



Activities Based On Non-Routine And Recreational Problems, To Improve The Learning Of School Mathematics.

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ABSTRACT

This article shows how to start the student in the representation of points in a coordinate axis system. A series of challenging problems were proposed with different levels of difficulty and of different kinds that are integrated into the curriculum. Additionally, four spaces were opened for students to interact with recreational math activities in order to provide opportunities for all students to access the construction of robust meaning of the concepts involved. In the end it describes how these components are interrelated and are directly involved in learning mathematics. The topics covered were: whole number order, representation of points in the Cartesian plane, translations of geometric figures and slope of a straight line.

Keywords : Challenge problems, recreational math, robust thinking.

INTRODUCTION

This work is part of an investigation carried out in a public school in the city of Bogotá Colombia in the sixth grade, the objective is to promote analysis, exploration and inquiry in the context of solving non-routine and challenging problems. Allow the development of mathematical thinking through conjecture and argumentation by students [1]. Motivate the student towards the study of mathematics through recreational mathematics. Encourage inclusion through recreational math activities that allow the student to develop their imagination. For this, activities were created in which students explore the Cartesian plane and thus build a robust knowledge of it.

Methodology

At the beginning of this unit, the material (complete unit) was given to each student. The introduction of the activity (Recreational Mathematics) was carried out. This recreational math activity aims to open a different space in which students could explore, investigate and have fun with their classmates. The choice of this game is based on its relation to the number order theme.

To carry out this activity, groups of three students were formed freely and then proceeded to carry out the following assignment: each group chose a student in charge of keeping time and reminding the group of the activities to work on, another

student was in charge of managing materials and resources, and a third student was in charge of taking notes or making a video of the most important ideas, processes and questions that arose in the group.

At the end of the game time, they proceeded to socialize the experiences they had with the game, in addition each group chose a member to publicize the answers to the two questions that were formulated regarding the game.

Results and discussions

Activity 1

For the following problem taken from [4]. In the next row of boxes are located 6 tiles in disorder numbered from 1 to 7 with the exception of the number 4 that is located in the center of the row.

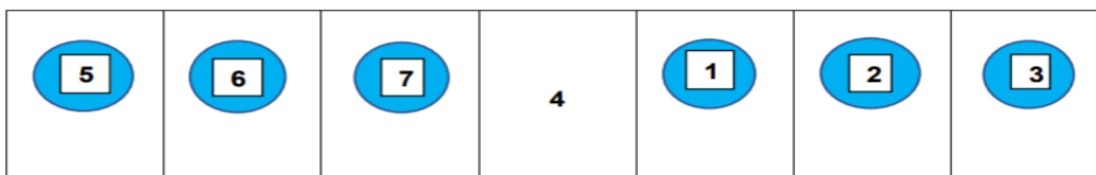


Figure 1. [3]

The game consists of exchanging the numbers placing them in their respective order, the tiles move only forward. This means that you only have to move the tiles from the right to the left and vice versa. A move consists of moving a piece to the next empty square, or jumping onto a next piece in the empty square immediately behind it. At the end of the game the box with the number 4 must be empty.

Answer the following questions: What is the minimum number of moves that can be made to arrange the tiles? Is there a method to make the exchange in the fewest number of moves?

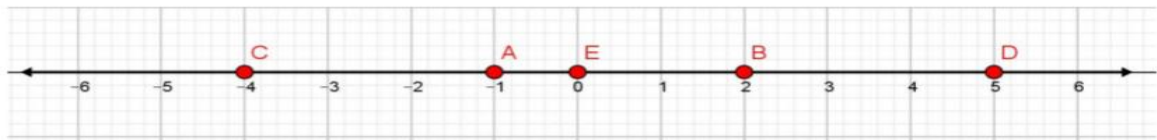
Activity 2

In this activity, the teacher will introduce the concept of the Cartesian plane by formulating an exercise based on [2]. It is intended that with this initial exercise of locating whole numbers, students can begin to build the bases to locate a point in the Cartesian plane, then two simple problems are posed that aim to generate a significant knowledge of the Cartesian plane, this activity is developed in a individual.

It is expected that in the development of this activity all the questions that may arise based on the problem will be investigated and socialized.

During this time the teacher will be permanently monitoring the progress made by each student in order to see their ways of facing the new concept and solving a problem, additionally the material will be collected and then socialized.

GRAPH ON THE COORDINATE PLANE (Based and translated from MATH CONNECTIONS Editorial Mc Graw Hill, Course 1) Identify the number marked by each point on the number line.



1. A 2. B 3. C 4. D 5. E

Los puntos en un mapa usualmente se ubican usando pares de coordenadas. En la cuadrícula que está al lado la cual representa un mapa, la ciudad de Dexter está ubicada en el punto (3,E). Las coordenadas de la ubicación son 3 y E.

Usa un método similar para ubicar puntos en un **plano de coordenadas** como el presentado aquí abajo.

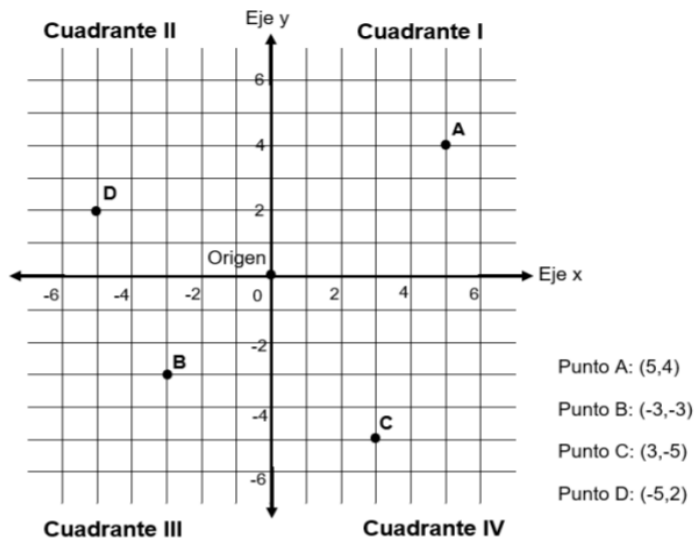


Figure 2. [3]

The coordinate plane is divided by a horizontal number line called the x-axis and by a vertical number line called the y-axis. The two axes divide the plane into 4 parts called quadrants. The point (0,0), at which the axes intersect, is called the origin.

Example: Describe how to locate the point represented by the ordered pair (6, -5) on the coordinate plane.

From the origin, move 6 units to the right, since 6 is positive. Then move down 5 units, since 5 is negative.

Is the point (3,4) equal to the point (4,3)? Explain.

Think and discuss: Go through the lesson and answer each question.

1. Explain how the point (5,4) differs from the point (-5, -4).

2. Decide in which quadrant a point with two negative coordinates should appear. Explain your answer.

Draw and label a coordinate plane, and locate the following points:

F (-7.3) L (-4.6) Y (-1.3) A (-4, -4)

GEOMETRY. Locate the points (2,3), (2, -3), and (-4,3) on a coordinate plane. What ordered pair tells us the location of a fourth point to complete the figure of a square? What is the area of that square? What is its perimeter?

Activity No. 3 (Problem).

This activity consists of a challenging problem (Triangular numbers) selected to be tackled in groups of three students chosen by lottery. Initially, the following assignment will be made: Each group will choose a student in charge of keeping time and reminding the group of the activities to work on, another student will be in charge of managing materials and resources, and a third student will be in charge of taking notes or making a video. of the most important ideas, processes and questions that arise in the group.

In this activity the teacher will be monitoring the progress of each group. It is expected that during the development of this activity there will be an exchange of ideas by the students in each group. Given the case that a group requests help from the teacher, he will do it in a very subtle way by asking guiding questions so that the students are directed to the solution of the problem. Additionally, students are expected to discover the sequence of triangular numbers.

Once the activity is finished, the material produced will be collected and if the group does not finish the activity, they can take it home and continue working in such a way that time is not a pressure factor for the students and that each student can think freely about solutions to problems.

Once all the groups have finished the activity, a presentation will be made in which each group will discuss their procedures. These arguments will be the instrument that will allow the student to build a robust meaning of the concept of triangular number.

Activity N^o 4 (Recreational Mathematics).

This recreational math activity aims to open a different space for students to explore, inquire and have fun with their classmates. The choice of this game is based on its relationship with the issue of number order, its simple implementation and the level of challenge it represents for students.

To carry out this activity, groups of three students formed by the teacher will be formed through a lottery and then the following assignment will be carried out: Each group will choose a student in charge of keeping time and reminding the group of the activities to work on, another student will be In charge of managing materials and resources, and a third student will be in charge of taking notes or making a video of the most important ideas, processes and questions that arise in the group.

At the end of the game time, the experiences that were had with the game will be socialized.

The following table contains the numbers 1 through 8 in disarray. You must find a way to fold the squares (numbers) of the grid in such a way that the numbers are in order.

1	8	7	4
2	3	6	5

Figure 3. [3]

Cutting the numbers is not allowed.

Activity N° 5 (Management of the Cartesian plane).

This activity consists of recognizing and locating coordinates in the Cartesian plane using a grid. This activity will be carried out individually since it is expected that each student can recognize and locate a set of points on the Cartesian plane. In this activity the teacher will be monitoring the progress of each student. It is expected that during the development of this activity there will be an exchange of ideas by the students. Given the case that a student requests help from the teacher, he will do it in a very subtle way by asking guiding questions to guide the student to the solution of the problem.

Once the activity is finished, the material produced will be collected and if the student does not finish the activity, they can take it home and continue working in such a way that time is not a pressure factor and that each student can think freely about the solution of the activity.

1. Write the coordinates of the points that are the vertices of each figure on the coordinate plane of the diagram. (Recommendation to use letters of the alphabet A, B, C, D,... to name the points.)

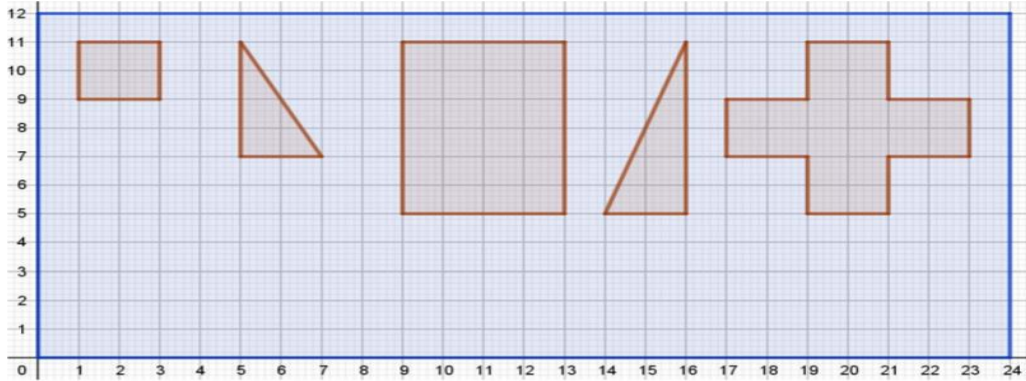


Figure 4. [3]

2. Draw the figures if their sides are tapered to half the original length. You can place them anywhere on the map below.

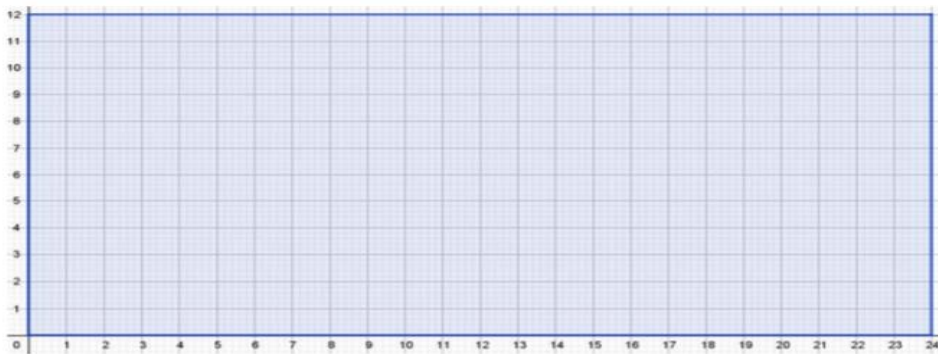


Figure 5. [3]

Write down the coordinates of the vertices of the new figures.

Activity No. 6 (Inclination or slope of a straight line).

This activity was approached through two practices where the concept of the slope of a straight line was introduced through simple practices in the classroom with materials for daily use (books, paper and ruler).

For its implementation, groups of three students were formed freely and then the following assignment will be carried out: Each group chose a student in charge of keeping time and reminding the group of the activities to work on, another student was in charge of managing materials and resources. , and a third student was in charge of taking notes or making a video of the most important ideas, processes and questions that arose in the group.

In this activity, students discovered the concept of slope through inquiry, manipulation, and visualization, and developed mathematical thinking based on their guesses and conclusions from practice. The teacher was permanently monitoring the

progress made by each group in order to see their ways of dealing with the new concept, additionally the material was collected and then socialized, there were cases in which the group did not finish the activity, so they were allowed to lead home to continue working in such a way that time was not a pressure factor for the students and thus each student could think freely about the questions that were formulated for the practice.

Once all the groups finished the activity, a presentation was made in which each group will discuss their procedures. These arguments are the instrument that allowed the student to build a robust meaning of the concept worked.

Instructions:

In the following problem taken from [4] your individual table, some books and a ruler should be used.

1. Make a pile with 3 books one on top of the other.
2. Then support the ruler on the books forming a ramp.
3. Lay the ruler on the table.

Next, a graph of the previous procedure is shown

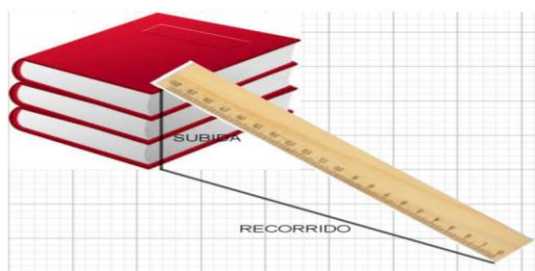


Figure 6. [3]

- a) Measure and record in the table the data corresponding to the ascent and the path of the ramp. Then calculate the relationship and record it in the first row of the table.

subida	recorrido	$\frac{\text{subida}}{\text{recorrido}}$
		—
		—
		—
		—
		—
		—

Figure 7. [1]

- b) Now, keeping the same route, change the length of rise qu

- c) itating a book and then removing two books. Record the data.
- d) In the same way as in the previous step, keep the same path and add a book and record the data, then add 2 books and record the data.
- e) Now explain how the relationships in the last column of the table change when the ramp is steeper?
- f) What happens when the route stays the same and the length of the climb increases?

Now, according to the previous ideas, draw the diagonals of the square and the rectangle starting from the lower left vertex, then do the same with the figures that you drew with half the length of each side in literal a) of point 1 and complete the table

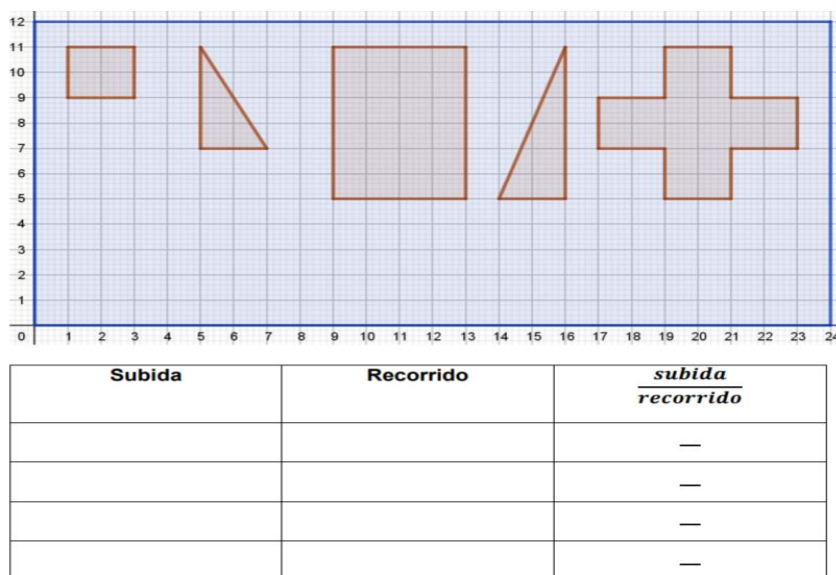


Figure 8. [1]

It then compares the results of the original square and rectangle with the results of the square and rectangle that have half the lengths. What can be said about them? Are they proportional? Justify your answers.

Activity No. 7 (Movements in the plane)

For the following activity taken from [5], 10 practices were approached where movements in the Cartesian plane will be carried out (reflections, translations, dilations and contractions), additionally the students will be put to the test when they face practices in which they must apply the concept of slope discussed in the previous activity and acquire a robust knowledge of it. For its implementation, groups of three students formed by the teacher were formed through a lottery and then the following assignment will be carried out: Each group chose a student in charge of keeping time and reminding the group of the activities to work on,

another student was in charge of manage materials and resources, and a third student was tasked with taking notes or videotaping the ideas,

In this activity, students are expected to visualize and construct mathematical thinking based on their conjectures and conclusions from practices. The teacher will be permanently monitoring the progress made by each group in order to see their ways of facing the new concepts, additionally the material will be collected and then socialized, in the event that the group does not finish the activity, they can take it home to to be able to continue working in such a way that time is not a pressure factor for the students and that each student can think freely about the questions that were formulated for the practice.

Once all the groups have finished the activity, a presentation will be made in which each group will discuss their procedures. These arguments will be the instrument that will allow the student to build a robust meaning of the concept worked.

Instructions: 1. Reflect the figures in the lower part of the horizontal axis and write the coordinates of the points of the reflected figures.

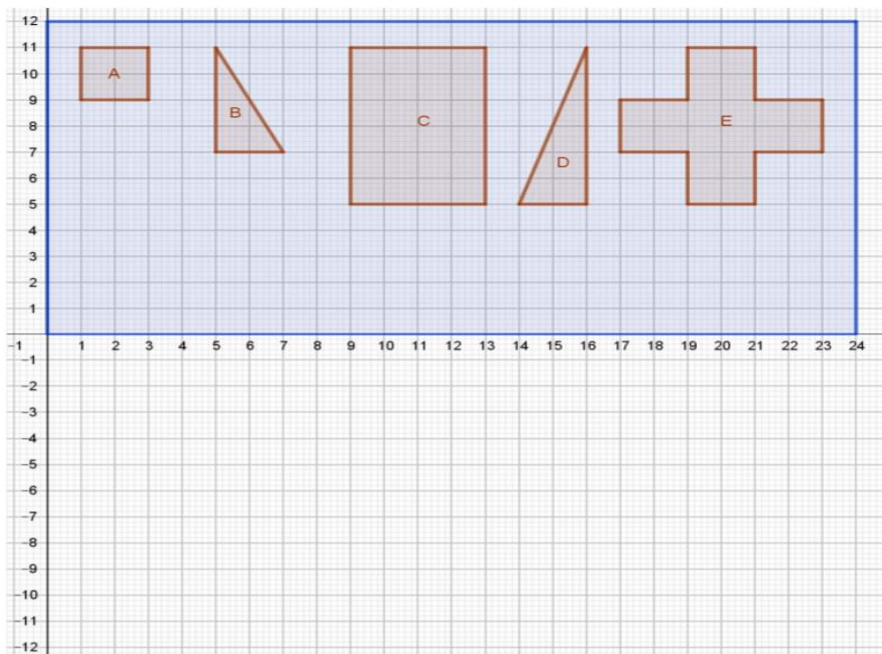


Figure 9. [1]

Now translate the figures as follows, writing the points with their respective coordinates:

1. Figure A: 6 units left and 14 down.

Figure B: 6 units down and 18 to the left.

Figure C: 5 units on the right and 5 below.

Figure D: 25 on the left and 15 on the bottom.

Figure E: 13 below and 15 on the right

2. Find the slopes of the sides of Figure B and Figure D. Are they proportional? Justify your answer.

3. Find the slopes of Figure A and Figure C. Are they proportional? Justify your answer.

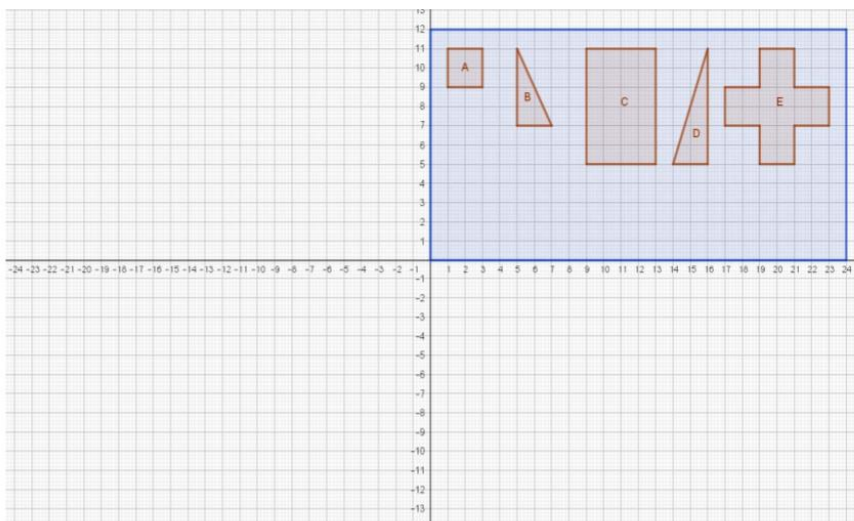


Figure 10. [1]

In the following plane you can see a baby giraffe, it is predicted that it will be 3 times larger when it is an adult. Draw the adult giraffe anywhere on the plane and write the coordinates of its vertices.



Figure 11. [1]

The giraffe in the graph needs to feed and to do this it must move to the top of a tree canopy. Which is the closest tree canopy to the giraffe's mouth?

to. What is the distance the giraffe must travel to reach the top of the crown of each tree?

b. Write the translation movements that the giraffe must make to reach tree A

c. Write the translational movements that the giraffe must make to reach tree B.

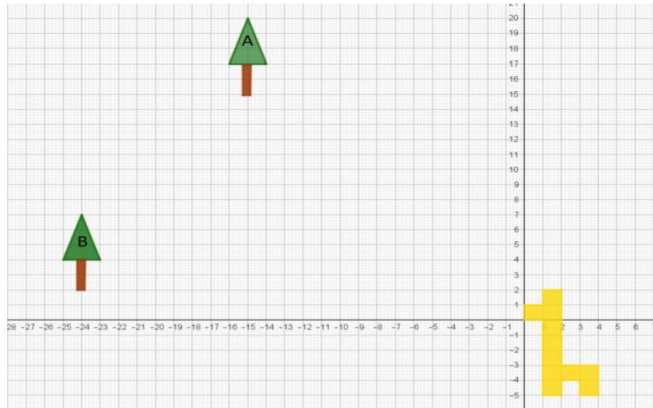


Figure 12. [1]

4 squares are shown on the map. Find the pattern that was taken into account for its construction and draw the next two squares, write the coordinates of their vertices. Determine if the slopes of the diagonals of the squares are proportional to each other. Justify your answer.

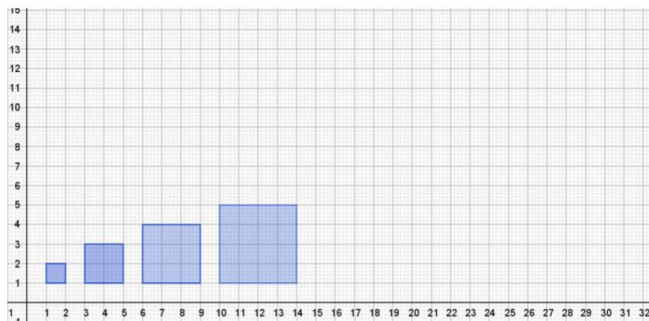


Figure 13. [1]

Now you must transfer the squares to the third quadrant, you must use point A as a vertex of square N°4, point B as a vertex of square N°3 and so on. What will be the coordinates of squares number 1 and 2?

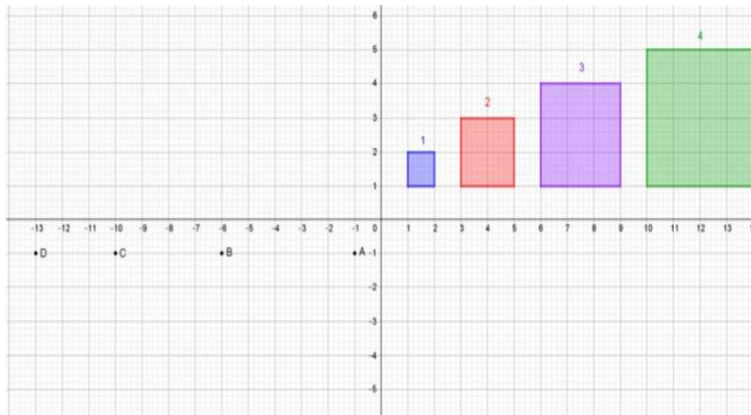


Figure 14. [1]

The following problem is taken from [6]. A coffee shop needs to make an advertisement, for this the dimensions of the first coffee granite shown in the graph must be doubled. Which of the granites would you choose for the advertisement?

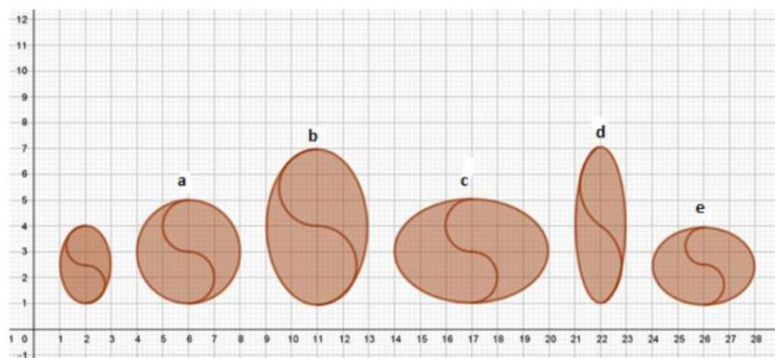


Figure 15. [1]

Activity N° 8 (Recreational Mathematics).

This recreational math activity aims to open a different space for students to explore, inquire and have fun with their classmates. The choice of this game is based on its relationship with the Cartesian plane theme, its simple implementation and the level of challenge it represents for students.

To carry out this activity, groups of three students will be formed freely and then the following assignment will be carried out: Each group will choose a student in charge of keeping time and reminding the group of the activities to work on, another student will be in charge of managing materials and resources, and a third student will be in charge of taking notes or making a video of the most important ideas, processes and questions that arise in the group. It should be noted that because this activity is designed for two people, a third student will be in charge of keeping score and at the

end of a game they will take turns in the same way so that everyone can interact with the game.

The teacher will be permanently monitoring the activity in order to see the ways in which the students face the game. At the end of the game time, they will proceed to socialize the experiences they had with the game and answer the two questions that were formulated at the end of the activity.

Instructions:

The game is for two players and consists of catching the animals that escaped from the zoo and are located in the zooplano. To do this, you must throw a pair of dice of different color (blue and red) and the result will give you the opportunity to catch the animal that is at that point. The blue die moves horizontally and the red die vertically. If you don't catch any animals on your turn, it's up to the other player and so on. The game ends when 6 animals have been caught.

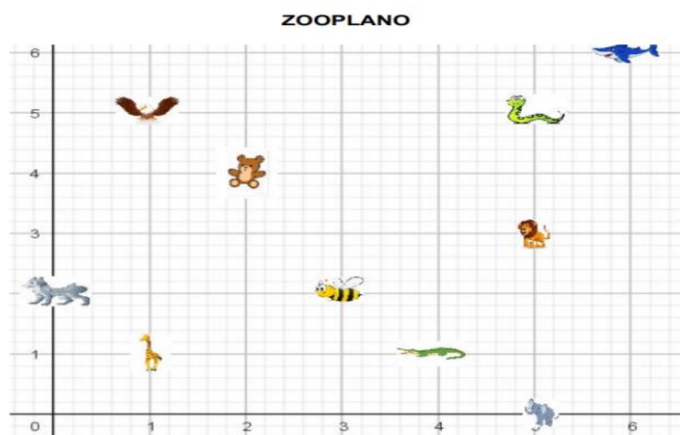


Figure 16. [1]

Now solve the following questions:

1. Are there animals that are impossible to catch? Because?
2. Do all animals have the same chance of being trapped?

In this unit called Cartesian Plane, several solutions of the proposed activities were obtained, evidencing various ways of approaching the problems. The students faced the problems through natural and spontaneous ways, all of them had the opportunity to participate in the activities and some of them participated by making a video as an exposition of a problem or a game proposed in the unit.

The results of this test can be seen in the following graph:

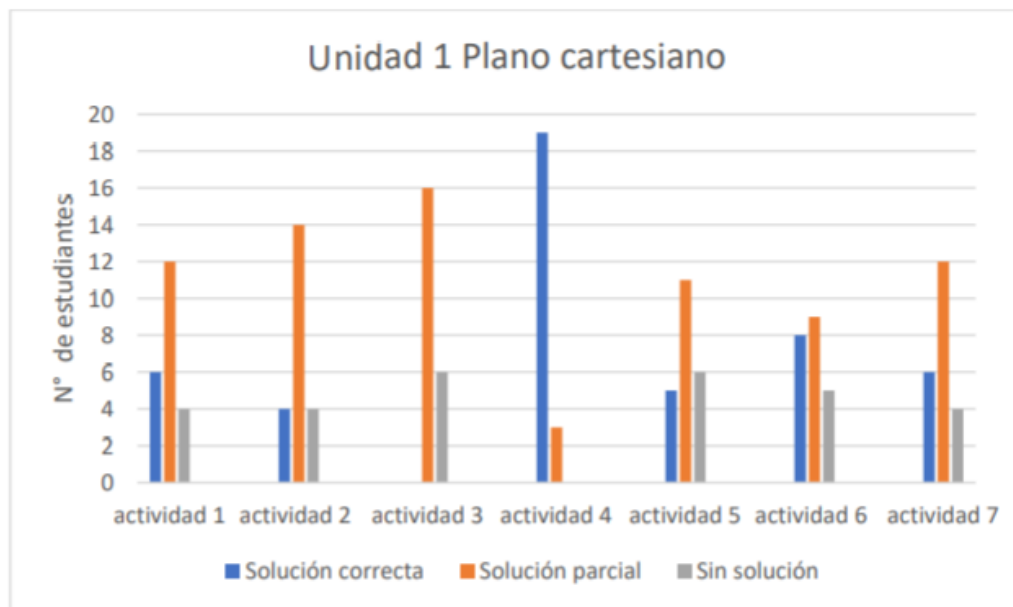


Figure 17. Percentage of solutions of the activity.

Conclusions

This material showed excellent results and specific evidence of their participation and inclusion in all activities. Additionally, it is evidenced that the videos made by the students favored their autonomy when thinking mathematically and that the solutions that emerged did not depend on methods used or designed by others, but were devised by the students themselves when faced with novel situations.

In terms of theory, the design, implementation and analysis of the implemented instruments reflect the existence of a very close relationship between the use of recreational mathematics as a tool that favors equity, inclusion and motivation, together with positive expectations. of parents and teachers, on the one hand, and the learning of mathematics and development of autonomy in mathematical thinking at the level of basic education, on the other. This use of recreational mathematics also provides solid elements to make the curriculum more flexible and thus build a more challenging curriculum designed based on the needs of the students and their possibilities for mathematical development.

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