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Impact Of Awareness On The Spread Of Hiv Infection

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Abstract

In this paper, a non-linear mathematical model is proposed to study the impact of awareness on the dynamics of HIV. It is assumed that due to awareness susceptible are taking necessary care to avoid the contact with HIV infected while infected population are isolated to prevent contact for the further spread of disease. It is assumed that the susceptible and infected classes are becoming aware at different rates. The non-linear compartmental model is analyzed by using the stability theory of differential equations and numerical simulation. This results in dual impact on the susceptible as well as infected population.

1. Introduction

The Mathematical modeling is very useful tool to study and control the spread of infectious diseases in the populations. The classical models governing the spread of infectious diseases depend mainly on the interactions between susceptible and invectives. However, there are other factors, such as vaccination, awareness, migration of population etc., which also affect the spread of infectious diseases. In particular, awareness plays an important role on the dynamics of the diseases.

It is observed that the spread of the infectious or communicable diseases in the population make the people to change their behavior and attitude in such a way that the effect of the disease onto themselves is minimized to prevent themselves and others from contracting the disease (Hays, 2006). These changes in the behavior may be called awareness. The level of awareness not only depends on the behavioral changes imposed by public authorities, but also depends on responses driven by risk and fear of the given disease due to its effects on the life, duration of the sickness due to the disease and most importantly availability of the effective treatment of the disease. Kristiansen et al. (2007) in his work explored the usage of face masks to avoid airborne diseases. Rubin et al. (2009) showed the importance of using better hygiene. Laver et al. (2001) in their study of malaria considered importance of preventive medicine. Ahituv et al. (1996) explored the demand of practicing safer sex using condoms. These actions can change the transmission patterns of the disease. For people to react in some way, they do not necessarily need to have seen the effects of the disease themselves, but they may have heard of it through some sources like media. These, however, usually focus on high-profile diseases like HI V/AIDS, Tuberculosis etc. This kinds of awareness can be developed in the people by hearing about someone having fallen ill by not following some necessary precautions, As the information about the presence of a disease spreads in the population, people adapt their behavior as a result of their awareness of the disease (e.g., Stone burner and Low-Beer,2004).

It is the awareness which make people to take precautions such as vaccination, screening of donated blood to prevent blood borne diseases, adapting to protected sex to prevent sexually transmitted diseases. Therefore, to predict the spread of an infectious disease the effect of the awareness must be considered in the modeling process.

It has been observed in statistical analysis on AIDS awareness programs that public awareness can play an appreciable role in preventing the AIDS epidemic [16]. Some researchers have proposed and analyzed compartmental models with the assumption that the awareness plays a vital role to reduce the contact rate. Liu et al. [3] have considered and analyzed a model with the psychological impact on epidemic outbreaks in his work. Misra et al.[17] have proposed and analyzed a non-linear mathematical model for the effects of awareness programs on the spread of infectious diseases such as flu has been proposed and analyzed.

2. Mathematical Model

The total population of interest divided into four mutual disjoint compartments, susceptible class (C_1), aware class) without HIV infection whose members are taking sufficient precautionary actions to o protect themselves from HIV infection, HIV infected class (C_3) whose members are unaware about their own infection or even if they are aware about their infection then also they are not taking sufficient action to stop further spread of HIV infection and the fourth compartment (C_4) consists of those HIV infected individuals who are aware about their HIV infection and interact carefully with others in such way that there is no further spread of HIV infection by their infection. Let S, S_h, I_1 and I_2 be the number of individuals in the compartments C_1, C_2, C_3 and C_4 respectively. Let N be the total population size at time t such that $N = S + S_h + I_1 + I_2$.

The compartmental model is developed according to the schematic diagram shown in figure 1. The following assumptions are made:

The susceptible become HIV infected following contact with the HIV infective at the contact rate α.

The susceptible population is getting awareness at the rate m_1 so they take sufficient precaution such that they cannot get the contact with HIV infectives.
HIV invectives are becoming aware at the rate m_2 so that they are taking necessary action to stop further spread of infection by them.

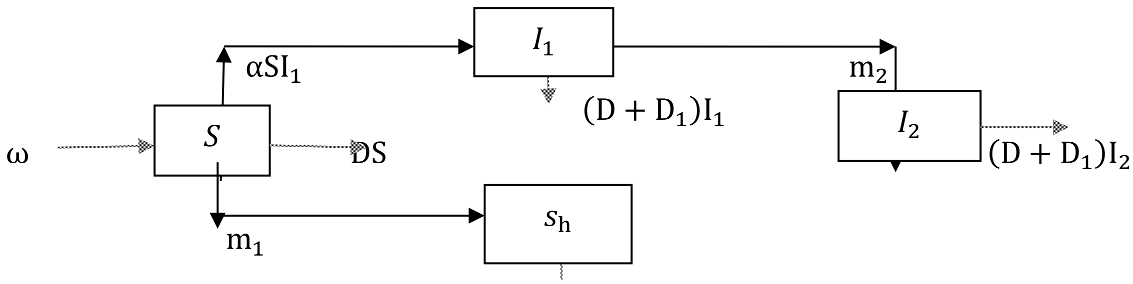

Figure 1 Schematic diagram showing interaction in different compartments.

$$\begin{aligned}
\frac{dS}{dT} &= \omega - \alpha S I_1 - m_1 S - DS \\
\frac{dS_h}{dT} &= m_1 S - DS_h \\
\frac{dI_1}{dT} &= \alpha S I_1 - m_2 I_1 - (D + D_1) I_1 \\
\frac{dI_2}{dT} &= m_2 I_1 - (D + D_1) I_2
\end{aligned} \tag{2.1}$$

All parameters are assumed to be nonnegative. The initial conditions associated with system are:

$$S(0) = S_0 \geq 0, I_1(0) = I_{10} \geq 0, I_2(0) = I_{20} \geq 0, S_h(0) = S_{h0} \geq 0 \tag{2.2}$$

For non-dimensional form of the system, consider the following new dimensionless state variables and parameters:

$$\tau = DT,\ s = \frac{DS}{\omega},\ i_1 = \frac{DI_1}{\omega},\ i_2 = \frac{DI_2}{\omega},\ s_h = \frac{DS_h}{\omega},\ d_1 = \frac{D_1}{D}, A = \frac{\alpha\omega}{D^2},\ \mu_1 = \frac{m_1}{D},\ \mu_2 = \frac{m_2}{D}$$

The dimensionless form of the system (2.1) and the initial conditions (2.2) are

$$\frac{ds}{d\tau} = 1 - Asi_1 - \mu_1 s - s$$
$$\frac{ds_h}{d\tau} = \mu_1 s - s_h$$
$$\frac{di_1}{d\tau} = Asi_1 - \mu_2 i_1 - (1+d_1)i_1 \qquad (2.3)$$
$$\frac{di_2}{d\tau} = \mu_2 i_1 - (1+d_1)i_2$$
$$s(0) = s_0 \ge 0, i_1(0) = i_{10} \ge 0, i_2(0) = i_{20} \ge 0, s_h(0) = s_{h0} \ge 0 \qquad (2.4)$$

3. Mathematical Analysis

3.1 Positivity and boundedness of solution

It is observed that all solutions of the system (2.2) with nonnegative initial data will remain nonnegative for all $\tau > 0$. The proof is straightforward application of Nagumo's theorem. [6]

Theorem 3.1 The solution $(s(\tau), s_h(\tau), i_1(\tau), i_2(\tau))$ of the system (2.3) with the initial condition (2.4) is bounded.

Proof: Adding all the equations of system (2.2) and using the fact $n = s + s_h + i_1 + i_2$

$$\frac{dn}{d\tau} = 1 - n - d_1 i_1 - d_2 i_2$$
$$\frac{dn}{d\tau} \le 1 - n$$
$$n \le 1 \qquad (3.1)$$
$$n(\tau) \le 1 - (1 - n(0))e^{-\tau}$$

As $\tau \to \infty$, $n(\tau) \le 1$.

Thus, the total population $n(\tau)$ is bounded. ■

Further, $s(\tau)$, $i_1(\tau)$, $i_2(\tau)$ and $s_h(\tau)$ must be less or equal to 1. It may be concluded that there exists a topological region $B = \{(s, i_1, i_2, i_3) : 0 \le s + s_h + i_1 + i_2 \le 1\}$ such that all solution trajectories once enter into it remains in it for all τ.

3.2 Basic Reproduction Number

The basic reproduction number is defined as the number of secondary infections produced by a single infectious individual during his or her entire infectious period.

The basic reproduction number for HIV is obtained when $i_2 = i_3 = 0$ in the system (2.3)

$$R_0 = \frac{A}{(1+\mu_1)(\delta + \mu_2)}, \qquad \text{where, } \delta = 1 + d_1 \qquad (3.2)$$

3.3 Equilibrium points

The following Equilibrium points of the non-linear dynamical system (2.3) are obtained:

i) The disease free equilibrium point is $E_0(s^*, s_h^*, 0, 0) = E_0\left(\frac{1}{(1+\mu_1)}, \frac{\mu_1}{(1+\mu_1)}, 0, 0\right)$.

The disease free equilibrium point E_0 always exists.

ii) The endemic equilibrium $E_1(\bar{s}, \bar{s}_h, \bar{i}_1, \bar{i}_2)$ is obtained by solving the four simultaneous equations:

$$1 - Asi_1 - \mu_1 s - s = 0 \tag{3.3a}$$

$$\mu_1 s - s_h = 0 \tag{3.3b}$$

$$As - \mu_2 - \delta = 0 \tag{3.3c}$$

$$\mu_2 i_1 - (1+d_1) i_2 = 0 \tag{3.3d}$$

Solving (3.3c), gives $s = \frac{\delta + \mu_2}{A}$. This substitution in equation (3.3b) yields

$$s_h = \frac{\mu_1(\delta + \mu_2)}{A}$$

Substitution of the values of s in equation (3.3a) gives

$$i_1 = \frac{(1+\mu_1)(R_0 - 1)}{A}$$

Finally, the value of i_2 is obtained from the (3.5d) as

$$i_2 = \frac{\mu_2(1+\mu_1)(R_0 - 1)}{A(1+d_2)}$$

Observe that, i_1 and i_2 are positive when $R_0 > 1$.

$$E_1(\bar{s}, \bar{s}_h, \bar{i}_1, \bar{i}_2) = E_1\left(\frac{\delta + \mu_2}{A}, \frac{\mu_1(\delta + \mu_2)}{A}, \frac{(1+\mu_1)(R_0 - 1)}{A}, \frac{\mu_2(1+\mu_1)(R_0 - 1)}{A(1+d_2)}\right).$$

The endemic $E_1(\bar{s}, \bar{s}_h, \bar{i}_1, \bar{i}_2)$ equilibrium point exists if

$$\mathrm{R}_0 > 1 \tag{3.4}$$

3.4 Stability Analysis

The stability of different equilibrium points can be discussed on the basis of stability matrix at E (s , s_{hhh} , i_2 , i_3). It is computed as

$$J = \begin{bmatrix} -1-Ai_1-\mu_1 & -As & 0 & 0 \\ Ai_1 & As-\mu_2-\delta & 0 & 0 \\ 0 & \mu_2 & -(1+d_2) & 0 \\ \mu_1 & 0 & 0 & -1 \end{bmatrix}$$

The stability results are stated in the form of theorems.

Theorem 3.2 The disease-free equilibrium $E_0(s^*, s_h^*, 0, 0) = E_0\left(\frac{1}{(1+\mu_1)}, \frac{\mu_1}{(1+\mu_1)}, 0, 0\right)$.is locally asymptotically stable when

$$R_0 < 1 \tag{3.7}$$

Proof: The eigenvalues of the stability matrix J at E_0 are computed as $-1, -(1+d_2), -(1+\mu_1), -\left(\delta+\mu_2-\frac{A}{1+\mu_1}\right)$.

Clearly, all eigenvalues are negative if $\delta+\mu_2-\frac{A}{1+\mu_1} > 0$ or $R_{01} < 1$

Hence, the result is proved. ■

Remark: It may be noted from (3.4) that the local stability of E_0 ensures the non-existence of equilibrium points E_1. Therefore, when $R_0 < 1$, one may expect E_0 to be globally stable which is proved by using Lyapunov's second method in form of following theorem.

Theorem 3.3 The locally stable disease-free equilibrium, $E_0(1, 0, 0, 0)$ is always globally asymptotically stable

Proof: Consider a function

$$V_1(s, s_h, i_1, i_2) = (s-s^*)^2 + \frac{4(1+\mu_1)}{\mu_1^2}(s_h - s_h^*)^2 + \frac{q}{(1+\mu_1)} i_1 + i_2$$

Where, the positive constant q can be chosen later.

Note: $V_1(s, i_1, i_2, i_3) = 0$ only at $E_0(s^*, s_h^*, 0, 0)$ and $V_1 > 0$ for all points in the region B except E_0.

Computing derivative of $V_1(s, i_1, i_2, i_3)$ using system (2.2) gives

$$\frac{dV_1}{d\tau} = (s - s^*)\left\{1 - (\mu_1 + 1)s - Asi_1\right\} + \frac{4(1+\mu_1)}{\mu_1^{\,2}}(s_h - s_h^{\,*})(\mu_1 s - s_h)$$

$$+ \frac{q}{\mu_2 + \delta}\left\{As - (\mu_2 + \delta)\right\} i_1 + (\mu_2 i_1 - (1+d_2)i_2)$$

$$= -(1+\mu_1)\left\{-(s-s^*)^2 - \frac{2(s_h - s_h^{\,*})^2}{\mu_1}\right\}^2 - As^2 i_1 - (1+d_2)i_2$$

$$+ s^* Asi_1 + q\left\{\frac{A}{(\mu_2+\delta)(1+\mu_1)} - 1\right\} i_1 + \left\{\mu_2 i_1\right\}$$

$$\frac{dV_1}{d\tau} = -(1+\mu_1)\left\{-(s-s^*)^2 - \frac{2(s_h - s_h^{\,*})^2}{\mu_1}\right\}^2 - As^2 i_1 - (1+d_2)i_2 + \phi$$

where,

$$\phi = s^* Asi_1 + \mu_2 i_1 + q\left\{\frac{A}{(\mu_2+\delta)(1+\mu_1)} - 1\right\} i_1$$

$$\le \left(\frac{A}{(1+\mu_1)} + \mu_2 - q(1-R_0)\right) i_1$$

Since, $R_0 < 1$ there exists a constant q such that $A + \mu_2 - q(1-R_0) < 0$. For such value, $\frac{dV_1}{d\tau} < 0$ in the domain B.

Therefore, $\frac{dV_1}{d\tau} < 0$, for $R_0 < 1$

$V_1(s, i_1, i_2, i_3)$ is a Lyapunov function.

Hence, the result is proved. ■

Clearly, if $R_0 < 1$ then no other equilibrium point exists and only disease free equilibrium point exists and it remains globally stable. This means there will be no disease in the population which is most desirable situation. But suppose if this condition is violated then disease free equilibrium point becomes unstable and equilibrium point E_1 comes into the existence.

When $R_0 > 1$, the endemic equilibrium $E_1(\bar{s}, \bar{s}_h, \bar{i}_1, \bar{i}_2)$ exists.

Theorem 3.3 The endemic equilibrium $E_1(\bar{s}, \bar{s}_h, \bar{i}_1, \bar{i}_2)$, if it exists, is locally asymptotically stable

Proof: The characteristic equation of stability matrix at E_1 is obtained as

$$(\lambda - 1)(\lambda - \delta)(\lambda^2 + a_0\lambda + a_1) = 0,$$

where, $a_0 = \frac{A}{\delta + \mu_2} > 0; a_1 = A - (1+\mu_1)(\delta + \mu_2) > 0$

It is observed that the quadratic factor gives two eigenvalues having negative real parts. Hence, the result is proved. ■

Clearly, when $R_0 > 1$, the disease free equilibrium point becomes unstable and the endemic equilibrium $E_1(\bar{s}, \bar{s}_h, \bar{i}_1, \bar{i}_2)$ comes into existence. So the only one locally endemic equilibrium $E_1(\bar{s}, \bar{s}_h, \bar{i}_1, \bar{i}_2)$ is globally stable this is also justified by using Lyapunov second method in the following result.

Theorem 3.4 The locally stable equilibrium point $E_1(\bar{s}, \bar{s}_h, \bar{i}_1, \bar{i}_2)$ is globally asymptotically stable.

Proof: Consider a function

$$V_1(s, s_h, i_1, i_2) = m_1(s - \bar{s})^2 + (s_h - \bar{s}_h)^2 + m_2(i_1 - \bar{i}_1 - \bar{i}_1 \log \frac{i_1}{\bar{i}_1})$$

Note: For arbitrarily chosen positive constants m1 and m2, the function $V_1 > 0$ for all points in the region B except at $E_1(\bar{s}, \bar{i}_1, \bar{i}_2, \bar{i}_3)$ and $V_1(E_1) = 0$

Computing derivative of $V_1(s, i_1, i_2, i_3)$ using system (2.2) gives

$$\frac{dV_1}{d\tau} = m_1(s - s^*)\{1 - (\mu_1 + 1)s - Asi_1\} + (s_h - s_h^*)(\mu_1 s - s_h) + m_2 \frac{(i_1 - \bar{i}_1)}{i_1}\{As - (\mu_2 + \delta)\} i_1$$

$$1 - (\mu_1 + 1)\bar{s} - A\bar{s}\bar{i}_1 = 0, \qquad 1 = (\mu_1 + 1)\bar{s} + A\bar{s}\bar{i}_1$$

Clearly, $A\bar{s} - (\mu_2 + \delta) = 0,$ means $A\bar{s} = (\mu_2 + \delta)$

$$\mu_1 \bar{s} - \bar{s}_h = 0 \qquad \mu_1 \bar{s} - \bar{s}_h = 0$$

$$\frac{dV_1}{d\tau} = m_1(s - \bar{s})\left\{-(\mu_1 + 1)(s - \bar{s}) - A(i_1 - \bar{i}_1)\bar{s} - A(s - \bar{s})\right\} + (s_h - \bar{s}_h)\left\{\mu_1(s - \bar{s}) - (s_h - \bar{s}_h)\right\}$$
$$+ m_2(i_1 - \bar{i}_1)A(s - \bar{s})$$

Simplification gives

$$\frac{dV_1}{d\tau} = -m_1(s - \bar{s})^2\{(1 + \mu_1 + Ai_1)\} - m_1 A(s - \bar{s})(i_1 - \bar{i}_1)\bar{s} + \left\{\mu_1(s_h - \bar{s}_h)(s - \bar{s}) - (s_h - \bar{s}_h)^2\right\}$$
$$+ m_2(i_1 - \bar{i}_1)A(s - \bar{s})$$

Choosing the arbitrary constants as $m_1 = \dfrac{\mu_1^{\,2}}{4(1 + \mu_1)}$ and $m_2 = \dfrac{\mu_1^{\,2}(\delta + \mu_2)}{4(1 + \mu_1)A}$ gives

$$\frac{dV_1}{d\tau} = -\frac{\mu_1^{\,2}}{4}(s - \bar{s})^2 + \mu_1(s_h - \bar{s}_h)(s - \bar{s}) - (s_h - \bar{s}_h)^2 - \frac{\mu_1^{\,2} A(s - \bar{s})^2 i_1}{4(1 + \mu_1)}$$

Or $$\frac{dV_1}{d\tau} = -\left[\frac{\mu_1}{2}(s - \bar{s}) - (s_h - \bar{s}_h)\right]^2 - \frac{\mu_1^{\,2} A(s - \bar{s})^2 i_1}{(1 + \mu_1)} \le 0$$

Therefore, $\dfrac{dV_1}{d\tau} < 0$ in the domain B for $R_0 > 1$

$V_1(s, i_1, i_2, i_3)$ is a Lyapunov function.
Therefore, the result is proved. ■

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