of the results are discussed.

Keywords: Hierarchical linear modeling, mathematics achievement, TIMSS 2011

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INTRODUCTION

International large-scale assessments such as the Trends in International Mathematics and Science Study (TIMSS) and the Program for International Student Assessment (PISA) are assessments organized by the International Association for Evaluation of Educational Assessment (IEA) and the Organization for Economic Co-operation and Development (OECD), respectively. In addition to providing information about the achievement trends of the countries (Johansson, 2016), these assessments lead to identifying strengths and weaknesses of education systems of these countries by comparing their relative rankings, and shaping their educational policies for improving the quality of their education systems. Because countries need citizens with high competence in mathematics and science for surviving in the global economy, it is important to analyze such international assessments by detecting similarities and differences across the countries in terms of their students' mathematics and science achievement. In addition to measuring students' achievement in mathematics and science, the variables that are associated with these achievements can also be examined for each country.

Considering the fact that countries which consistently have high performance (i.e., top rankings) indicate evidence for high quality education systems, it is important to analyze the variables that influence mathematics and science achievement of students in these countries and to compare the effects of these variables on achievement with those in other countries which need to improve their education systems. Based on the rankings in such international large-scale assessments, three of the top-performing countries are Finland, South Korea, and Taiwan (e.g., Mullis, Martin, & Foy, 2008; Mullis, Martin, Foy, & Arora, 2012). Compared to these countries, one country, whose students' mathematics and science achievement is below the average on TIMSS assessments, is Turkey (e.g., Mullis et al., 2012). Several studies have focused on explaining the effects of student-level and school-level variables on Turkish eighth grade students' mathematics achievement (e.g., Aksu, Guzeller, & Eser, 2017; Ozberk, Atalay Kabasakal,

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& Boztunc Ozturk, 2017; Sulku & Abdioglu, 2015; Yavuz, Demirtasli, Yalcin, & Dibek, 2017). In one of those studies, Yavuz et al. (2017) examined the effects of student-level and school-level variables on Turkish eighth grade students' mathematics achievement using the TIMSS 2011 dataset. Yavuz et al. found that Turkish students' mathematics achievement was associated with bullying at school, liking mathematics, self-confidence in mathematics, and teachers' emphasis on academic success. However, their achievement was found to be not related to valuing mathematics, teachers' working conditions, and teachers' collaborations to enhance teaching.

Despite numerous studies on examining the effects of student-level and school-level variables on mathematics achievement of students in Turkey, there exists few studies (e.g., Akyuz, 2014; Akyuz & Berberoglu, 2010) that have compared mathematics achievement of Turkish students with other top-performing countries. For example, in one recent study using TIMSS 2011 dataset, Akyuz (2014) examined the effects of student-level and school-level variables on eighth grade students' mathematics achievement in Finland, Singapore, USA, and Turkey. Akyuz found that educational resources at home, school composition by students' socioeconomic background and self-confidence in mathematics were significantly related to students' mathematics achievement in all four countries. Therefore, comparing the effects of student-level and school-level variables on Turkish students' mathematics achievement with those on students' mathematics achievement in top-performing countries have the potential to inform what are the main effects that are critical at the student-level and school-level so that it is possible to develop policies in Turkey by focusing on those particular areas.

Student-level and School-level Variables

Mathematics achievement is a multidimensional construct and refers to one's performance in mathematics. Several student-level and school-level variables contribute to the explanation of mathematics achievement. In terms of student-level variables, education levels of parents (i.e., *Mother Education* and *Father Education*) can be important for contributing to students' mathematics achievement (Sandefur, Meier, & Campbell, 2006). Parents with higher education levels have the potential for helping their children's homework and providing better opportunities such as access to high quality schools. While several studies have shown a positive relationship between parents' education levels and students' achievement (e.g., Tomul & Savasci, 2012), some other studies have found that students' mathematics achievement was not related to parents' education levels (Liu, Wu, & Zumbo, 2006). In addition to parents' education levels, educational resources at home (i.e., *Home Resources*) such as the number of books, educational aids including computer and study desk might also contribute to mathematics achievement of students (Akyuz & Berberoglu, 2010).

Bullying is another variable that might help explain mathematics achievement. Roman and Murillo (2011) reported that elementary school students who experienced bullying at school demonstrated worse mathematics performance than those students who did not. Moreover, affective variables such as enjoyment of learning mathematics (i.e., *Like Math*), value of learning mathematics (i.e., *Value Math*), and self-confidence in mathematics (i.e., *Confidence Math*) can make a difference regarding students' mathematics achievement. Affective variables are evaluated in the domain of mathematics attitudes, and there exists, in general, a positive relationship between mathematics attitudes and mathematics achievement (Kloosterman, 1991; Ma & Kishor, 1997). In a study using the TIMSS 2011 dataset, Kim, Park, Park, and Kim (2013) examined the effects of school and students' educational contexts in fourth and eighth grade students in South Korea, Singapore, and Finland. Kim et al. found that while self-confidence in mathematics (i.e., *Confidence Math*) significantly contributed to mathematics achievement in all three countries, valuing mathematics (i.e., *Value Math*) was positively related to mathematics achievement of students only in South Korea and Singapore. In another study, Ker (2016) examined the effects of student-level and school-level variables using the TIMSS 2011 dataset for Singapore, Taiwan, and USA. Ker found that while self-confidence in mathematics (i.e., *Confidence Math*) was significantly associated with students' mathematics achievement in all three countries, valuing mathematics (i.e., *Value Math*) significantly contributed to students' mathematics achievement only in Taiwan. Similarly, in Arikan, van de Vijver, and Yagmur's

(2016) study that compared mathematics achievement of eighth grade students between Turkey and Australia using TIMSS 2007 and TIMSS 2011 datasets, they reported significant relationship between self-confidence in mathematics (i.e., *Confidence Math*) and mathematics achievement in both countries.

In terms of school-level variables, an income level in the nearby areas of a school (i.e., *Income Area*) might be important because a high income level may indicate that students come from families with high socioeconomic status as opposed to a low income level that likely indicates that students come from families with low socioeconomic status. Similarly, teachers' job satisfaction (i.e., *Job Satisfaction*), intimidation or verbal abuse among students (i.e., *Intimidation*), occurrence of physical injuries to students resulting from fights (i.e., *Fights*), and *Lack of Resources* related to mathematics instruction might be other important variables that contribute to students' mathematics achievement. Furthermore, emphasis of schools on academic achievement might influence students' mathematics achievement because school settings that encourage hard work and put high emphasis on academic standards have the potential of providing students better mathematics performance. In Kim et al.'s (2013) study, they found that schools' emphasis on academic standards (i.e., *School Emphasis*) significantly contributed to students' mathematics achievement in Finland and Singapore, but not in South Korea. Finally, school discipline and safety characteristic of a school (i.e., *School Discipline*) might enable students to focus more on improving their mathematics performance.

Due to the complexity of mathematics achievement as a construct, many variables have been used to explain it such as variables related to students, teachers, and schools. As a result, most of the studies in the literature have developed different models believed to explain mathematics achievement. In addition, past research so far has provided contradictory results about the effects of student, home-family background, and school related variables on students' mathematics achievement. For example, in terms of student related variables, some studies (e.g., Papanastasiou, 2002) suggested that students' attitudes towards mathematics and their beliefs about success in mathematics did not have a significant effect on their mathematics achievement. On the other hand, some other studies (e.g., Berberoglu, Celebi, Ozdemir, Uysal, and Yayan, 2003) documented that variables such as students' perception of failure or success in mathematics greatly influenced students' mathematics achievement. Similarly, regarding home-family background related variables, some studies (e.g., Yayan & Berberoglu, 2004) documented significant relationships among mathematics achievement and home-family background variables such as parents' education levels (i.e., *Mother Education* and *Father Education*), home educational resources (i.e., Home Resources), and socioeconomic status, but some other studies (e.g., Yavuz, 2009) found indirect effect of family income on students' mathematics achievement. In terms of school related variables, while some studies (e.g., Chiu, 2010) pointed out that school variables have little role in contributing to students' mathematics achievement, some other studies (e.g., Edmonds, 1979) reported that with good principal leadership and high expectations for achievement, school makes a difference on achievement.

Considering the past research, the present study analyzed the TIMSS 2011 dataset by comparing the effects of both student and school variables on eighth grade students' mathematics achievement in four countries including Finland, South Korea, Taiwan, and Turkey. The results of this study contribute to the literature by helping reconcile the contradictory results reported so far about the effects of student-level and school-level variables on students' mathematics achievement. Furthermore, identifying such variables contributing to mathematics achievement in each country is important because such an identification will enable policy-makers in each country (especially, in Turkey) to distinguish similarities and differences in the four countries and to improve the quality of education in their countries. In this study, the following research questions were addressed:

1. Which student-level variables contribute to mathematics achievement of eighth grade students in Finland, South Korea, Taiwan, and Turkey?
2. Which school-level variables contribute to mathematics achievement of eighth grade students in Finland, South Korea, Taiwan, and Turkey?

METHODS

Participants

A two-stage sampling design of the TIMSS was used in this study. At the first stage, a sample of schools was obtained. At the second stage, one or two classes were randomly sampled in each school. Table 1 presents the descriptive statistics for each country including the number of schools, the number of students, rankings, and mean mathematics achievement scores. Based on Table 1, while Finland, South Korea, and Taiwan were the top-scoring countries, Turkey had relatively low ranking in this assessment.

Table 1. *Descriptive statistics for country*

Country	Number of schools	Number of students	Rank	Mean Score
Finland	131	3626	8	514
South Korea	147	5015	1	613
Taiwan	149	4958	3	609
Turkey	236	6502	24	452

Instruments

The data for this study consisted of eighth grade students' responses to the TIMSS 2011 Student Questionnaire and the Mathematics Test, and school principals' responses to the TIMSS 2011 School Questionnaire. TIMSS is an international assessment conducted by the IEA in every four years since 1995. While the Mathematics Test measured mathematics achievement of students on numbers, algebra, geometry, and data and chance, the Student Questionnaire asked students questions about their home and school lives, and their attitudes towards mathematics and science. Additionally, the School Questionnaire asked school principals questions about school environment such as teachers' performance, school resources for instruction, parental involvement, and roles of the principals (Mullis et al., 2012).

All instruments used in TIMSS 2011 were reliable. As one indicator of reliability, Cronbach's alpha reliability was used at the instrument booklet level and the international median reliability coefficient was 0.87 for the Mathematics Test at the eighth grade. In particular, the reliability coefficients for this test at the eighth grade were 0.86, 0.91, 0.94, and 0.92 for Finland, South Korea, Taiwan, and Turkey, respectively (Foy, Martin, Mullis, & Stanco, 2012, p. 20). Psychometric characteristics of every item for all instruments used in TIMSS 2011 were checked for quality insurance and low quality items were removed from the database (See Foy et al., 2012 for details about reliability and validity of the instruments used in TIMSS 2011). Table 2 and Table 3 present detailed explanations for both student-level and school-level variables.

Table 2. *Explanations of Student-level Variables*

Mother Education (BSBG06A)	This index was based on students' responses related to the highest education level of their mother (or stepmother or female guardian). [1=did not finish elementary school or middle school or did not go to school, 2=finished middle school, 3=finished high school, 4=finished junior college, 5=finished university, 6=finished graduate school, 7=Beyond graduate school]
Father Education (BSBG06B)	This index was based on students' responses related to the highest education level of their father (or stepfather or male guardian). [1= did not finish elementary school or middle school or did not go to school, 2= finished middle school, 3= finished high school, 4=finished junior college, 5=finished university, 6=finished graduate school, 7=Beyond graduate school]
Home Resources (BSDGHER)	This index was based on students' responses regarding number of books at home, educational aids such as computer, study table and dictionary at home, and parents' education. [1=Few resources, 2=Some resources, 3=Many resources]
Bullying (BSDGSBS)	This index was based on students' responses regarding the following behaviors: a) I was made fun of or called names, b) I was left out games or activities by other students, c) Someone spread lies about me, d) Something was stolen from me, e) I was hit or hurt by other student(s) (e.g., shoving, hitting, kicking), f) I was made to do things I didn't want to do by other students. [1=Almost never, 2=About monthly, 3=About weekly]
Like Math (BSDGSLM)	This index was based on students' responses to the following statements: a) I enjoy learning mathematics, b) I wish I did not have to study mathematics, c) Mathematics is boring, d) I learn many interesting things in mathematics, e) I like mathematics, f) It is important to do well in mathematics. [1=Like learning math, 2=Somewhat like learning math, 3=Do not like learning math]
Value Math (BSDGSVM)	This index was based on students' responses to the following statements: a) I know what my teacher expects me to do, b) I think of things not related to the lesson, c) My teacher is easy to understand, d) I am interested in what my teacher says, e) My teacher gives me interesting things to do. [1=Value, 2=Somewhat value, 3=Do not value]
Confidence Math (BSDGSVM)	This index was based on students' responses to the following statements: a) I usually do well in mathematics, b) Mathematics is more difficult for me than for many of my classmates, c) Mathematics is not one of my strengths, d) I learn things quickly in mathematics, e) Mathematics makes me confused and nervous, f) I am good at working out difficult mathematics problems, g) My teachers think I can do well in mathematics (i.e., programs, classes, lessons) with difficult materials, h) My teacher tells me I am good at mathematics, i) Mathematics is harder for me than any other subject. [1=Value, 2=Somewhat value, 3=Do not value]

Note. Round brackets indicate the actual codes from the TIMSS 2011 database.

As can be seen in Table 2, the student-level variables of this study were *Mother Education* (i.e., Education levels of mothers), *Father Education* (i.e., Education levels of fathers), *Home Resources* (i.e., Educational home resources), *Bullying* (i.e., Bullying at school), *Like Math* (i.e., Like learning mathematics), *Value Math* (i.e., Value learning mathematics), and *Confidence Math* (i.e., Self-confidence in mathematics).

In addition, the school-level variables, as shown in Table 3, were *Income Area* (i.e., Schools' immediate area), *Job Satisfaction* (i.e., Teachers' job satisfaction), *Intimidation* (i.e., Intimidation or verbal abuse among students), *Fights* (i.e., Fights or physical injuries to other students), *Lack of Resources* (i.e., Lack of resources affecting mathematics instruction), *School Emphasis* (i.e., Schools' emphasis on academic standards), and *School Discipline* (i.e., School discipline and safety).

Table 3. *Explanations of school-level variables*

Income Area ((BCBG05C)	This index was based on school principals' responses related to the average income level of the school's immediate area. [1=High, 2=Medium, 3=Low]
Job Satisfaction (BCBG11A)	This index was based on school principals' responses related to teachers' job satisfaction within their schools. [1=Very high, 2=High, 3=Medium, 4=Low, 5=Very low]
Intimidation (BCBG12AH)	This index was based on school principals' responses related to intimidation or verbal abuse among students (including texting, emailing etc.) in their schools. [1=Not a problem, 2=Minor problem, 3=Moderate problem, 4=Serious problem]
Fights (BCBG12AI)	This index was based on school principals' responses related to physical injury to other students. [1=Not a problem, 2=Minor problem, 3=Moderate problem, 4=Serious problem]
Lack of Resources (BCDGMRS)	This index was based on school principals' responses related to lack of resources for mathematics instruction in the following areas: a) Teachers with a specialization in mathematics, b) Computers for mathematics instruction, c) computer software for mathematics instruction, d) library materials relevant to mathematics instruction, e) audio-visual resources for mathematics instruction, f) calculators for mathematics instruction. [1=Not affected, 2=Somewhat affected, 3=Affected a lot]
School Emphasis (BCDGEAS)	This index was based on school principals' responses related to school emphasis on academic success to the following statements: a) Teachers' understanding of the school's curricular goals, b) Teachers' degree of success in implementing the school's curriculum, c) Teachers' expectations for student achievement, d) Parental support for student achievement, e) Students' desire to do well in school. [1=Very high emphasis, 2=High emphasis, 3=Medium emphasis]
School Discipline (BCDGDAS)	This index was based on school principals' responses on school discipline and safety to the following statements: a) I usually do well in mathematics, b) Mathematics is more difficult for me than for many of my classmates, c) Mathematics is not one of my strengths, d) I learn things quickly in mathematics, e) Mathematics makes me confused and nervous, f) I am good at working out difficult mathematics problems, g) My teachers think I can do well in mathematics (i.e., programs, classes, lessons) with difficult materials, h) My teacher tells me I am good at mathematics, i) Mathematics is harder for me than any other subject. [1=Hardly any problems, 2=Minor problems, 3=Moderate problems]

Note. Round brackets indicate the actual codes from the TIMSS 2011 database.

Data Analysis

For analyzing TIMSS and PISA assessments to date, many statistical techniques have been used to explain students' mathematics and science achievement including analysis of variance (ANOVA; e.g., Pahlke, Hyde, & Mertz, 2013), multiple regression (e.g., Sulku & Abdioglu, 2015), diagnostic classification models (DCMs; e.g., Lee, Park, & Taylan, 2011), structural equation modeling (SEM; e.g., Kalender & Berberoglu, 2009; Papanastasiou & Zembylas, 2002), and hierarchical linear models (HLM; e.g., Akyuz, 2014; Wang, Osterlind, & Bergin, 2012). In this study, HLM was preferred as a statistical technique because TIMSS and PISA datasets are nested in nature (i.e., students are nested within schools) and HLM is an appropriate technique to account for the nesting of students within schools. Data files used in this study were obtained from the IEA website (http://timssandpirls.bc.edu/timss2011/internationaldatabase). In order to analyze the data, model building approach using HLM (Raudenbush & Bryk, 2002) was followed to study the incremental contribution of student-level and school-level variables in explaining mathematics achievement. In this approach, HLM models are built sequentially.

As the first step, unconditional models were estimated for each country. Unconditional models do not contain any student-level (i.e., level-1) or school-level (i.e., level-2) variables, and they aim at partitioning the variance as within-schools and between-schools using intra-class correlation (ICC) for the degree of non-independence in the data. The ICC can be defined as the proportion of mathematics achievement score variation among schools and as how two students' mathematics achievement scores are related to each other within the same school

(Peugh, 2010). Thus, the ICC value provides information about the need for HLM in the analysis. A zero value of ICC indicates no average mathematics achievement score variation among schools, implying that all score variations exist among students. In this situation, HLM analysis is not needed and other statistical techniques such as ANOVA and regression can be used to analyze the particular dataset (Peugh, 2010). The second step estimated was the random coefficients models, which contain only student-level (level-1) variables. In this step, random coefficients that do not have variability across school-level units are fixed prior to the third and final step. Finally, the third step estimated was the full contextual models, which contain both student-level (level-1) and school-level (level-2) variables. In the full contextual models, the extent to which student-level and school-level variables contribute to mathematics achievement was determined. Therefore, the full contextual models include the student-level and school-level variables that have statistically significant relationships with students' mathematics achievement for each country. These three steps were applied to the four countries separately.

In each step, model building approach was pursued through model comparisons using Likelihood Ratio Test (or Chi-square Difference Test). Likelihood Ratio Test was used to compare deviance values of two nested models (i.e., models in which one model is a subset of the other model). Deviance represents the badness of fit of a given model, and subtracting the deviance of the simpler model ($2LL_1$) with df_1 degrees of freedom from the deviance of the more complex model ($2LL_2$) with df_2 degrees of freedom demonstrates the change in the deviance values. If this difference value is larger than the critical value of chi-square with $df_1 - df_2$ degrees of freedom, then the more complex model is preferred as part of the model building approach (McCoach, 2010). HLM 7.00 software (Raudenbush, Bryk, & Congdon, 2011) was used to build a two-level HLM model. To handle missing data, listwise deletion was performed before starting to the analysis. Lastly, the present study used all plausible values to indicate students' mathematics achievement in parameter estimations.

During the analysis, several assumptions including normality assumptions at both levels, homogeneity of variances at the student level and assumption of independence of errors were checked and these assumptions were met.

RESULTS

Unconditional Models

Unconditional models (i.e., One-way ANOVA with random effects model) were the same for each country as follows:

L1 (i.e., level-1) $\text{MathAch}_{ij} = \beta_{0j} + r_{ij}$

L2 (i.e., level-2) $\beta_{0j} = \gamma_{00} + u_{0j}$

where $MathAch_{ij}$ is the mathematics achievement score for student *i* in school *j*; β_{0j} is the mean mathematics achievement of school *j*; r_{ij} is the variability of random error at the student-level (level-1); γ_{00} is the grand mean of mathematics achievement of all students in a country; and u_{0j} is the random error at the school level (level-2). While the variance of r_{ij}, namely σ^2, is the variability of mathematics achievement between students within each school, the variance of u_{0j}, namely τ_{00}, indicates the variability of mathematics achievement between schools. Thus, the ICC (denoted as ρ) is defined as τ_{00} / $(\tau_{00} + \sigma^2)$ and ICC demonstrates relative school differences for each unconditional model. Estimates of γ_{00}, σ^2, τ_{00}, and ρ are given in Table 4.

Table 4. *Results from unconditional models by country*

	Finland	South Korea	Taiwan	Turkey
Grand mean, γ_{00}	514.57**	612.25**	609.85**	453.97**
	(2.16)	(2.51)	(4.17)	(4.08)
Within-school variance, σ^2	3506.27	7194.87	8220.94	8172.70
	[59.21]	[84.82]	[90.67]	[90.39]
Between-school variance, τ_{00}	472.36	712.51	2342.67	3607.01
	[21.73]	[26.69]	[48.40]	[60.04]
Intra-class correlation, ρ	0.12	0.09	0.22	0.31

Note. Round brackets indicate standard errors; Square brackets indicate standard deviations
** $p < 0.001$; otherwise, not significant.

Based on Table 4, the highest and the smallest grand means belonged to South Korea and Turkey, respectively. Regarding ICC values, the largest variance between schools was in Turkey (31%), and the smallest variance was in South Korea (9%). Variance within the same schools was 69%, 78%, 88%, and 91% for Turkey, Taiwan, Finland, and South Korea, respectively. These results confirm the need to use HLM analysis for each country in the present study because there exists average mathematics achievement score variations across schools in each country.

Conditional Models

Random Coefficients Models

Model building at the student level started in each country by adding student-level variables one by one to the original model and then by comparing each model's deviance values through Likelihood Ratio Test. In particular, the student-level variables entered into the models sequentially were *Mother Education*, *Father Education*, *Home Resources*, *Bullying*, *Like Math*, *Value Math*, and *Confidence Math*. Table 5 presents the results from random coefficients models.

Table 5. *Fixed Effects of Student-level Variables with No School-level Variables*

	Finland	South Korea	Taiwan	Turkey
Grand mean, γ_{00}	672.7**(5.1)	905.3**(5.6)	872.6**(7.2)	704.6**(7.3)
Mother Education, γ_{10}	—	—	—	5.3**(0.6)[0.05]
Father Education, γ_{20}	—	—	—	—
Home Resources, γ_{30}	20.3**(2.1)[0.32]	41.3**(1.9)[0.47]	36.6**(2.4)[0.36]	32.7**(2.1)[0.30]
Bullying, γ_{40}	-4.9**(1.6)[-0.08]	—	—	-4.0*(1.4)[-0.04]
Like Math, γ_{50}	—	13.4**(2.0)[0.15]	21.9**(2.2)[0.21]	—
Value Math, γ_{60}	2.5*(1.2)[0.04]	18.0**(1.7)[0.20]	10.8**(1.9)[0.11]	—
Confidence Math, γ_{70}	47.5**(1.3)[0.75]	57.5**(2.2)[0.65]	43.6**(2.4)[0.42]	64.7**(1.4)[0.60]

Note. — not statistically significant, and significantly worse model regarding the Likelihood Ratio Test, therefore removed from the original model; * $p < 0.01$; ** $p < 0.001$; otherwise, not significant; Round brackets indicate standard errors; Square brackets indicate effect sizes for significant variables.

Based on Table 5, *Father Education* did not contribute to students' mathematics achievement in any country. *Mother Education* influenced mathematics achievement only in Turkey, but not in the other countries. For Turkey, one point increase in *Mother Education* is associated with 5.3 points increase in mathematics achievement on average. In all four countries, *Home Resources* and *Confidence Math* were found to influence students' mathematics achievement. The more educational resources such as the number of books were available at home and the more self-confidence students had, the better their mathematics achievement was. A one point increase in the variables of *Home Resources* and *Confidence Math*, led to increases of 20.3 and 47.5 points, respectively, in average mathematics achievement of students in Finland; increases of 41.3 and 57.5 points, respectively, in average mathematics achievement of students in South Korea; increases of 36.6 and 43.6 points, respectively, in average mathematics achievement of students in Taiwan; and increases of 32.7 and 64.7 points, respectively, in average mathematics achievement of students in Turkey. Moreover, there was a significant negative relationship between *Bullying* and mathematics achievement in Finland and Turkey, but not in South Korea and Taiwan. Similarly, *Like Math* did not contribute to students' mathematics achievement in Finland and Turkey, as opposed to those in South Korea and Taiwan. In addition, *Value Math* contributed to students' mathematics achievement in Finland, South Korea, and Taiwan, but not in Turkey.

Full Contextual Models (Final Mathematics Achievement Models)

Model building continued in each country by adding school-level variables one by one to the final random coefficients model and then by making model comparisons using the Likelihood Ratio Test. Table 6 presents final mathematics achievement models for each country.

Table 6. *Final Mathematics Achievement Models for Each Country*

Finland	L1	$\text{MathAch}_{ij} = \beta_{0j} + \beta_{1j}(\text{HomeResources}) + \beta_{2j}(\text{Bullying}) + \beta_{3j}(\text{ValueMath}) + \beta_{4j}(\text{ConfidenceMath}) + r_{ij}$
	L2	$\beta_{0j} = \gamma_{00} + \gamma_{01}(\text{Intimidation}) + \gamma_{02}(\text{SchoolEmphasis}) + \gamma_{03}(\text{SchoolDiscipline}) + u_{0j}$
		$\beta_{1j} = \gamma_{10} + u_{1j}$
		$\beta_{2j} = \gamma_{20}$
		$\beta_{3j} = \gamma_{30}$
		$\beta_{4j} = \gamma_{40}$
South Korea	L1	$\text{MathAch}_{ij} = \beta_{0j} + \beta_{1j}(\text{HomeResources}) + \beta_{2j}(\text{LikeMath}) + \beta_{3j}(\text{ValueMath}) + \beta_{4j}(\text{ConfidenceMath}) + r_{ij}$
	L2	$\beta_{0j} = \gamma_{00} + \gamma_{01}(\text{IncomeArea}) + \gamma_{02}(\text{JobSatisfaction}) + \gamma_{03}(\text{SchoolEmphasis}) + u_{0j}$
		$\beta_{1j} = \gamma_{10}$
		$\beta_{2j} = \gamma_{20}$
		$\beta_{3j} = \gamma_{30}$
		$\beta_{4j} = \gamma_{40} + u_{4j}$
Taiwan	L1	$\text{MathAch}_{ij} = \beta_{0j} + \beta_{1j}(\text{HomeResources}) + \beta_{2j}(\text{LikeMath}) + \beta_{3j}(\text{ValueMath}) + \beta_{4j}(\text{ConfidenceMath}) + r_{ij}$
	L2	$\beta_{0j} = \gamma_{00} + \gamma_{01}(\text{IncomeArea}) + \gamma_{02}(\text{SchoolEmphasis}) + u_{0j}$
		$\beta_{1j} = \gamma_{10}$
		$\beta_{2j} = \gamma_{20} + u_{2j}$
		$\beta_{3j} = \gamma_{30}$
		$\beta_{4j} = \gamma_{40} + u_{4j}$
Turkey	L1	$\text{MathAch}_{ij} = \beta_{0j} + \beta_{1j}(\text{MotherEducation}) + \beta_{2j}(\text{HomeResources}) + \beta_{3j}(\text{Bullying}) + \beta_{4j}(\text{ConfidenceMath}) + r_{ij}$
	L2	$\beta_{0j} = \gamma_{00} + \gamma_{01}(\text{SchoolEmphasis}) + \gamma_{02}(\text{SchoolDiscipline}) + u_{0j}$
		$\beta_{1j} = \gamma_{10} + u_{1j}$
		$\beta_{2j} = \gamma_{20}$
		$\beta_{3j} = \gamma_{30} + u_{3j}$
		$\beta_{4j} = \gamma_{40} + u_{4j}$

Based on the final mathematics achievement models (Table 6), Table 7 presents the results from fixed effects of these models. The school-level variables, which were entered into the model sequentially, were *Income Area*, *Job Satisfaction*, *Intimidation*, *Fights*, *Lack of Resources*, *School Emphasis*, and *School Discipline*.

Table 7. *Fixed effects of student-level variables with school-level variables*

	Finland	South Korea	Taiwan	Turkey
Grand mean, γ_{00}	701.2**(10.6)	948.6**(7.4)	975.8**(17.0)	807.3**(17.5)
Student Level				
Mother Education, γ_{10}	—	—	—	5.2**(0.8)[0.05]
Father Education, γ_{20}	—	—	—	—
Home Resources, γ_{30}	20.1**(2.1)[0.32]	40.1**(1.9)[0.45]	35.6**(2.3)[0.35]	30.3**(2.1)[0.28]
Bullying, γ_{40}	-4.8**(1.4)[-0.08]	—	—	-3.8*(1.6)[-0.03]
Like Math, γ_{50}	—	13.5**(2.0)[0.15]	22.3**(2.3)[0.22]	—
Value Math, γ_{60}	2.5*(1.2)[0.04]	18.0**(1.7)[0.20]	10.6**(1.9)[0.10]	—
Confidence Math, γ_{70}	47.5**(1.2)[0.75]	57.8**(2.3)[0.65]	44.9**(2.4)[0.44]	63.7**(1.6)[0.59]
School Level				
Income Area, γ_{01}	—	12.3**(2.3)[0.14]	29.6**(6.9)[0.29]	—
Job Satisfaction, γ_{02}	—	-1.7(2.2)	—	—
Intimidation, γ_{03}	-6.7(3.6)	—	—	—
Fights, γ_{04}	—	—	—	—
Lack of Resources, γ_{05}	—	—	—	—
School Emphasis, γ_{06}	8.0*(3.5)[0.13]	6.7*(2.7)[0.08]	20.1**(5.8)[0.20]	33.1**(5.5)[0.31]
School Discipline, γ_{07}	13.8**(4.0)[0.22]	—	—	12.5*(3.9)[0.12]

Note. — not statistically significant, and significantly worse model regarding the Likelihood Ratio Test, therefore removed from the original model; * $p < 0.01$; ** $p < 0.001$; otherwise, not significant; Round brackets indicate standard errors; Square brackets indicate effect sizes for significant variables.

Regarding the school-level variables in Table 7, *Income Area* (i.e., income level in the nearby school area) was found to influence students' mathematics achievement in South Korea and Taiwan, but not in Finland and Turkey. *Job Satisfaction* (i.e., teachers' job satisfaction) was related to mathematics achievement only in South Korea. Moreover, *Intimidation* (or verbal abuse among students) was found to make a difference (but not a significant difference) in Finland on mathematics achievement. On the other hand, *Fights* (i.e., physical injuries to other students) and *Lack of Resources* (i.e., lack of resources for mathematics instruction) did not have significant effects on mathematics achievement in any of the four countries. Regarding *School Emphasis*, there was a significant relationship between *School Emphasis* and mathematics achievement in all four countries. The higher emphasis schools placed on academic standards, the better students' mathematics achievement was in these schools. A one point increase in *School Emphasis* led to increase of 8.0 points in average mathematics achievement of students in Finland; increase of 6.7 points in average mathematics achievement of students in South Korea; increase of 20.1 points in average mathematics achievement of students in Taiwan; and increase of 33.1 points in average mathematics achievement of students in Turkey. Hence, the largest relationship between *School Emphasis* and mathematics achievement belonged to Turkey, and the smallest relationship occurred in South Korea. Finally, *School Discipline* was related to students' mathematics achievement in Finland and Turkey, but not in South Korea and Taiwan.

DISCUSSION and CONCLUSIONS

The present study examined the effects of student-level and school-level variables on mathematics achievement in four countries including Finland, South Korea, Taiwan, and Turkey using TIMSS 2011 dataset. With respect to the research questions in this study, similarities and differences were found across the countries in terms of the effects of student-level and school-level variables on students' mathematics achievement.

In Finland, the final mathematics achievement model consisted of *Home Resources*, *Bullying*, *Value Math*, and *Confidence Math* at the student level; *Intimidation*, *School Emphasis*, and *School Discipline* at the school-level. While the strongest contribution for the explanation of students' mathematics achievement in Finland was from *Confidence Math*, the weakest contribution came from *Value Math*. Based on the results, *Bullying* and intimidation or verbal abuse among students (i.e., *Intimidation*) such as texting and emailing might prevent Finnish students from spending less time with mathematics. For example, such negative experiences at school might make students pay less attention to mathematics lessons and feel less safe in a school environment. The findings of the present study confirm the findings of Kim et al.'s (2013) study that analyzed the same TIMSS 2011 dataset and found that self-confidence in mathematics (i.e., *Confidence Math*) and *School Emphasis* significantly contribute to fourth and eighth grade students' mathematics achievement in Finland.

In South Korea, the student-level variables in the final mathematics achievement model were *Home Resources*, *Like Math*, *Value Math*, and *Confidence Math*; and the school-level variables that explain mathematics achievement of South Korean students were *Income Area*, *Job Satisfaction*, and *School Emphasis*. The results suggest that affective characteristics such as enjoyment of learning mathematics (i.e., *Like Math*), value of learning mathematics (i.e., *Value Math*), and self-confidence in mathematics (i.e., *Confidence Math*) are important variables for South Korean students to perform better in mathematics. Similarly, having educational resources at home (i.e., *Home Resources*), the immediate area where the school is located (i.e., *Intimate Area*), and the schools' emphasis on academic standards (i.e., *School Emphasis*) are other important variables that have effects on students' mathematics achievement in South Korea. While the findings of the present study align with the findings of Kim et al.'s (2013) reporting that self-confidence in mathematics (i.e., *Confidence Math*) and *Value Math* were associated with students' mathematics achievement in South Korea, there was a contradictory finding about the effect of *School Emphasis* on mathematics achievement. As opposed to Kim et al.'s study, the present study found a significant positive relationship between *School Emphasis* and mathematics achievement in South Korea.

In Taiwan, the student-level variables were found to be the same as those of South Korea. In addition, the school-level factors were almost same except the variable of *Job Satisfaction*. Considering the fact that *Job Satisfaction* did not have a significant effect on South Korea, it can be concluded that mathematics achievement models of South Korea and Taiwan were identical. The findings of the present study confirm the findings of Ker's (2016) study that analyzed the same TIMSS 2011 dataset and found significant contributions of *Confidence Math* and *Value Math* to students' mathematics achievement in Taiwan. Furthermore, having the same student-level and school-level variables in both South Korea and Taiwan's mathematics achievement models can inform researchers and policy-makers in those countries in terms of developing common programs in their education systems. Further research is needed with more countries from the same continent of South Korea and Taiwan to see whether the variables influencing mathematics achievement in those countries are due to the effects of living in similar cultures.

In Turkey, the final mathematics achievement model included *Mother Education*, *Home Resources*, *Bullying*, and *Confidence Math* at the student-level; *School Emphasis* and *School Discipline* at the school level. While the strongest predictors of mathematics achievement of

Turkish students were *Confidence Math* and *School Emphasis*, the weakest predictors were *Bullying* and *Mother Education*. This result implies that more focus should be given on placing high academic standards at school and increasing students' confidence levels about mathematics to improve mathematics achievement of Turkish students. Despite being a weak predictor in comparison to other variables, education levels of parents, especially those of mothers (i.e., *Mother Education*), are still important for Turkish students' better performance in mathematics. Parents with higher education levels can provide better opportunities to their children such as students' having educational resources at home, their enrollment to schools with higher quality of education, and involvement of parents in their children's school related activities. For example, Engelhard (1990) pointed out that there exists a positive relationship between education levels of mothers, and their children's attitudes towards mathematics and their mathematics performances. The findings of the present study mostly align with the findings of Yavuz et al.'s (2017) study that examined the effects of student-level and school-level variables on Turkish eighth grade students' mathematics achievement using the same TIMSS 2011 dataset. Specifically, as similar to the present study, Yavuz et al. (2017) found significant contributions of *Confidence Math*, *Value Math*, and *Bullying* on mathematics achievement. On the other hand, Yavuz et al. found significant positive relationship between *Like Math* and mathematics achievement, in contrast to the present study. Furthermore, the findings of the present study confirm the findings of Akyuz (2014) that reported significant contributions of *Home Resources*, *Bullying*, *Confidence Math*, *School Emphasis*, and *School Discipline* to students' mathematics achievement in Turkey.

Finally, *Home Resources*, *Confidence Math*, and *School Emphasis* were the common variables that were significantly associated with students' mathematics achievement in all four countries. Although it is not possible to make causal claims in this study about the relationships among mathematics achievement and these three variables, the consistent positive relationship among them in each country suggests that students' having educational resources at home (i.e., *Home Resources*), their self-confidence in mathematics (i.e., *Confidence Math*), and schools' emphasis on academic achievement (i.e., *School Emphasis*) are necessary characteristics of mathematics achievement. On the other hand, there was no significant effects of *Father Education* in terms of student-level variables; of *Fights* and *Lack of Resources* in terms of school-level variables in any of the four countries, after controlling for other variables included in the models.

Results from the present study offer several implications for improving students' mathematics achievement in all countries. First, it is important to provide students educational resources at home such as computer to increase their achievement in mathematics (Akyuz & Berberoglu, 2010). For example, parents could be informed about creating a home environment where their children can use computers for educational purposes. Second, given that self-confidence in mathematics is a key variable in improving students' mathematics achievement and shaping their career goals (e.g., Ker, 2016; Zeldin, Britner, & Pajares, 2008), teachers should develop students' self-confidence in mathematics through classroom activities such as presenting problem situations about mathematics from simple to more complex (Ölmez & Cohen, 2018). Third, considering the fact that schools are the places where students receive instruction and spend most of their daily time, schools' placing high academic standards is critical for enhancing students' mathematics achievement (e.g., Akyuz, 2014; Hoy, Tarter, & Kottkamp, 1991).

Due to the characteristics of the TIMSS 2011 dataset, the present study has few limitations. First, the results are correlational, and thus do not indicate causal comparisons. Despite strong evidence for the relationships among several variables and mathematics achievement, it is not possible to provide any causal directions among the variables reported in this study. Additional research that conducts experimental studies is needed to test the causal claims about the effects of student-level and school-level variables on students' mathematics achievement.

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