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Modelling Of An Inventory System Of Supply Chain Under Fuzzy Environment

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ABSTRACT

This paper considers deteriorating production inventory model of a supply chain system considering transportation cost depending upon quantity ordered and distance from supplier's production unit to retailer's warehouse. First a crisp model is developed and considering some parameters of fuzzy nature affecting inventory cost, a corresponding fuzzy model is developed. In most inventory modelling separate model has been developed by many researchers for retailer and supplier and rarely transportation cost is taken into account. In the present modelling total average inventory cost of supply chain including retailers and supplierstogether is minimized under crisp and fuzzy environment. Demand parameter of retailer's is considered as inventory level dependent. During production and storage, constant deterioration occurs at retailer's and supplier's warehouse. The objective of this paper is to derive an inventory model that together minimize the total average inventory cost of supply chain using signed Distance Method to defuzzyfy fuzzy numbers. Numerical example is presented to validate the model applicability and sensitivity analysis has been performed on fuzzy parameters in case of fuzzy model.

Keywords: Triangular Fuzzy Number, Signed Distance Methods, Inventory level dependent demand,distance and ordered quantity dependent transportation cost.

1.0 INTRODUCTION

Inventory is the most important commodity of business and in the system of business many steps are involved when commodities are distributed during business process. Most of the inventory models are developed to minimize the inventory cost of a businessman involved at single level of business process. In the business there are mainly four level of distribution of commodities which are Producer/manufacturer of products/commodities, Distributers/Suppliers of commodities and Retailers of Products/commodities and end users that is consumers. All stockist are involved in the business process. Most of research papers involved in modelling inventory system of single item or multi-items are to minimize the cost of inventory system for either retailers, suppliers or manufacturer. In the field supply chain system where many businessmen at various level are involved, researchers have developed model of SCN (Supply Chain Network) considering transportation of commodities among the businessman and end users. Researchers have considered various factors affecting inventory system or SCN including cost parameters which are the most important factors to be modelled in a such way that minimized the total inventory cost or cost of supply chain network and hence making profit for the business. Various inventory modelconsidered constant rate of demand, time dependent demand, exponential and stock dependent demand. But these are not always fruitful in optimizing inventory system.

During the marketing process of commodities in a business, items are produced /manufactured at various level and finished goods are brought in the market to be used by end consumers. In this process, at various level either during production process, stock holding process and inventory in transit system, some items are deteriorated and some are in good condition to be used until it reaches to end consumers. Deterioration is the process in which either biochemical losses of items or physical damages during all process of supply chain causing loss to the businessman.Therefore, deteriorationis the key factor affecting the inventory costduring the storage period and in transit period and sohas drawn attention ofresearchers since past many decades.

During the business process,uncertainty in the market causes inflation in the price and also in the production system. The uncertainty of the market cannot be dealt with parameters having certain and fixed value. Forecasting cannot be considered as exact quantity of requirement and may vary any time due to various uncertain causes in the country. Similarly, inflation in the cost parameters is also uncertain depending on the situation and crisis of present time. Therefore, there is need to study the uncertain situation of future and model the system accordingly so that uncertainty of the market scenario can be observed closely to the crisp quantity of commodities. In this area, many research papers have been published to deal with the uncertain situation and quantity.This type of uncertaintywas initially observedand studied by the L.A. Zadeh [1] in seventeenth century considering interval-based membership function describing a graded value. Using fuzzy set theory in fuzzy environment many papers are available [2,3,4,5,6,7,8,9,10,11,12]. Few recent researchers in their papers [13,14,15,16,17,18,19,20,21] have considered fuzzy situation of market scenario and developed models of inventory system accordingly.In these papers they have considered only inventory modelling of an inventory system in fuzzy environment and most of these have not included the transportation activities and cost of transportation. In current scenario many researchers are working in modelling supply chain system considering transportation network of commodities but still very few are considering the fuzzy environment to deal with future uncertain situations that are not exact but may be very close to the situation in demand during the time. Dincer Konur [22] has developed a Carbon constrained integrated inventorycontrol and truckload transportation with heterogeneous freight trucks by including the transportation system in inventory modelling. A tri-level location model for forward/reverse supply chain has been proposed by Amir Mohammad et. al. [23]. OptimizingSupply chain network for perishable products using improved bacteria foraging algorithm (IBFA) is developed by Amit Kumar Sinha and Ankush Anand [24] and compared the result of IBFA with existing BFA. However, these models have not considered in optimizing total inventory cost of supply chain together in a fuzzy environment.

Motivated by various research papers of inventory modelling and model of supply chain system of transportation involving commodities in the present market scenario of competitive business era, an inventory model is proposed for supply chain system involving supplier and retailer to optimize total average inventory cost of both together involved in the business process.First, a crisp model has been developed under inventory level demand rate and corresponding fuzzy model is developed to deal with uncertain situation of production rate, demand rate, deterioration rate and purchasing rate. It is assumed that production and supply are instantaneous and have no shortages of demand. Fuzzy parameters consider triangular fuzzy numbers which are defuzzified using Signed Distance Method.The study includes only single item. Sensitivity analysis have been performed on fuzzy parameters in case of fuzzy models.

2.0 Preliminaries of Fuzzy Set

Fuzzy set theory has emerged as a very powerful tool to quantitatively represent and manipulate the imprecise data that is used to govern the decision- making process. Mostly fuzzy numbers are used in the inventory modelling to encounter the imprecise data by setting the values of input parameters to be function of fuzzy triplet of trapezoidal or other fuzzy numbers. Some definition fuzzy set are presented here.

Fuzzy Set: A fuzzy set $\tilde{V}$ on a given universal set X is denoted and defined by

$$\tilde{V} = \{(x, \lambda_{\tilde{V}}(x)) : x \,\varepsilon\, X\}$$

Where $\lambda_{\tilde{V}} : X \to [0,1]$, is the membership function and $\lambda_{\tilde{V}}(x)$ describes degree of x in $\tilde{V}$.

Fuzzy Triangular Number: A fuzzy number is specified by the triplet $(a_1 a_2 a_3)$ is known as triangular fuzzy if$a_1 < a_2 < a_3$ and is defined by its continuous membership function $\lambda_{\tilde{V}}$: $X \to [0,1]$as follows:

$$\lambda_{\tilde{V}}(x) = \begin{cases} \frac{x - a_1}{a_2 - a_1} & \text{if } a_1 \le x \le a_2 \\ \frac{a_3 - x}{a_3 - a_2} & \text{if } a_2 \le x \le a_3 \\ 0 & \text{otherwise} \end{cases}$$

Signed Distance: Let $\tilde{V}$ be the fuzzy set defined on the $\Re$ (set of real numbers), then the signed distance of $\tilde{V}$ is defined as

$$d(\tilde{V},0) = \frac{1}{2}\int_0^1 [V_L(\alpha) + V_R(\alpha)]\, d_\alpha$$

where $V_\alpha = [V_L(\alpha) + V_R(\alpha)]$= [a + b- a) α, d -(d-c) , a] , a ε [0,1] is a a-cut of a fuzzy set $\tilde{V}$.

If $\tilde{V}$ = $(a_1 a_2 a_3)$ is a triangular fuzzy number then the signed distance of $\tilde{V}$ is defined as

$d(\tilde{V}, 0) = \frac{1}{4}(a_1 + 2\ a_2 + a_3)$

3.0 Assumptions and Notations

The formulation of model is based on the following assumptions and notations: -

3.1 Assumptions

1. The replenishment rate is instantaneous
2. Supply lead time is negligible.
3. Shortages are not allowed.
4. Rate of deterioration is constant at warehouses.
5. Rate of holding cost is constant at both warehouses.
6. Deteriorated units neither repaired nor reproduced during the period under review.
7. Deterioration occurs as soon as items are received into inventory management system.
8. Parameters are considered to be triangular fuzzy number in case of fuzzy model.
9. The demand (D) of supplier is such that D=Q/T; where Q is ordered quantity of retailer in period T.
10. Demand rate of retailer is deterministic and is the function of in hand inventory of retailer at any time t and is illustrated as
    $d(t) = \alpha + \beta\, I_R(t) \qquad 0 \le t \le T$
    such that $\alpha > 0, \beta > 0$

3.2 Notations

O_c Cost of ordering of retailer per order
S_c Set up cost of supplier
C_s Production cost of supplier per unit item
C_o Ordering cost per order
d_r Deterioration cost of retailer per unit item
d_s Deterioration cost of supplier per unit item
h_r Inventory holding cost of retailer per unit item
h_s Inventory holding cost of supplier per unit item
p_r Purchasing cost of retailer per unit item
a Fix transportation cost per trip of transportation
b Variable transportation cost per ordered quantity per unit distance
d Distance of retailer's warehouse from production unit of supplier
γ Rate of deterioration of inventory at both warehouse of supply chain
T Cycle length of retailers
T_1 Production time period of supplier
T_2 Time at which supplier's inventory vanishes
$I_R(t)$ Inventory level of retailer at any time t in the interval $0 \le t \le T$
$I_{Si}(t)$ Inventory level of supplier at any time t in the interval $0 \le t \le T_2$ where $i = 1,2$

N Number of deliveries supplied by supplier to the retailers
Q Order size per order of retailer
$TCI_{csr}(N, T, T_1, T_2)$ Total Average inventory cost for crisp model
$\tilde{\alpha}$ Fuzzy fixed demand parameter of retailer
$\tilde{\beta}$ Fuzzy variable demand parameter base on inventory level
$\tilde{\gamma}$ Fuzzy deterioration rate in both warehouses
$\tilde{p}_r$ Fuzzy purchasing cost parameter of retailer
$\tilde{S}_c$ Fuzzy set up cost parameter of supplier
$\tilde{P}$ Fuzzy production rate of supplier
$\widetilde{TCI}_{csr}(N, T, T_1, T_2)$ Total Average inventory cost for fuzzy model
{~ Sign represent the fuzziness of the parameters}

4.0 Formulation of Mathematical Model (Crisp Model)

In this section models are developed for retailer and supplier separately and effect of parameters on combined supply chain is illustrated by numerical example in the later section.
4.1: Retailer's Inventory model for j^{th} cycle $(j-1)T \le t \le jT$
Graphical scenario is illustrated in the figire-1and level of inventory is described by the differential equation. At the fist instant Q quantity of inventory entered into the inventory system which is stocked in a warehouse and decline due to demand of consumers and deteriorationin theperiod$(j-1)T \le t \le jT$. The following differential equation govern the level of inventory:

$$\frac{dI_R(t)}{dt} = -\gamma I_R(t) - d(t); \qquad (j-1)T \le t \le jT \qquad (1)$$

At the beginning of inventory system$I_R((j-1)T) = Q$, using as boundary condition solution of eq. (1) is

$$I_{Rj}(t) = \left(Q + \frac{\alpha}{\beta+\gamma}\right) e^{-(\beta+\gamma)(t-(j-1)T)} - \frac{\alpha}{\beta+\gamma}; \qquad (j-1)T \le t \le jT \qquad (2)$$

Where,

$$Q = e^{(\beta+\gamma)T} - 1$$

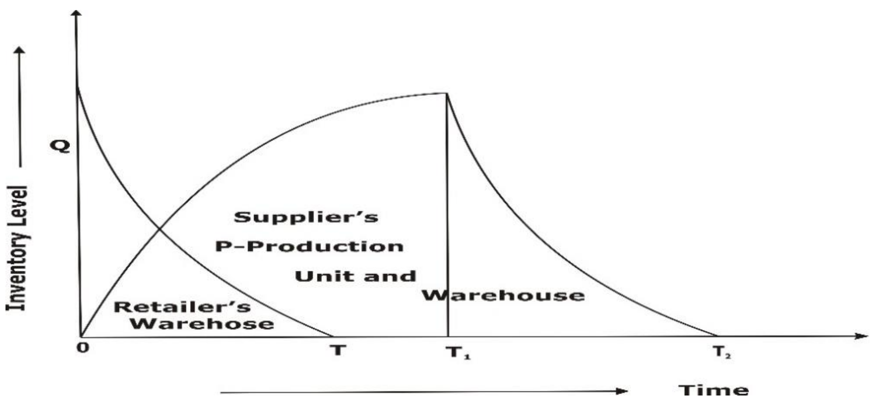

Figure-1: Graph representing depletion of inventory level in warehouses

The retailer's inventory system consists following costs to be included in minimizing average total inventory cost

- Ordering cost
- Inventory holding cost
- Deterioration cost
- Purchasing cost

Now above costs are illustrated as under:

Ordering cost for j^{th}cycle$(j-1)T \leq t \leq jT$

$$CO_{Ij} = j\, o_r \tag{3}$$

Ordering cost for complete N–cycle

$$CO_{In} = \sum_{j=1}^{n} j\, o_r \tag{4}$$

Inventory holding cost for j^{th}cycle$(j-1)T \leq t \leq jT$

$$IH_{crj} = h_r \left\{ \int_{(j-1)T}^{jT} I_{Rj}(t)\, dt \right\}$$

$$= h_r \left\{ \int_{(j-1)T}^{jT} \left(Q + \frac{\alpha}{\beta+\gamma} \right) e^{-(\beta+\gamma)(t-(j-1)T)} - \frac{\alpha}{\beta+\gamma}\, dt \right\} \tag{5}$$

Inventory holding cost for complete N-cycle

$$IH_{crn} = \sum_{j=1}^{n} IH_{crj} \tag{6}$$

Inventory deterioration cost for j^{th}cycle$(j-1)T \leq t \leq jT$

$$ID_{crj} = d_r \left\{ \int_{(j-1)T}^{jT} \gamma\, I_{Rj}(t)\, dt \right\}$$

$$= d_r \left\{ \int_{(j-1)T}^{jT} \gamma \left(Q + \frac{\alpha}{\beta+\gamma} \right) e^{-(\beta+\gamma)(t-(j-1)T)} - \frac{\alpha}{\beta+\gamma}\, dt \right\} \tag{7}$$

Inventory deterioration cost for complete N-cycle

$$ID_{crn} = \sum_{j=1}^{n} ID_{crj} \tag{8}$$

Inventory purchase cost forjthcycle

$$IP_{crj} = p_c\, (Q_j)$$

$$= P_c\, (e^{(\beta+\gamma)T} - 1) \tag{9}$$

Inventory purchase cost for completeN-cycle

$$IP_{crn} = \sum_{j=1}^{n} IP_{crj} \tag{10}$$

Hence the total average inventory cost per unit of time of retailer for complete n-cycle is given by

$$TI_{cr}(N,T) = \frac{1}{T}[CO_{In} + IH_{crn} + ID_{crn} + IP_{crn}]$$

$$= \frac{1}{T}\left[\sum_{j=1}^{n} j\, o_r + \sum_{j=1}^{n} IH_{crj} + \sum_{j=1}^{n} ID_{crj} + \sum_{j=1}^{n} IP_{crj}\right] \tag{11}$$

4.2: Supplier’s Inventory Model

Graphical scenario for the level of inventory produced and supplied to retailer is illustrated in the figure-1. At the first instant P quantity of inventory produced and is reduced due tosupplier’s demand and deterioration in the period $(0 \le t \le T_1)$ and remaining products decline due to demand of supplier and deterioration in theperiod$(T_1 \le t \le T_2)$. The following differential equation govern the level of inventory during the entire period:

$$\frac{dI_{s1}(t)}{dt} = P - d(t) - \gamma I_s(t); \qquad (0 \le t \le T_1) \tag{12}$$

$$\frac{dI_{s2}(t)}{dt} = -\gamma I_R(t) - d(t); \qquad (T_1 \le t \le T_2) \tag{13}$$

At the beginning of inventory system$I_{s1}(0) = 0$, using this condition as boundary condition solution of eq. (12) is

$$I_{s1}(t) = \left(\frac{P-D}{\gamma}\right)(1 - e^{-\gamma t}); \qquad (0 \le t \le T_1) \tag{14}$$

Due to continuity at $t = T_1$,$I_{s2}(T_2) = I_{s1}(T_1)$, using this condition solution of eq. (13) is obtained as

$$I_{s2}(t) = \left(I_{s1}(T_1) + \frac{D}{\gamma}\right) e^{-\gamma(t-T_1)} - \frac{D}{\gamma}; \qquad (T_1 \le t \le T_2) \tag{15}$$

The supplier’s inventory system consists following costs to be included in minimizing average total inventory cost

- Setup cost
- Inventory holding cost
- Deterioration cost
- Production cost
- Transportation cost

Now above costs are illustrated as under:

Setup cost

$$S_{sc} = s_c \tag{16}$$

Inventory holding cost

$$IH_{cs} = h_s \left\{ \int_0^{T_1} I_{s1}(t)\, dt + \int_{T_1}^{T_2} I_{s2}(t)\, dt \right\}$$

$$= h_s \left\{ \int_0^{T_1} \left(\frac{P-D}{\gamma}\right)(1 - e^{-\gamma t})\, dt + \int_{T_1}^{T_2} \left(\left(I_{s1}(T_1) + \frac{D}{\gamma} \right) e^{-\gamma(t-T_1)} - \frac{D}{\gamma} \right) dt \right\} \quad (17)$$

Inventory deterioration cost

$$ID_{cs} = d_s \left\{ \int_0^{T_1} \gamma\, I_{s1}(t)\, dt + \int_{T_1}^{T_2} \gamma\, I_{s2}(t)\, dt \right\}$$

$$= d_s \left\{ \int_0^{T_1} \gamma \left(\frac{P-D}{\gamma}\right)(1 - e^{-\gamma t}) dt + \int_{T_1}^{T_2} \gamma \left(\left(I_{s1}(T_1) + \frac{D}{\gamma} \right) e^{-\gamma(t-T_1)} - \frac{D}{\gamma} \right) dt \right\} \quad (18)$$

Inventory production cost

$$IP_{cs} = C_p \int_0^{T_2} P\, dt \quad (19)$$

Inventory transportation cost for completeN-cycle

$$IT_{csn} = N\,(a + b\,Q\,d) \quad (20)$$

Hence the total average inventory cost per unit of time for complete n-cycle is given by

$$TI_{cs}(N, T_1, T_2) = \frac{1}{T}[S_{sc} + IH_{cs} + ID_{cs} + IP_{cs} + IT_{csn}]$$

$$= \frac{1}{T}\Bigg[S_c + h_s \left\{ \int_0^{T_1} \left(\frac{P-D}{\gamma}\right)(1 - e^{-\gamma t})\, dt + \int_{T_1}^{T_2} \left(\left(I_{s1}(T_1) + \frac{D}{\gamma} \right) e^{-\gamma(t-T_1)} - \frac{D}{\gamma} \right) dt \right\} + d_s \left\{ \int_0^{T_1} \gamma \left(\frac{P-D}{\gamma}\right)(1 - e^{-\gamma t}) dt + \int_{T_1}^{T_2} \gamma \left(\left(I_{s1}(T_1) + \frac{D}{\gamma} \right) e^{-\gamma(t-T_1)} - \frac{D}{\gamma} \right) dt \right\} + C_p \int_0^{T_2} P\, dt + N\,(a + b\,Q\,d) \Bigg] \quad (21)$$

Present worth of total inventory cost of supply chain taken together is given by

$$TIC_{csr}(N, T, T_1, T_2)$$

$$= \frac{1}{T}\Bigg[\sum_{j=1}^{n} j\, o_r + \sum_{j=1}^{n} IH_{crj} + \sum_{j=1}^{n} ID_{crj} + \sum_{j=1}^{n} IP_{crj} + S_{sc} + h_s \left\{ \int_0^{T_1} \left(\frac{P-D}{\gamma}\right)(1 - e^{-\gamma t})\, dt + \int_{T_1}^{T_2} \left(\left(I_{s1}(T_1) + \frac{D}{\gamma} \right) e^{-\gamma(t-T_1)} - \frac{D}{\gamma} \right) dt \right\} + d_s \left\{ \int_0^{T_1} \gamma \left(\frac{P-D}{\gamma}\right)(1 - e^{-\gamma t}) dt + \int_{T_1}^{T_2} \gamma \left(\left(I_{s1}(T_1) + \frac{D}{\gamma} \right) e^{-\gamma(t-T_1)} - \frac{D}{\gamma} \right) dt \right\} + C_p \int_0^{T_2} P\, dt + N\,(a + b\,Q\,d) \Bigg]$$

(22)

The combined inventory average cost is function of T, T_1, T_2 only and the optimal value of T, T_1, T_2 can be obtained by equating $\frac{\partial TIC_{csr}(T,T_1,T_2)}{\partial T} = 0; \frac{\partial TIC_{csr}(T,T_1,T_2)}{\partial T_1} = 0; \frac{\partial TIC_{csr}(T,T_1,T_2)}{\partial T_2} = 0$ and necessary condition of Hessian matrix H which are as under satisfied.

$$H = \begin{pmatrix} \frac{\partial TIC_{csr}(T,T_1,T_2)}{\partial T^2} & \frac{\partial^2 TIC_{csr}(T,T_1,T_2)}{\partial T\,\partial T_1} & \frac{\partial^2 TIC_{csr}(T,T_1,T_2)}{\partial T\,\partial T_2} \\ \frac{\partial^2 TIC_{csr}(T,T_1,T_2)}{\partial T_1\,\partial T} & \frac{\partial TIC_{csr}(T,T_1,T_2)}{\partial {T_1}^2} & \frac{\partial^2 TIC_{csr}(T,T_1,T_2)}{\partial T_1\,\partial T_2} \\ \frac{\partial^2 TIC_{csr}(T,T_1,T_2)}{\partial T_1\,\partial T_2} & \frac{\partial^2 TIC_{csr}(T,T_1,T_2)}{\partial T_2\,\partial T} & \frac{\partial TIC_{csr}(T,T_1,T_2)}{\partial {T_2}^2} \end{pmatrix}$$

The necessary condition for T, T_1, T_2 to be optimal is that the first principal minor determinant of H , $H_{11} > 0$, second principal minor $H_{22} > 0$ and third principal minor $H_{33} > 0$ and optimal value of N (discrete value) is found in such a way that optimal value of $TIC_{csr}(T^*, T_1^*, T_2^*)$is achieved.Here $T^*, T_1^*, T_2^*$are optimal values.

5.0 Formulation of Inventory Model (Fuzzy Model)

Uncertain situation cannot dealwith crisp nature of parameters. In the present global market scenario, the value of parameters like different cost, demand rate, production rate and deterioration rate may fluctuate due to the several reasons and uncertainty like natural hazards etc. The fluctuation in parameters at any time cannot be pre-determined until one reaches the situation of that time. Therefore, the only possibility is to consider the possible range of fluctuation. To deal with such type of uncertain situation, a corresponding fuzzy model has been developed, considering vagueness of some parameter affecting the total inventory cost. Parameters affecting inventory cost is considered as triangular fuzzy numbers. TheSigned Distance Method is used to solve the developed model to minimize the total inventory costand sensitivity of the fuzzy parameters are performed on fuzzy model. Also, sensitivity is performed and deviation is observed.

Using equation (22) and fuzzy parameters we have,

$\tilde{\alpha}_r = (\tilde{\alpha}_1, \tilde{\alpha}_2, \tilde{\alpha}_3)$,$\tilde{\beta}_r = (\tilde{\beta}_1, \tilde{\beta}_2, \tilde{\beta}_3)$,$\tilde{\gamma}_{rs} = (\tilde{\gamma}_1, \tilde{\gamma}_2, \tilde{\gamma}_3)$,$\tilde{S}_{sc} = (\tilde{S}_1, \tilde{S}_2, \tilde{S}_3)$,

$\tilde{p}_r = (\tilde{p}_1, \tilde{p}_2, \tilde{p}_3)$and $\tilde{P} = (\tilde{P}_1, \tilde{P}_2, \tilde{P}_3)$

Therefore, fuzzy model is given by

$\widetilde{TIC}_{csr}(\tilde{T}, \tilde{T}_1, \tilde{T}_2) = (\widetilde{TIC}_{1csr}(\tilde{N}, \tilde{T}, \tilde{T}_1, \tilde{T}_2), \widetilde{TIC}_{2csr}(\tilde{N}, \tilde{T}, \tilde{T}_1, \tilde{T}_2), \widetilde{TIC}_{3csr}(\tilde{N}, \tilde{T}, \tilde{T}_1, \tilde{T}_2))$

(23)

Where,

$$\widetilde{TIC}_{1csr}(\tilde{T},\tilde{T_1},\tilde{T_2})=\frac{1}{T}\Bigg[\sum_{j=1}^{n} j\, o_r + \sum_{j=1}^{n} IH_{crj} + \sum_{j=1}^{n} ID_{crj} + \sum_{j=1}^{n} IP_{crj} + \tilde{S_1} + h_s\left\{\int_0^{T_1}\left(\frac{\tilde{P_1}-D}{\tilde{\gamma_1}}\right)(1-e^{-\tilde{\gamma_1}t})\,dt + \int_{T_1}^{T_2}\left(\left(\left(Q+\frac{\tilde{\alpha_1}}{\tilde{\beta_1}+\tilde{\gamma_1}}\right)e^{-(\tilde{\beta_1}+\tilde{\gamma_1})(T_1-(j-1)T)} - \frac{\tilde{\alpha_1}}{\tilde{\beta_1}+\tilde{\gamma_1}} + \frac{D}{\tilde{\gamma_1}}\right)e^{-\tilde{\gamma_1}(t-T_1)} - \frac{D}{\tilde{\gamma_1}}\right)dt\right\} + d_s\left\{\int_0^{T_1}\tilde{\gamma_1}\left(\frac{\tilde{P_1}-D}{\tilde{\gamma}}\right)(1-e^{-\tilde{\gamma_1}t})dt + \int_{T_1}^{T_2}\tilde{\gamma_1}\left(\left(\left(Q+\frac{\tilde{\alpha_1}}{\tilde{\beta_1}+\tilde{\gamma_1}}\right)e^{-(\tilde{\beta_1}+\tilde{\gamma_1})(T_1-(j-1)T)} - \frac{\tilde{\alpha_1}}{\tilde{\beta_1}+\tilde{\gamma_1}} + \frac{D}{\gamma}\right)e^{-\tilde{\gamma_1}(t-T_1)} - \frac{D}{\tilde{\gamma_1}}\right)dt\right\} + C_p\int_0^{T_2}\tilde{P_1}dt + N\,(a+b\,Q\,d\,)\Bigg]$$

$$\widetilde{TIC}_{2csr}(\tilde{T},\tilde{T_1},\tilde{T_2})=\frac{1}{T}\Bigg[\sum_{j=1}^{n} j\, o_r + \sum_{j=1}^{n} IH_{crj} + \sum_{j=1}^{n} ID_{crj} + \sum_{j=1}^{n} IP_{crj} + \tilde{S_2} + h_s\left\{\int_0^{T_1}\left(\frac{\tilde{P_2}-D}{\tilde{\gamma_2}}\right)(1-e^{-\tilde{\gamma_2}t})\,dt + \int_{T_1}^{T_2}\left(\left(\left(Q+\frac{\tilde{\alpha_2}}{\tilde{\beta_2}+\tilde{\gamma_2}}\right)e^{-(\tilde{\beta_2}+\tilde{\gamma_2})(T_1-(j-1)T)} - \frac{\tilde{\alpha_2}}{\tilde{\beta_2}+\tilde{\gamma_2}} + \frac{D}{\tilde{\gamma_2}}\right)e^{-\tilde{\gamma_2}(t-T_1)} - \frac{D}{\tilde{\gamma_2}}\right)dt\right\} + d_s\left\{\int_0^{T_1}\tilde{\gamma_2}\left(\frac{\tilde{P_2}-D}{\tilde{\gamma_2}}\right)(1-e^{-\tilde{\gamma_2}t})dt + \int_{T_1}^{T_2}\tilde{\gamma_2}\left(\left(\left(Q+\frac{\tilde{\alpha_2}}{\tilde{\beta_2}+\tilde{\gamma_2}}\right)e^{-(\tilde{\beta_2}+\tilde{\gamma_2})(T_1-(j-1)T)} - \frac{\tilde{\alpha_2}}{\tilde{\beta_2}+\tilde{\gamma_2}} + \frac{D}{\gamma}\right)e^{-\tilde{\gamma_2}(t-T_1)} - \frac{D}{\tilde{\gamma_2}}\right)dt\right\} + C_p\int_0^{T_2}\tilde{P_2}dt + N\,(a+b\,Q\,d\,)\Bigg]$$

$$\widetilde{TIC}_{3csr}(\tilde{T},\tilde{T_1},\tilde{T_2})=\frac{1}{T}\Bigg[\sum_{j=1}^{n} j\, o_r + \sum_{j=1}^{n} IH_{crj} + \sum_{j=1}^{n} ID_{crj} + \sum_{j=1}^{n} IP_{crj} + \tilde{S_1} + h_s\left\{\int_0^{T_1}\left(\frac{\tilde{P_3}-D}{\tilde{\gamma_3}}\right)(1-e^{-\tilde{\gamma_3}t})\,dt + \int_{T_1}^{T_2}\left(\left(\left(Q+\frac{\tilde{\alpha_3}}{\tilde{\beta_3}+\tilde{\gamma_3}}\right)e^{-(\tilde{\beta_3}+\tilde{\gamma_3})(T_1-(j-1)T)} - \frac{\tilde{\alpha_3}}{\tilde{\beta_3}+\tilde{\gamma_3}} + \frac{D}{\tilde{\gamma_3}}\right)e^{-\tilde{\gamma_3}(t-T_1)} - \frac{D}{\tilde{\gamma_3}}\right)dt\right\} + d_s\left\{\int_0^{T_1}\tilde{\gamma_3}\left(\frac{\tilde{P_3}-D}{\tilde{\gamma_3}}\right)(1-e^{-\tilde{\gamma_3}t})dt + \int_{T_1}^{T_2}\tilde{\gamma_3}\left(\left(\left(Q+\frac{\tilde{\alpha_3}}{\tilde{\beta_3}+\tilde{\gamma_3}}\right)e^{-(\tilde{\beta_3}+\tilde{\gamma_3})(T_1-(j-1)T)} - \frac{\tilde{\alpha_3}}{\tilde{\beta_3}+\tilde{\gamma_3}} + \frac{D}{\gamma}\right)e^{-\tilde{\gamma_3}(t-T_1)} - \frac{D}{\tilde{\gamma_3}}\right)dt\right\} + C_p\int_0^{T_2}\tilde{P_3}dt + N\,(a+b\,Q\,d\,)\Bigg]$$

Using Signed Distance Method total average inventory can be calculatedby

$$\widetilde{TIC}_{csr}(\tilde{T},\tilde{T_1},\tilde{T_2}) = \frac{1}{4T}\left[\widetilde{TIC}_{1csr}(\tilde{T},\tilde{T_1},\tilde{T_2}) + 2\widetilde{TIC}_{2csr}(\tilde{T},\tilde{T_1},\tilde{T_2}) + \widetilde{TIC}_{3csr}(\tilde{T},\tilde{T_1},\tilde{T_2})\right] \quad (24)$$

The objective is to

$$\text{minimize:}\widetilde{TIC}_{csr}(\tilde{T},\tilde{T_1},\tilde{T_2})$$
$$\text{Subject to:}(\tilde{T} > 0, \tilde{T_1} > 0, \tilde{T_2} > 0)$$

The combined inventory average cost is function of $\tilde{T}, \tilde{T}_1, \tilde{T}_2$ only and the optimal value of $\tilde{T}, \tilde{T}_1, \tilde{T}_2$ can be obtainedby equating $\frac{\partial \widetilde{TIC}_{csr}(\tilde{T}, \tilde{T}_1, \tilde{T}_2)}{\partial \tilde{T}} = 0$; $\frac{\partial \widetilde{TIC}_{csr}(\tilde{T}, \tilde{T}_1, \tilde{T}_2)}{\partial \tilde{T}_1} = 0$; $\frac{\partial \widetilde{TIC}_{csr}(\tilde{T}, \tilde{T}_1, \tilde{T}_2)}{\partial \tilde{T}_2} = 0$ under the necessary condition of Hessian matrix $\tilde{H}$ which are as under is satisfied.

$$\tilde{H} = \begin{pmatrix} \frac{\partial \widetilde{TIC}_{csr}(\tilde{T}, \tilde{T}_1, \tilde{T}_2)}{\partial \tilde{T}^2} & \frac{\partial^2 \widetilde{TIC}_{csr}(\tilde{T}, \tilde{T}_1, \tilde{T}_2)}{\partial \tilde{T}\, \partial \tilde{T}_1} & \frac{\partial^2 \widetilde{TIC}_{csr}(\tilde{T}, \tilde{T}_1, \tilde{T}_2)}{\partial \tilde{T}\, \partial \tilde{T}_2} \\ \frac{\partial^2 \widetilde{TIC}_{csr}(\tilde{T}, \tilde{T}_1, \tilde{T}_2)}{\partial \tilde{T}_1\, \partial \tilde{T}} & \frac{\partial \widetilde{TIC}_{csr}(\tilde{T}, \tilde{T}_1, \tilde{T}_2)}{\partial \tilde{T}_1^2} & \frac{\partial^2 \widetilde{TIC}_{csr}(\tilde{T}, \tilde{T}_1, \tilde{T}_2)}{\partial \tilde{T}_1\, \partial \tilde{T}_2} \\ \frac{\partial^2 \widetilde{TIC}_{csr}(\tilde{T}, \tilde{T}_1, \tilde{T}_2)}{\partial \tilde{T}_1\, \partial \tilde{T}_2} & \frac{\partial^2 \widetilde{TIC}_{csr}(\tilde{T}, \tilde{T}_1, \tilde{T}_2)}{\partial \tilde{T}_2\, \partial \tilde{T}} & \frac{\partial \widetilde{TIC}_{csr}(\tilde{T}, \tilde{T}_1, \tilde{T}_2)}{\partial \tilde{T}_2^2} \end{pmatrix}$$

The necessary condition for $\tilde{T}, \tilde{T}_1, \tilde{T}_2$ to be optimal is that the first principal minor determinant of $\tilde{H}$, $\tilde{H}_{11} > 0$, second principal minor $\tilde{H}_{22} > 0$and third principal minor $\tilde{H}_{33} > 0$and optimal value of N (discrete value) is found in such a way that optimal value of $TIC_{csr}(N, \tilde{T}^*, \tilde{T}_1^*, \tilde{T}_2^*)$is achieved. Here $\tilde{T}^*, \tilde{T}_1^*, \tilde{T}_2^*$ are optimal values.

6.0 Numerical Illustration for models:

To analyse the model, following example is taken. The exponential function has been solved up to second approximation. The values of parameters are not collected from any real-life case study but these values are realistic and chosen randomly to illustrate and validate the model. Considering the value of parameters in an appropriate unit (displayed in Table-1 for crisp model and in Table-3 for fuzzy model) and using suitablemathematical software, the optimal average inventory cost has been obtained which are displayed in Table-2& Table-4 for two models separately. Sensitivity analysis has been performed only for fuzzy model onfuzzy parameter considering change upper bound of triangular fuzzy numbers.

Table-1

Paramete r	h_r	h_s	d_r	d_s	α	B	γ	o_r	p_r	s_c	a	b	d	P	C_s	N
Example	0. 8	0. 7	0. 9	0.9 5	5 0	0. 2	0.0 2	9 0	2 0	20 0	4 0	0.0 3	9 5	15 0	5. 0	2

Table-2: Crisp Model

T^*	T_1^*	T_2^*	$TIC_{csr}(T^*, T_1^*, T_2^*)$
1.0689	3.0965	39.7485	191.795

Table-3:

Paramete	h_r	h_s	d_r	d_s	α	B	γ	o_r	p_r	s_c	a	b	d	P	C_s	N

r																
Example	0.8	0.7	0.9	0.95	50	0.2	0.02	90	20	180	40	0.03	95	150	5.0	2

Parameter	α	B	γ	p_r	s_c	P
Triangular fuzzy number	(49, 50,51)	(0.19,0.2, 0.21)	(0.01, 0.02,0.03)	(15,20,25)	(180,200,220)	(100, 150,200)

Table-4: Fuzzy Model

Method	T^*	T_1^*	T_2^*	$\widetilde{TIC}_{csr}(T^*, T_1^*, T_2^*)$
Signed Distance	0.9412	2.5961	41.4928	190.183

7. Sensitivity Performance

Fuzzy Model

Table-5.1: Variation in total inventory cost with respect to production unit

Variation in total inventory cost with respect to $\tilde{P}$ (100,150,250)				
Parameter→ Method ↓	$\hat{T}*$	$\hat{T}1*$	$\hat{T}2*$	$\widetilde{TIC}csr\hat{T}*,\hat{T}1*,\hat{T}2*$
Signed Distance	0.08193	2.2814	41.426	207.937

Table-5.2: Variation in total inventory cost with respect to purchase cost

Variation in total inventory cost with respect to $\tilde{pr}$(15,20,30)				
Parameter→ Method ↓	$\hat{T}*$	$\hat{T}1*$	$\hat{T}2*$	$\widetilde{TIC}csr\hat{T}*,\hat{T}1*,\hat{T}2*$
Signed Distance	0.9814	2.6708	40.835	212.11

Table-5.3: Variation in total inventory cost with respect to fixed demand

Variation in total inventory cost with respect to $\tilde{\alpha}(49,50,60)$				
Parameter→ Method ↓	T^*	T_1^*	T_2^*	$\widetilde{TIC}_{csr}(T^*, T_1^*, T_2^*)$
Signed Distance	0.9847	2.7614	40.9868	213.509

Table-5.4: Variation in total inventory cost with respect to scale factor of demand

Variation in total inventory cost with respect to $\tilde{\beta}(0.19, 0.20, 0.23)$				
Parameter→ Method ↓	T^*	T_1^*	T_2^*	$\widetilde{TIC}_{csr}(T^*, T_1^*, T_2^*)$
Signed Distance	0.9814	2.6743	40.8189	187.208

Table-5.5: Variation in total inventory cost with respect to deterioration rate

Variation in total inventory cost with respect to $\tilde{\gamma}(0.01, 0.02,0.035)$				
Parameter→ Method ↓	T^*	T_1^*	T_2^*	$\widetilde{TIC}_{csr}(T^*, T_1^*, T_2^*)$
Signed Distance	0.9116	2.5303	40.1561	191.458

Result analysis of Crisp and Fuzzy model

When all the given conditions and constraints are satisfied, the optimal solution is obtained. Table-2 & Table-4, reveals that the average total relevant average inventory cost in an appropriate unit is minimum in case of fuzzy model as compared to crisp model and cycle length of retailer and production time period of supplier are lower but total cycle length of supplier moderately higher.Therefore, fuzzy model is more valuable in dealing with uncertain situation due to flexibility on choosing a range of values of parameters in respect of future planning. Trend of change in the value of total average inventory cost of supply chain in case of fuzzy inventory model is illustrated by 2-D graphical representation in the figure- 3 for some selected parameters.

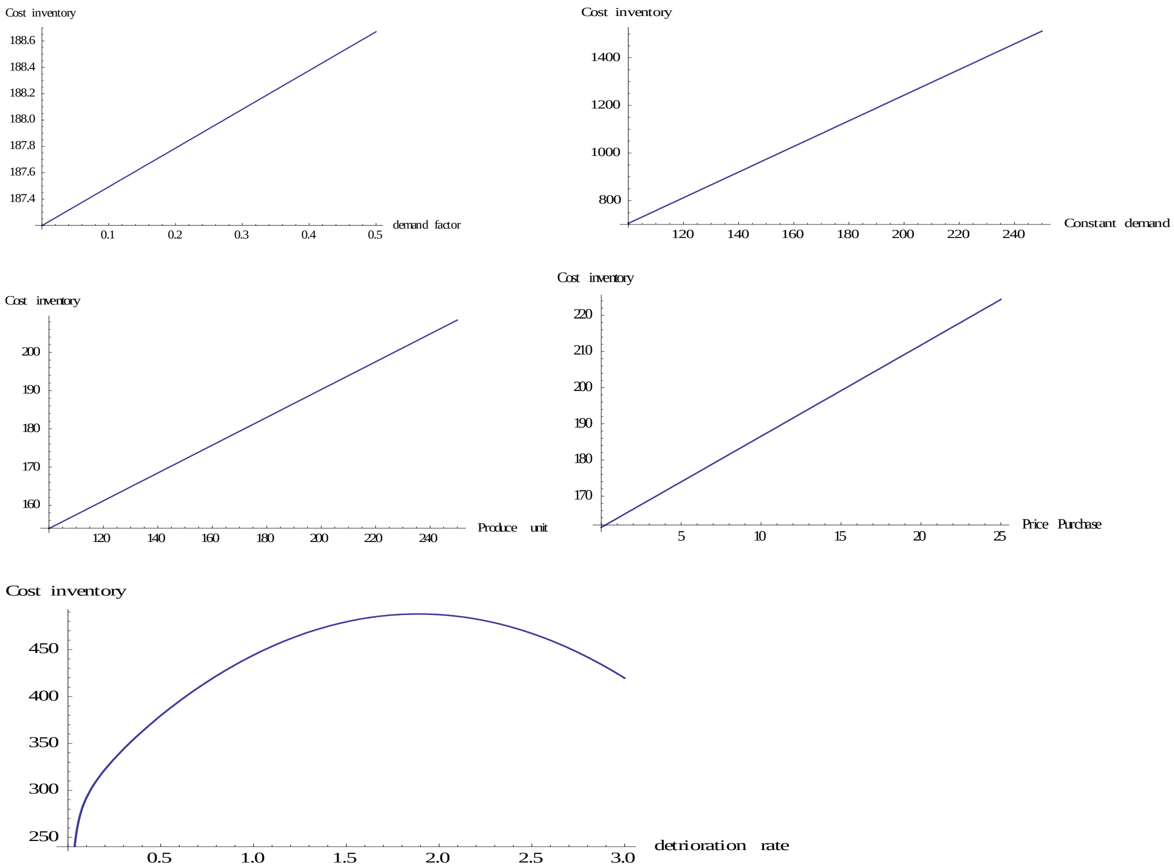

Figure-3: Representing trend of change in Average inventory cost

8. Sensitivity analysis and observance:

From sensitivity performance, thefollowing are observations have been made: -

1. If there is increase in the upper value of triangular fuzzy number of fixed demand parameter, keeping other same, the cycle length of retailer and production period of supplier and total cycle length of supplier slightly increases but the average inventory cost moderately increases.
2. If there is increase in the upper value of triangular fuzzy number of scale parameter of demand, keeping other same, the cycle length of retailer and production period of supplier and total cycle length of supplier slightly increases but the average inventory cost moderately decreases.
3. If there is increase in the upper value of triangular fuzzy number of deterioration parameter, keeping other same, the cycle length of retailer and production period of supplier and total cycle length of supplier slightly decreases but the average inventory cost overall have no effect.
4. If there is increase in the upper value of triangular fuzzy number of purchased cost parameter, keeping other same, the cycle length of retailer and production period of supplier have no more effect and total cycle length of supplier slightly increases but the average inventory cost moderately increases.

5. If there is increase in the upper value of triangular fuzzy number of production parameter, keeping other same, the cycle length of retailer moderately decreases and there is slight decrease in the production period of supplier but total cycle length of supplier have no effect while average inventory cost moderately increases.

9.0 Conclusion:

This paper presents an integrated supply chain model to optimize total inventory cost of supply chain in which retailer and supplier are involved. Deterioration rate for both are taken to be constant and same. Demand of retailer vary depending upon the inventory level while per order demand is fixed. Crisp and a fuzzy model has been developed and models are validated through numerical example. A suitable mathematical software has been used to solve the optimization problem and results have been compared. Sensitivity analysis has been performed on some fuzzyparameters considering change in the upper value of triangular fuzzy number. It has been observed that fuzzy model is more useful than crisp model to deal with future uncertainty. Further this model can be extended by considering other demand pattern and trade credit payment with different combination of deterioration rates with inflation.

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