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Buckling Response of Laminated Composite Plate with Random System Properties in Thermal Environment

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Abstract

This work uses a micromechanical technique to show how random system features affect the buckling response of laminated composite plates in temperature conditions. Independent random variables are used to simulate the system attributes, including foundation parameters, fiber volume fractions of the corresponding fiber and matrix ingredients, and thermo-material properties. The temperature field is thought to consist of consistent temperature distributions over the thickness and surface of the plate. The composite's material properties are influenced by temperature variations and are determined using a micromechanical model. The basic formulation is based on higher order shear deformation plate theory and general von-Karman types of nonlinearity. A direct iterative based C0 nonlinear finite element method in conjunction mean centered first order perturbation technique is out lined and solved the stochasticlinear generalized Eigen value problem. The developed stochastic procedure is usefully used for thermally induced problem based on micromechanical approach with a reasonable accuracy. Parametric studies are carried out to see the effect of volume fractions, amplitude ratios, temperature increments, temperature distributions geometric parameters, lay-ups, boundary conditions and foundation parameters on the mean and variance of plate frequency. The present outlined approach has been validated with those available results in literatures and independent Monte-Carlo simulation.

Keywords: Thermal Buckling, Random Material Properties, Stochastic Finite Element, Perturbation Technique

INTRODUCTION

Laminated composite plates are increasingly used as critical structural members in aerospace and many other applications due to gaining wide popularity as light weight components, ability to tailor structural properties through appropriate lamination scheme for achieving high strength and stiffness to weight ratio and durability and corrosion resistant characteristics combined with low density, make it more attractive compared to conventional materials.

BUCKLING EQUATIONS FOR LAMINATED PLATES

A plate buckles when the in-plane load gets so large that the originally flat equilibrium state is no longer stable and the plate deflects into a non flat configuration. The load at which the departure from flat state takes place is called the buckling load. Analysis of plates buckling under in-plane loading involves solution of eigenvalue problem as opposed to the boundary value problem of equilibrium analysis. The distinctions between boundary value problems and eigenvalue problems are too involved to treat here. Instead, the buckling differential equations governing the buckling behavior from a membrane prebuckled state (prebuckling deformations are ignored) are,

$$\delta N_{x,x} + \delta N_{x,y,y} = 0 \qquad (1)$$
$$\delta N_{xy,x} + \delta N_{y,y} = 0 \qquad (2)$$

$$\delta M_{x,xx} + 2\delta M_{xy,yy} + \delta M_{x,x} + \delta N_{y,y,y} + N_x \delta w_{,xx} + 2N_{xy}\,\delta w_{,xy} + N_y \delta w_{,yy} = 0 \quad (3)$$

Where the δ denotes a variation of the principal symbol from its value in the prebuckled equilibrium state. Thus the terms δN_x.................δM_x are variations of
forces and moments, respectively, from a membrane prebuckled equilibrium state. The term δu and δv are variations in displacement from the same flat prebuckled state. In appearance, the buckling differential equations resemble the equilibrium differential equations except for variational notation.
An important consequence of this type of problem is that the magnitude of the deformations after buckling cannot be determined without resort to large deflection considerations; i.e. the deformations are indeterminate when only Eqs are available.
The variations in force and moment resultants are

$$\begin{Bmatrix} \delta N_x \\ \delta N_y \\ \delta N_{xy} \end{Bmatrix} = \begin{bmatrix} A11 & A12 & A16 \\ A12 & A22 & A26 \\ A16 & A26 & A66 \end{bmatrix} \begin{Bmatrix} \delta \xi_x \\ \delta \xi_Y \\ \delta \gamma_{XY} \end{Bmatrix} + \begin{bmatrix} B11 & B11 & B11 \\ B12 & B22 & B26 \\ B16 & B26 & B66 \end{bmatrix} \begin{Bmatrix} \delta K_x \\ \delta K_Y \\ \delta K_{XY} \end{Bmatrix} \quad (4)$$

$$\begin{Bmatrix} \delta M_x \\ \delta M_y \\ \delta M_{xy} \end{Bmatrix} = \begin{bmatrix} B11 & B12 & B16 \\ B12 & B22 & B26 \\ B16 & B26 & B66 \end{bmatrix} \begin{Bmatrix} \delta \xi_x \\ \delta \xi_Y \\ \delta \gamma_{XY} \end{Bmatrix} + \begin{bmatrix} D11 & D12 & D16 \\ D12 & D22 & D26 \\ D16 & D26 & D66 \end{bmatrix} \begin{Bmatrix} \delta K_x \\ \delta K_Y \\ \delta K_{XY} \end{Bmatrix} \quad (5)$$

Where the variations in extensional strains and changes in curvature are related to the variations in displacements by

$$\delta\xi_x = \delta u_{,x} \quad \delta\xi_y = \delta v_{,y} \quad \delta\gamma_{XY} = \delta u_{,y} + \delta v_{,x} \quad (6)$$

$$\delta K_x = \delta w_{,xx} \quad \delta K_y = -\delta w_{,yy} \quad \delta K_{xy} = - - 2\delta w_{,xy} \quad (7)$$

The boundary conditions for buckling problems are applied only to the buckling deformations since the prebuckling deformations are assumed to be a membrane state. One of the distinguishing features are homogeneous, ie., zero. Thus during buckling, the simply supported edge and clamped edge boundary conditions are

SSSS1: $\delta w = 0$ $\delta M_n = 0$ $\delta U_n = 0$ $\delta U_t = 0$
SSSS2: $\delta w = 0$ $\delta M_n = 0$ $\delta N_n = 0$ $\delta U_t = 0$
SSSS3: $\delta w = 0$ $\delta M_n = 0$ $\delta U_n = 0$ $\delta N_{nt} = 0$ (8)

SSSS4: $\delta w = 0$ $\delta M_n = 0$ $\delta N_n = 0$ $\delta N_{nt} = 0$
CCCC1: $\delta w = 0$ $\delta W_n = 0$ $\delta U_n = 0$ $\delta U_t = 0$
CSCS1: $\delta w = 0$ $\delta W_n = 0$ $\delta U_n = 0$ $\delta U_t = 0$ (9)

The boundary conditions could be different for each edge of plate, so the number of combinations of possible boundary conditions is enormous, as it was with equilibrium problems.

A. The displacement field becomes

The displacement vector for the modified models is

$$\{\Lambda\} = \{u \quad v \quad w \quad \theta_y \quad \theta_x \quad \psi_y \quad \psi_x\}^T \qquad (10)$$

Strain-displacements relations with von Karmantype geometric nonlinear elasticity are expressed as:

$$
\begin{aligned}
\varepsilon_{xx} &= \varepsilon_1 = \bar{u},_x + \varepsilon_1^0 + z(k_1^0 + z^2 k_1^2) + \frac{1}{2}\bar{w}^2,_x \\
\varepsilon_{yy} &= \varepsilon_2 = \bar{v},_x + \varepsilon_2^0 + z(k_2^0 + z^2 k_2^2) + \frac{1}{2}\bar{w}^2,_y \\
\gamma_{xy} &= \varepsilon_6 = \bar{u},_y + \bar{v},_x = \varepsilon_6^0 + z(k_6^0 + z^2 k_6^2) + \bar{w},_{xx} + \bar{w},_{yy} \\
\gamma_{yz} &= \varepsilon_4 = \bar{v},_z + \bar{w},_y = \varepsilon_4^0 + z^2 k_4^2 \\
\gamma_{xz} &= \varepsilon_5 = \bar{u},_z + \bar{w},_x = \varepsilon_5^0 + z^2 k_5^2
\end{aligned}
\qquad (11)
$$

Where

$$
\begin{aligned}
&\varepsilon_1^0 = u,_x, \quad k_1^0 = \psi_{x,x}, \quad k_1^2 = -4/3h^2 (\psi_{x,x} + w,_{xx}) \\
&\varepsilon_2^0 = v,_y, \quad k_2^0 = \psi_{y,y}, \quad k_2^2 = -4/3h^2 (\psi_{y,y} + w,_{yy}) \\
&\varepsilon_6^0 = u,_y + v,_x, \quad k_6^0 = \psi_{x,y} + \psi_{y,x}, \quad k_6^2 = -4/3h^2(\psi_{x,y} + \psi_{y,x} + 2w,_{xy}) \\
&\varepsilon_4^0 = \psi_y + w,_y, \quad k_4^2 = -4/h^2 (\psi_x + w,_y) \\
&\varepsilon_5^0 = \psi_x + w,_x, \quad k_5^2 = -4/h^2 (\psi_x + w,_x)
\end{aligned}
$$

Where, comma (,) denotes the partial differential.The mid plane strain vector for the model is

$$
\{\varepsilon\} = \left(\varepsilon_1^0 \ \varepsilon_2^0 \ \varepsilon_6^0 \ k_1^0 \ k_2^0 \ k_6^0 \ k_1^2 \ k_2^2 \ k_6^2 \ \varepsilon_4^0 \ \varepsilon_5^0 \ k_4^2 \ k_5^2\right)^T \qquad (12)
$$

The constitutive law thermal elasticity for the materials under consideration relates the stresses with strainsin a plane stress state for the kth lamina oriented as anarbitrary angle with respect to the reference axis for an orthotropic layer is given by (Reddy 1997;Robert 1975)

$$
\{\sigma\}_k = [\bar{Q}]_k \{\varepsilon\}_k \text{ or}
$$

$$
\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \\ \sigma_{yz} \\ \sigma_{xz} \end{Bmatrix}_k =
\begin{bmatrix}
\bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} & 0 & 0 \\
\bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} & 0 & 0 \\
\bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} & 0 & 0 \\
0 & 0 & 0 & \bar{Q}_{44} & \bar{Q}_{45} \\
0 & 0 & 0 & \bar{Q}_{45} & \bar{Q}_{55}
\end{bmatrix}_k
\begin{Bmatrix} \varepsilon_{xx} - \alpha_{xx}\Delta T \\ \varepsilon_{yy} - \alpha_{yy}\Delta T \\ \varepsilon_{xy} - \alpha_{xy}\Delta T \\ \varepsilon_{yz} \\ \varepsilon_{xz} \end{Bmatrix}_k
\qquad (13)
$$

Where $\bar{Q}$, $\{\sigma\}_k$ and $\{\varepsilon\}_k$ are transformed stiffness matrix, stress and strain vectors of the kth lamina, respectively and (α_x, α_y) are the thermal expansion coefficients along x, y, z, direction, $(\alpha_{xy}, \beta_{xy})$ are shear coefficients in x,y plane respectively. This can be obtained from the thermal coefficients in the longitudinal (α_l, β_1) and transverse (α_2, β_2) directions of the fibres using transformation matrix. T (X, Y, Z) is the uniform temperature and moisture field distribution. Substituting Eq. (12) into Eq. (13) and integrating through thickness gives a relationship between stress resultants and mid-plane strainare given as Reddy [1997].

$$\begin{Bmatrix} N_i \\ M_i \\ P_i \end{Bmatrix} = \begin{bmatrix} A_{ij} & B_{ij} & E_{ij} \\ B_{ij} & D_{ij} & F_{ij} \\ E_{ij} & F_{ij} & H_{ij} \end{bmatrix} \begin{Bmatrix} \varepsilon^0_j \\ k^0_j \\ k^2_j \end{Bmatrix} - \begin{Bmatrix} N^T_i \\ M^T_i \\ P^T_i \end{Bmatrix} \quad (i, j=1, 2, 6) \quad (14)$$

$$\begin{Bmatrix} Q_2 \\ Q_1 \end{Bmatrix} = \begin{bmatrix} A_{4j} & D_{4j} \\ A_{5j} & D_{5j} \end{bmatrix} \begin{Bmatrix} \varepsilon^0_j \\ k^2_j \end{Bmatrix}; \begin{Bmatrix} R_2 \\ R_1 \end{Bmatrix} = \begin{bmatrix} D_{4j} & F_{4j} \\ D_{5j} & F_{5j} \end{bmatrix} \begin{Bmatrix} \varepsilon^0_j \\ k^2_j \end{Bmatrix} \quad (j=4, 5) \quad (15)$$

$$\{N_i\} = \begin{bmatrix} N_x & N_y & N_{xy} \end{bmatrix}^T, \{M_i\} = \begin{bmatrix} M_x & M_y & M_{xy} \end{bmatrix}^T \; \& \; \{P_i\} = \begin{bmatrix} P_x & P_y & P_{xy} \end{bmatrix}^T \quad (16)$$

Expressed in terms of mid plane strains and curvatures (Robert 1975; Bhagwan et al.1990). The thermal stress and moments results per unit length due to temperature change are calculated

B. Micromechanical Approach

The material properties of the fibre composite at different temperature are evaluated using micromechanical model. Since the effect of temperature is dominant in matrix material. The degradation of the fibre composite material properties is estimated by degrading the matrix propertyonly. The matrix mechanical property retention ratio is expressed as Upadhyay et al. (2010).The elastic constants are obtained from the following equations Upadhyay et al. (2010).

$$E_{11} = E_{f1} V_f + F_m E_m V_m \quad (17)$$

$$E_{22} = \left(1 - \sqrt{V_f}\right) F_m E_m + \frac{F_m E_m \sqrt{V_f}}{1 - \sqrt{V_f}\left(1 - \frac{F_m E_m}{E_{f2}}\right)} \quad (18)$$

$$G_{12} = \left(1 - \sqrt{V_f}\right) F_m G_m + \frac{F_m G_m \sqrt{V_f}}{1 - \sqrt{V_f}\left(1 - \frac{F_m G_m}{G_{f12}}\right)} \quad (19)$$

$$\nu_{12} = \nu_{f12} V_f + \nu_m V_m \quad (20)$$

Where "V" is the volume fraction and subscripts "f" and "m" are used for fibre and matrix, respectively. The effect of increased temperature on the coefficients of thermal expansion (α) is opposite from the corresponding effect on strength and stiffness. The matrix thermal property

$$F_h = \frac{1}{F_m}$$

Coefficients of thermal expansion are expressed asUpadhyay et al. (2010).

$$\alpha_{11} = \frac{E_{f1}V_f\alpha_{f1} + F_m E_m V_m F_h \alpha_m}{E_{f1}V_f + F_m E_m V_m} \quad (21)$$

$$\alpha_{22} = (1+\nu_{f12})V_f\alpha_{f2} + (1+\nu_m)V_m F_h \alpha_m - \nu_{12}\alpha_{11}$$

The elastic constants are obtained from thefollowing equations Shen, Hui-Shen.(2001).

$$E_{11} = VfEf + VmEm \quad (22)$$

$$\frac{1}{E_{22}} = \frac{Vf}{Ef} + \frac{Vm}{Em} - VfVm \frac{Vf^2 \frac{Em}{Ef} + \upsilon m^2 \frac{Ef}{Em} - 2\upsilon f \upsilon m}{VfEf + VmEm} \quad (23)$$

$$\frac{1}{G_{12}} = \frac{Vf}{Ef} + \frac{Vm}{Gm} \quad (24)$$

$$\upsilon_{12} = Vf\upsilon f + Vm\upsilon m \quad (25)$$

Where “V” is the volume fraction and subscripts “f” and “m” are used for fibre and matrix, respectively. The effect of increased temperature and moisture concentration on the coefficients of thermal expansion (α) is opposite fromthe corresponding effect on strength and stiffness.
Coefficients of thermal expansion are expressed asShen, Hui-Shen.(2001).

$$\alpha_{11} = \frac{VfEf\alpha f + VmEm\alpha m}{VfEf + VmEm} \quad (26)$$

$$\alpha_{22} = (1+\upsilon f)Vf\alpha f + (1+\upsilon m)Vm\alpha m - \upsilon_{12}\alpha_{11} \quad (27)$$

Other constants are related as Shen, Hui-Shen.(2001).
  Vf f Vmm & Vm Vf 1

C. Perturbation Technique

In general, without any loss of generality any arbitrary random variable can be represented as the sum of its meanand zero mean random part, denoted by superscripts „d" and
„r", respectively (Singh et al. 2002).

$$K = K^d + K^r, \quad K_g = K_g^d + K_g^r$$
$$\lambda_i = \lambda_i^d + \lambda_i^r, \quad q_i = q_i^d + q_i^r \quad (28)$$

By substituting Eq. (27) in Eq. (28) and expanding the random parts in Taylor"s series keeping the first order terms and neglecting the second and higher order terms, collecting same order of the magnitude term, one obtains as (Yamin et al. 1996; Lin 1967).

1) *Zeroth order*

$$[K^d]\{q_i^d\} = \lambda_i^d [K_g^d]\{q_i^d\} \qquad (29)$$

2) *First order*

$$[K^d]\{q^r\} + [K^r]\{q^d\} = \lambda^d [K_g^r]\{q^d\} + \lambda^d [K_g^d]\{q^r\} + \lambda^r [K_g^d]\{q^d\} \qquad (30)$$

Eq. (29) is the deterministic equation relating to themean eigenvalues and corresponding mean eigenvectors, which can be determined by conventional Eigen solution procedures. Eq. (30) is the random equation, defining the stochastic nature of the thermal buckling which cannot be solved using conventional method. For this a further analysis is required.

RESULTS AND DISCUSSION

The stochastic finite element analysis has been applied to obtain the mean and dispersion considering different random parameters of the thermal buckling load of laminated composite plate of graphite epoxy material. Fig.2.The lamina coefficients of thermal expansion including geometric and material properties are modelled as basic random variables RVs. The mean and standard deviation of the thermal buckling load are obtained considering the all random material input variables and thermal expansion coefficients as well as plate thickness taking separately as basic random variables (RVs) as stated earlier. However, the results are only presented taking SD/mean of the system property equal to 0.10 (Liu et al.1986), as the nature of the SD (Standard deviation) variation is linear and passing through the origin. Hence, the presented results would be sufficient to extrapolate the results for other COV value keeping in mind the limitation of FOPT (Klieberet al.1992).The thickness of all the lamina is assumed to be constant and of same material without varying individual property of materials used. The results obtained have been compared with MCS and those available in the literature. For the present study a nine nodded serendipity element, which results in 63 degree of freedom (DOFs) per elementmodified form HSDT based C^0 finite element model has been used for discretizing the laminate. Based on convergence, a (4x4) mesh has been used throughout for evaluation of the results. Results have been computed by employing the full (3x3) integration rule for bending stiffness matrices, thermal load vector and the geometric stiffness matrices and the reduced (2x2) integration rule for computing the shear stiffness matrices to avoid shear locking in the thin plates. The boundary conditions for the plate are as shown in Fig. 3.

A. All edges simply supported (SSSS (S1))

$$u = w = \theta_y = \psi_y = 0,\ at\ x = 0, a;\quad v = w = \theta_x = \psi_x = 0\ at\ y = 0, b;$$

B. All edges simply supported (SSSS (S2))

$$v = w = \theta_y = \psi_y = 0,\ at\ x = 0, a;\quad u = w = \theta_x = \psi_x = 0\ at\ y = 0, b;$$

C. All edges clamped (CCCC)

$$u = v = w = \psi_x = \psi_y = \theta_x = \theta_y = 0,\ at\ x = 0, a\quad and\ y = 0, b;$$

The plate geometry used is characterized by aspect ratios (a/b) = 1and 2, side to thickness ratios (a/h) = 10, 30, 80 and 100. Coefficients of thermal expansion and material constants are used for computation. We now consider a second step as the elastic constants, thermal expansion coefficients of each layers are assumed to be linear function of temperature. The only exception is the Poisson‟s ratio which can reasonably be assumed as constant due to weakly dependency on temperature change. For the ratio of the SD to the mean of material and geometric properties varying from 0 to 20% with a/h= 10, 30, 80 and 100 have been considered. The thermo-elastic material properties such as

E_{11}, E_{22}, G_{12}, G_{13}, G_{23}, α_1, α_2, are modelled as basic random variables (RVs). The input random variables b_i is related as b_1= E_{11}, b_2=E_{22}, b3=G_{12}, b_4= G_{13}, b_5=G_{23}, b_6=α_1, b_7=α_2, For

the temperature dependent material properties (TD), the relation among elastic constants is given as:

$$E_{111} = E_{110}\left(1 + E_{111}T + E_{111}C\right),$$
$$E_{221} = E_{220}\left(1 + E_{221}T + E_{221}C\right)$$
$$G_{12} = G_{120}\left(1 + G_{121}T + G_{121}C\right),$$
$$G_{13} = G_{130}\left(1 + G_{131}T + G_{131}C\right)$$
$$G_{23} = G_{230}\left(1 + G_{231}T + G_{231}C\right),$$
$$\alpha_1 = \alpha_{10}\left(1 + \alpha_{11}T\right),$$
$$\alpha_2 = \alpha_{20}\left(1 + \alpha_{21}T\right),$$

Where the coefficients of temperature are defined as

$$E_{111} = -0.5 \times 10^{-3}$$
$$E_{221} = G_{121} = G_{131} = G_{231} = -0.2 \times 10^{-3}$$
$$\alpha_{11} = \alpha_{21} = 0.5 \times 10^{-3}$$

The thermal coefficients α_{11} are taken equal in magnitude for degradation of composite plate. All layers are of equal thickness for the temperature independent material properties (TID)E_{111}, E_{221}, G_{121} G_{131}, G_{231}, ?1, ?21 quantities are taken equal to zero. For validation purposes the analysis of Tables 1(a), 1(b) and 1(c) is presented. Tables from 2 to 6are for generation of results for various combinations. To establish the validity of the present study, the results are compared with the existing results those available in the literature.

D. Validation Study; Mean and Standard Deviations

In order to validate the proposed outlined approach the results for the mean and standard deviation are compared with those available in the literature and an independentMonte Carlo simulation technique.

Table 1: Validation Study; Mean and Standard Deviations

Lay-up	Environment Conditions	Buckling Load Nx (KN)					
		(Shen 2001)			Present		
		Vf= 0.5	Vf=0.6	Vf=0.7	Vf= 0.5	Vf= 0.6	Vf= 0.7
(0/90)s	0ΔT=0^0C	171.36	203.90	243.28	171.21	203.81	243.99

E. Effect of plate thickness ratio(a/h) and input random variables bi{i=1...8, 7-8, and 9 = 0.10} on the buckling loads N_x (KN) for angle ply $(\pm45^0)_{2T}$, square laminated composite plate,V_f=0.6, T_0=25^0C, Simple Support (S2)under Environmental Conditions

Effect of plate thickness ratio(a/h) and input random variables bi{i=1...8, 7-8, and 9 = 0.10} on the buckling loads N_x (KN) for angle ply $(\pm45^0)_{2T}$, square laminated composite plate,V_f=0.6, T_0=25^0C, Simple Support (S2) under Environmental Conditions shown in table 2. It is observed that for simple support (S2) the increases in amplitude ratio decreases the mean thermal postbucklingload and COV of combined random variables increases, whereas COV of plate thickness increases. On increase in temperature the mean thermal post buckling load decreases and coefficient of variations of thermal post buckling load also varies. It is further seen that without rise in temperature mean thermal post buckling load for simply supportsignificantly increases

Table 2: Effect of plate thickness ratio(a/h) and input random variables bi{i=1…8, 7-8, and 9 = 0.10} on the buckling loadsN_x (KN) for angle ply $(\pm 45^0)2_T$, square laminated composite plate, V_f=0.6, T_0=25^0C, Simple Support (S2) under Environmental Conditions

a/h	W_{max}/h	$\Delta T=0^0C$, Mean (KN)	Coefficient of variation (SD/ Mean)Bi (i=1…8)	(i=7.-8)	(i=9)	W_{max}/h	$\Delta T=100^0C$, Mean (KN)	Coefficient of variation (SD/ Mean)Bi (i=1…8)	(i=7.-8)	(i=9)
10	0.3	178.026	(0.0665)	(4.8943e-004)	(0.0187)	0.3	166.0780	(0.0592)	(0.0023)	(0.0233)
	0.6	177.806	(0.0667)	(4.9003e-004)	(0.0185)	0.6	166.8566	(0.0593)	(0.0023)	(0.0238)
	0.9	178.735	(0.0664)	(4.8784e-004)	(0.0187)	0.9	169.6873	(0.0611)	(0.0023)	(0.0236)
	□crl	171.226				□crl	161.5558			
30	0.3	20.0468	(0.0688)	(0.0015)	(0.0421)	0.3	19.6905	(0.0709)	(0.0067)	(0.0408)
	0.6	20.0554	(0.0689)	(0.0015)	(0.0423)	0.6	20.3595	(0.0672)	(0.0067)	(0.0364)
	0.9	32.4826	(0.0634)	(9.0411e-004)	(0.0568)	0.9	25.0384	(0.0591)	(0.0042)	(0.0425)
	□crl	17.7045				□crl	17.3034			
80	0.3	1.3751	(0.0787)	(0.0081)	(0.0395)	0.3	1.3746	(0.0862)	(0.0366)	(0.0390)
	0.6	1.5604	(0.0761)	(0.0071)	(0.0309)	0.6	1.4733	(0.0824)	(0.0341)	(0.0362)
	0.9	4.3655	(0.0666)	(0.0025)	(0.0545)	0.9	1.6826	(0.0774)	(0.0299)	(0.0383)
	□crl	1.0948				□crl	1.0869			
100	0.3	0.7237	(0.0811)	(0.0122)	(0.0323)	0.3	0.7220	(0.0330)	(0.0330)	(0.0330)
	0.6	0.9005	(0.0749)	(0.0098)	(0.0370)	0.6	0.8205	(0.0306)	(0.0306)	(0.0306)
	0.9	0.9048	(0.0731)	(0.0098)	(0.0376)	0.9	0.9039	(0.0380)	(0.0380)	(0.0380)
	□crl	0.5693				□crl	0.5657			

F. Validation result for random material properties

Validation result for random material propertiesmentioned earlier, no results are available in reported literature for normalized standard deviation for the nonlinear problems.The first order perturbation technique [FOPT] results for random response has been validated. normalized standard deviation, SD (i.e. the ratio of the standard deviation (SD) to the mean value) of thermal buckling load versus the SD to the Effects of input random variables bi, {(i =1 to 9) = 0.10} on the dimensionlised mean (□crl) and coefficient of variation (□cr) on the hygrothermally induced buckling load Nx(KN) for perfect cross ply (00/900) s square plates, plate thickness ratio (a/h=20), fiber volume fraction (Vf =0.6), initial temperature (T0 =250C), simple support (S2) under environmental conditions. The dimensionlised mean hygrothermal buckling loads are given in brackets (KN). □crl - Linear solution. (Micromechanical Model)

Table 3: Validation result for random material properties

(b_i)	(TID) COV, □cr $\Delta T=0^0C$	$\Delta T=150^0C$	(TD) COV, □cr $\Delta T=0^0C$	$\Delta T=150^0C$
E_{11} (i=1)	(36.3533)0.0706	(34.7229)0.0711	(35.9363) 0.0706	(32.0987) 0.0712
E_{22} (i=2)	0.0049	9.6007e-005	0.0025	3.8160e-004
G_{12} (i=3)	0.0059	0.0059	0.0059	0.0062
G_{13} (i=4)	0.0185	0.0188	0.0185	0.0180
G_{23}(i=5)	0.0030	0.0031	0.0030	0.0030

V12 (i=6)	0.0749	1.8699e-004	0.0038	6.9102e-004
α11 (i=7)	5.5279e-05	6.3605e-004	5.5909e-005	6.8144e-004
α22 (i=8)	0.0012	0.0074	0.0012	0.0083
h (i=9)	0.0363	0.0397	0.0361	0.0398

G. Parametric analysis of second order statistics

Table 4 shows Effect of boundary conditions and input random variables bi{i=1…9, 7-8, 9 and 10= 0.10}on the dimensionalised mean and dispersion (SD/mean) of thermal buckling load of cross ply $[0^0/90^0]_{2T}$ square laminated plate (a/h=30) ,V_f=0.6, T_0=25^{0}C subjected to uniform (U.T.) temperature and moisture distribution, in-plane uni-axial compression under $\Delta T=0^0C$, $\Delta C=0.00$ and $\Delta T=200^0C$. It is observed that on change og boundary conditions the expected mean thermal load changes significantly for plates with clamp (CCCC) support without rise in temperature and moisture condition. On change of environmental conditions the mean thermal load decreases, however there is small change of coefficient of variations of thermal load

Table 4: Validation result for random material properties

(b_i)	(TID)		(TD)	
	COV, $\square_{cr}$		COV, $\square_{cr}$	
	$\Delta T=0^0C$	$\Delta T=150^0C$	$\Delta T=0^0C$	$\Delta T=150^0C$
E11 (i=1)	(36.3533) 0.0706	(34.7229) 0.0711	(35.9363) 0.0706	(32.0987) 0.0712
E22 (i=2)	0.0049	9.6007e-005	0.0025	3.8160e-004
G12 (i=3)	0.0059	0.0059	0.0059	0.0062
G13 (i=4)	0.0185	0.0188	0.0185	0.0180
G23 (i=5)	0.0030	0.0031	0.0030	0.0030
V12 (i=6)	0.0749	1.8699e-004	0.0038	6.9102e-004
α11 (i=7)	5.5279e-05	6.3605e-004	5.5909e-005	6.8144e-004
α22 (i=8)	0.0012	0.0074	0.0012	0.0083
h (i=9)	0.0363	0.0397	0.0361	0.0398

CONCLUSIONS

The stochastic SFEM procedure outlined in the present study has been used to obtain the expected mean and coefficient of variation of the bucking load of the laminated composite plates subjected to uniform temperature rise with random system parameters. The followings are outcome of this limited study:

1) For various input random variables, environmental conditions for cross ply laminate, the increase in temperature effects significantly as the buckling load decreases. The effects are more prominent for temperature.

2) The combined effects of boundary conditions and random variables in thermal environment are studied; the plate is significantly affected by the thermal buckling load for clamp support conditions.

3) In general, the thermal effects are of due importance foranalyzing the structure made of composite materials. The negligence of thermal effects in analyzing the system behavior may leave the design unsafe for manufacturing and may prone to failure for reliable operational requirements.

4) The CCCC plate at slightly higher temperature compared to other supports. The buckling is more dominant in plates of temperature dependent material properties

as compared to temperature independent case.

5) The thick plate are less affected by random input variables and other input variables compared to thinplates.

FUTURE SCOPE

There is a wide scope to extend the present study. Some of the possible extensions are:

- The present study can be extended for other geometries like cylindrical, spherical, and conical.
- The present study can be extended to various shapes of cut-outs.
- The study can be extended to include smart layers in thelaminated composite plates.
- The study can be extended to functionally graded materials.
- In the present study we have used direct iterative method to handle geometric nonlinearity in the plate and elastic foundation. A more systematic approach may be used to handle the nonlinearity.
- Hygrothermal effects on laminated plate and shell.

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