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Approximation Of Generalized Compound Rayleigh Distribution With Bayes For General Entropy

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Abstract
This paper provides the Bayes estimate ofscale parameterα of Generalized Compound Rayleigh distribution under the General Entropyloss function (GELF). Lindley approximation procedure gives the Approximate Bayes estimate of parameter β of Generalized Compound Rayleigh distribution under the GELF. We have done the numerical comparison of the Approximate Bayes estimator of model by using by R-programming.

Keywords: Lindley Approximation, Generalized Compound Rayleigh distribution, General Entropy loss function, Approximate Bayes estimate of β.

1. INTRODUCTION

The Generalized Compound Rayleigh Distribution is a special case of the three-parameter Burr type XII distribution(Dubey(1968)) with probability density function (p.d.f.) with re-parameterized γ as $\frac{1}{\gamma}$

$$f(x;\alpha,\beta,\gamma)=\frac{\alpha}{\gamma}\beta^{\frac{1}{\gamma}}x^{(\alpha-1)}(\beta+x^{\alpha})^{-(\gamma+1)}\,; \qquad x,\alpha,\beta,\gamma>0 \quad (1.1)$$

with Probability Distribution Function

The symmetrical Loss function associates the equal importance to the losses due to overestimation and underestimation with equal magnitudes however in some estimation problems such an assumption may be inappropriate Ferguson.(1967)

In many practical situations, it appears to be more realistic to express the loss in terms of the ration $\frac{\hat{\theta}}{\theta}$. In this case Calabria and Pulcini (1994) points out that a useful asymmetric loss function is the Entropy loss

$$L(\delta)\ \alpha\ [\delta^{P}-p\log_e(\delta)-1 \quad ; \text{Where } \delta=\frac{\hat{\theta}}{\theta} \quad (1.2)$$

And whose minimum occurs at $\hat{\theta}=\theta$ where p>0, a positive error ($\hat{\theta}>\theta$) causes more serious consequences that a negative error and vice-versa. For small |p| value the function is almost symmetric, when both $\hat{\theta}$ and θ are measured in a logarithmic scale and approximately.

$L(\delta) = b[\delta - \log_e(\delta) - 1]; \quad b > 0; \qquad \text{Where } \delta = \frac{\hat{\theta}}{\theta}$ (1.3)

2.The Estimators

Let$x_1 \leq x_2 \leq \ldots \ldots \ldots \leq x_n$be the n failures in complete sample case. Thelikelihood function is given by

$L(\underline{x}|\alpha, \beta, \gamma) = \left(\frac{\alpha}{\gamma}\right)^n U e^{-T/\gamma}$ (2.1)

Where

$T = \sum_{j=1}^{n} \log\left[1 + \frac{x_j{}^{\alpha}}{\beta}\right]andU = \prod_{j=1}^{n} \frac{x_j{}^{\alpha-1}}{\beta + x_j{}^{\alpha}}$

from equation(2.1),the log likelihood function is

$\text{Log } L = n \log\alpha + \frac{n}{\gamma}\log\beta - n\log\gamma + (\alpha - 1)\sum_{j=1}^{n}\log x_j - \left(\frac{1}{\gamma} + 1\right)\sum_{j=1}^{n}\log\left(\beta + x_j{}^{\alpha}\right)$(2.2)

and differentiation of equation(2.3) with respect to α, β and γ yields the maximum likelihood estimators (MLE) of the parameters namely $\hat{\alpha}_{MLE}$, $\hat{\beta}_{MLE}$ and $\hat{\gamma}_{MLE}$ by using Newton Raphson Method.

3.Approximate Bayes Estimator of the unknown parametersβ (Lindley(1980),Solimon(2001)

The Joint prior density of the parameters α, β, γ is given by

$G(\alpha, \beta, \gamma) = g_1(\alpha) g_2(\beta) g_3(\gamma|\beta)$

$\qquad = \frac{c}{\delta\Gamma\xi}\beta^{-\xi}\gamma^{\xi+1}\exp\left[-\left(\frac{\gamma}{\beta} + \frac{\beta}{\delta}\right)\right]$ (3.1)

where

$g_1(\alpha) = c$(3.2)

$g_2(\beta) = \frac{1}{\delta}e^{-\frac{\beta}{\delta}}$(3.3)

$g_3(\gamma) = \frac{1}{\Gamma\xi}\beta^{-\xi}\gamma^{\xi+1}e^{-\frac{\gamma}{\beta}}$(3.4)

The Joint posterior combing the likelihood equation(2.1) and joint prior equation(3.1) is

$$h^*(\alpha, \beta, \gamma|\underline{x}) = \frac{\beta^{-\xi}\gamma^{\xi+1}\exp\left[-\left(\frac{\gamma}{\beta}+\frac{\beta}{\delta}\right)\right].L(\underline{x}|\alpha,\beta,\gamma)}{\int_{\alpha}\int_{\beta}\int_{\gamma}\beta^{-\xi}\gamma^{\xi+1}\exp\left[-\left(\frac{\gamma}{\beta}+\frac{\beta}{\delta}\right)\right].L(\underline{x}|\alpha,\beta,\gamma)d\alpha d\beta d\gamma} \quad (3.5)$$

The Approximate Bayes Estimator is given by

$$\cup(\Theta) = \cup(\alpha, \beta, \gamma) \qquad (3.6)$$

$$\hat{\cup}_{BS} = E(\cup|\underline{x}) = \frac{\int_{\alpha}\int_{\beta}\int_{\gamma}\cup(\alpha,\beta,\gamma)G^*(\alpha,\beta,\gamma)d\alpha d\beta d\gamma}{\int_{\alpha}\int_{\beta}\int_{\gamma}G^*(\alpha,\beta,\gamma)d\alpha d\beta d\gamma} \qquad (3.7)$$

Lindley Approximation Procedure

The Bayes estimators of a function $\mu = \mu(\theta, p)$ of the unknown parameter θ and p under squared error loss is the posterior mean

$$\hat{\mu}_{BS} = E(\mu|\underline{x}) = \frac{\iint \mu(\theta,p)h^*(\theta, p|\underline{x})d\theta dp}{\iint h^*(\theta, p|\underline{x})d\theta dp} \quad (3.7a)$$

The ratio of integrals in equation (3.7a) does not seem to take a closed form so we must consider the Lindley approximation procedure as

$$E(\mu(\theta, p)|\underline{x}) = \frac{\int \mu(\theta).e^{(l(\theta)+\rho(\theta))}d\theta}{\int e^{(l(\theta)+\rho(\theta))}.d\theta} \quad (3.7b)$$

Lindley developed approximate procedure for evaluation of posterior expectation of $\mu(\theta)$. Several other authors have used this technique to obtain Bayes estimators (see Sinha(1986),Sinha and sloan(1988),Soliman(2001)).The posterior expectation of Lindley approximation procedure to evaluate of $\mu(\theta)$ in equation(3.7a and 3.7b) under SELF, where where $\rho(\theta) = \log g(\theta)$, and $g(\theta)$ is an arbitrary function of θ and $l(\theta)$ is the logarithm likelihood function (Lindley (1980)).

The modified form of equation (3.7) is given by

$$E(\cup(\alpha, \beta, \gamma|\underline{x}) = \cup(\Theta) + \frac{1}{2}[A(\cup_1 \sigma_{11} + \cup_2 \sigma_{12} + \cup_3 \sigma_{13}) + B(\cup_1 \sigma_{21} + \cup_2 \sigma_{22} + \cup_3 \sigma_{23}) + P(\cup_1 \sigma_{31} + \cup_2 \sigma_{32} + \cup_3 \sigma_{33})] + (\cup_1 a_1 + \cup_2 a_2 + \cup_3 a_3 + a_4 + a_5) + 0\left(\frac{1}{n^2}\right) \quad (3.8)$$

Above equation is evaluated at MLE = $(\hat{\alpha}, \hat{\beta}, \hat{\gamma})$

where

$a_1 = \rho_1\sigma_{11} + \rho_2\sigma_{12} + \rho_3\sigma_{13}$ (3.9)

$a_2 = \rho_1\sigma_{21} + \rho_2\sigma_{22} + \rho_3\sigma_{23}$ (3.10)

$a_3 = \rho_1\sigma_{31} + \rho_2\sigma_{32} + \rho_3\sigma_{33}$ (3.11)

$a_4 = \cup_{12}\,\sigma_{12} + \cup_{13}\,\sigma_{13} + \cup_{23}\,\sigma_{23}$ (3.12)

$a_5 = \frac{1}{2}(\cup_{11}\,\sigma_{11} + \cup_{22}\,\sigma_{22} + \cup_{33}\,\sigma_{33})$; (3.13)

And

$A = [\sigma_{11}l_{111} + 2\sigma_{12}l_{121} + 2\sigma_{13}l_{131} + 2\sigma_{23}l_{231} + \sigma_{22}l_{221} + \sigma_{33}l_{331}]$(3.14)

$B = [\sigma_{11}l_{112} + 2\sigma_{12}l_{122} + 2\sigma_{13}l_{132} + 2\sigma_{23}l_{232} + \sigma_{22}l_{222} + \sigma_{33}l_{332}]$(3.15)

$P = [\sigma_{11}l_{113} + 2\sigma_{13}l_{133} + 2\sigma_{12}l_{123} + 2\sigma_{23}l_{233} + \sigma_{22}l_{223} + \sigma_{33}l_{333}]$(3.16)

To apply Lindley approximation on equation (3.8), we first obtain

$\sigma_{ij} = [-l_{ijk}]^{-1} i, j, k = 1,2,3$, where $l_{ijk}'s$ are the partial derivatives of α, β, γ of likelihood function.

Likelihood function from equation (3.2) is

$$L = \frac{\alpha^n}{\gamma^n}\beta^{\frac{n}{\gamma}}\prod_{j=1}^n x_j^{\alpha-1}\prod_{j=1}^n\left(\beta + x_j^{\alpha}\right)^{-\left(\frac{1}{\gamma}+1\right)} \quad ; \ (x, \alpha, , \gamma > 0)$$

Now

$$Log\ L = n\ log\alpha - nlog\gamma + \frac{n}{\gamma}log\beta + (\alpha - 1)\sum_{j=1}^n logx_j - \left(\frac{1}{\gamma} + 1\right)\sum_{j=1}^n \frac{x_j^{\alpha}logx_j}{\beta + x_j^{\alpha}} \quad (3.17)$$

The matrix of derivatives is given as

$$[-l_{ijk}] = -\begin{bmatrix} l_{111} & l_{112} & l_{113} \\ l_{221} & l_{222} & l_{223} \\ l_{331} & l_{332} & l_{333} \end{bmatrix} \quad (3.18)$$

$$= \begin{bmatrix} \frac{2n}{\alpha^3} - \left(\frac{1}{\gamma}+1\right)\omega_{133} & ,\left(\frac{1}{\gamma}+1\right)\omega_{123} & -\frac{\beta}{\gamma^2}\omega_{122} \\ -2\left(\frac{1}{\gamma}+1\right)\omega_{113} & ,\frac{2n\gamma}{\gamma\beta^3} - 2\left(\frac{1}{\gamma}+1\right)\delta_{13} & \frac{n}{(\gamma\beta)^2} - \frac{1}{\gamma^2}\delta_{12} \\ \frac{-2}{\gamma^3}\omega_{11} & ,\frac{-2}{\gamma^3}\left(\frac{n}{\gamma} - \delta_{11}\right) & ,-\frac{2n}{\gamma^3} - \frac{6n\,log\beta}{\gamma^4} + \frac{6}{\gamma^4}\delta_{10} \end{bmatrix}$$

$$\left[-l_{ijk}\right] = \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix}$$

$$\left[-l_{ijk}\right]^{-1} = \frac{(Adjoint\ of\ \left[-l_{ijk}\right])'}{D}$$

$$\left[-l_{ijk}\right]^{-1} = \begin{bmatrix} \frac{Y_{11}}{D} & \frac{Y_{12}}{D} & \frac{Y_{13}}{D} \\ \frac{Y_{21}}{D} & \frac{Y_{22}}{D} & \frac{Y_{23}}{D} \\ \frac{Y_{31}}{D} & \frac{Y_{32}}{D} & \frac{Y_{33}}{D} \end{bmatrix}$$

$$\left[-l_{ijk}\right]^{-1} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix}; (3.19)$$

Approximate Bayes Estimator

$\cup\,(\alpha,\beta,\gamma) = \cup$

$\widehat{\cup}_{AB} = E(\cup\,|\underline{x})$

evaluated from equation number and from joint prior density , we have

$G(\alpha,\beta,\gamma) = g(\alpha)g_2(\beta)g_3(\gamma|\beta)$

$= \frac{c}{\delta\Gamma\xi}\beta^{-\xi}\gamma^{\xi-1}exp\left[-\left(\frac{\gamma}{\beta}+\frac{\beta}{\delta}\right)\right];$

$\rho = log\,G = log\,C - log\delta - log\lceil\xi + (\xi-1)log\gamma - \xi log\beta$

$-\left(\frac{\gamma}{\beta}+\frac{\beta}{\delta}\right)$ (3.20)

logG$= constant - \xi log\beta + (\xi-1)log\gamma - \frac{\gamma}{\beta} - \frac{\beta}{\delta}$

$\rho_1 = \frac{\delta\rho}{\delta\alpha} = 0$(3.21)

$\rho_2 = \frac{\delta\rho}{\delta\beta} = \frac{-\xi}{\beta} + \frac{\gamma}{\beta^2} - \frac{1}{\delta}$(3.22)

$\rho_3 = \frac{\delta\rho}{\delta\gamma} = \frac{\xi-1}{\gamma} - \frac{1}{\beta}$(3.23)

Using equation(3.14) to equation(3.19), we have

$A = [\sigma_{11}l_{111} + 2\sigma_{12}l_{121} + 2\sigma_{13}l_{131} + 2\sigma_{23}l_{231} + \sigma_{22}l_{221} + \sigma_{33}l_{331}]$

$$= \sigma_{11}\left(\frac{2n}{\alpha^3} - \left(\frac{1}{\gamma}+1\right)\omega_{133}\right) + 2\sigma_{12}\left(\frac{1}{\gamma}+1\right)\omega_{123} + 2\sigma_{13}\left(\frac{\beta}{\gamma^2}\omega_{122}\right) + 2\sigma_{23}\left(-\frac{\omega_{14}}{\gamma^2}\right) + \sigma_{22}\left(-2\left(\frac{1}{\gamma}+1\right)\omega_{113}\right) + \sigma_{33}\left(-\frac{2}{\gamma^3}\omega_{11}\right)$$

$= \frac{1}{D}\Big[Y_{11}\left(\frac{2n}{\alpha^3} - \left(\frac{1}{\gamma}+1\right)\omega_{133}\right) + 2Y_{12}\left(\frac{1}{\gamma}+1\right)\omega_{123} + 2Y_{13}\frac{\beta}{\gamma^3}\omega_{122} - 2Y_{23}\frac{\omega_{14}}{\gamma^2} - 2Y_{22}\Big(\frac{1}{\gamma}+$
$1\Big)\omega_{113} - \frac{2}{\gamma^3}Y_{33}\omega_{11}\Big]$(3.24)
$B = [\sigma_{11}l_{112} + 2\sigma_{12}l_{122} + 2\sigma_{13}l_{132} + 2\sigma_{23}l_{232} + \sigma_{22}l_{222} + \sigma_{33}l_{332}]$

$$= \sigma_{11}\left(\frac{1}{\gamma}+1\right)\omega_{123} + 2\sigma_{12}\left(-2\left(\frac{1}{\gamma}+1\right)\omega_{113}\right) + 2\sigma_{13}\left(\frac{-\omega_{14}}{\gamma^2}\right) + 2\sigma_{23}\left(\frac{n}{(\gamma\beta^2)} - \frac{1}{\gamma^2}\delta_{12}\right) + \sigma_{22}\left(\frac{n}{(\gamma\beta)^2} - \frac{1}{\gamma^2}\delta_{12}\right) + \sigma_{33}\left(\frac{2}{\gamma^3}\left(\frac{n}{\beta} - \delta_{11}\right)\right)$$

$$= \frac{1}{D}\left[\left(\frac{1}{\gamma}+1\right)\omega_{123}Y_{11} - 4Y_{12}\left(\frac{1}{\gamma}+1\right)\omega_{113} - 2Y_{13}\left(-\frac{\omega_{14}}{\gamma^2}\right) + (Y_{22}+2Y_{23})\left(\frac{n}{(\gamma\beta)^2} - \frac{1}{\gamma^2}\delta_{12}\right) + Y_{33}\left(-\frac{2}{\gamma^3}\left(\frac{n}{\beta} - \delta_{11}\right)\right)\right] \quad (3.25)$$

$P = [\sigma_{11}l_{113} + 2\sigma_{12}l_{123} + 2\sigma_{13}l_{133} + 2\sigma_{23}l_{233} + \sigma_{22}l_{223} + \sigma_{33}l_{333}]$

$$= \sigma_{11}\frac{\beta}{\gamma^2}\omega_{122} + 2\sigma_{12}\left(-\frac{\omega_{14}}{\gamma^2}\right) + 2\sigma_{13}\left(-\frac{2}{\gamma^3}\omega_{11}\right) + 2\sigma_{23}\frac{2}{\gamma^3} + \left(\frac{n}{\beta} - \delta_{11}\right) + \sigma_{22}\left(\frac{n}{(\gamma\beta)^2} - \frac{1}{\gamma^2}\delta_{12}\right) + \sigma_{33}\left(-\frac{2n}{\gamma^3} - \frac{6n\,log\beta}{\gamma^4} + \frac{6}{\gamma^4}\delta_{10}\right)$$

$= \frac{1}{D}\Big[\frac{Y_{11}\beta}{\gamma^2}\omega_{122} - \frac{2Y_{12}\omega_{14}}{\gamma^4} - \frac{4Y_{13}\omega_{11}}{\gamma^3} + \frac{4Y_{23}}{\gamma^3}\left(\frac{n}{\beta} - \delta_{11}\right) + Y_{22}\left(\frac{n}{\gamma^2\beta^2} - \frac{1}{\gamma^2}\delta_{12}\right) + Y_{33}\Big(-\frac{2n}{\gamma^3} -$
$\frac{6n\,log\beta}{\gamma^4} + \frac{6}{\gamma^4}\delta_{10}\Big)\Big]$ (3.26)

Now
$\hat{\cup}_{AB} = E(\cup\,|\underline{x})$

$$E(\cup\,|\underline{x}) = u + (u_1a_1 + u_2a_2 + u_3a_3 + a_4 + a_5) + \frac{1}{2}[A(u_1\sigma_{11} + u_2\sigma_{12} + u_3\sigma_{13}) + B(u_1\sigma_{21} + u_2\sigma_{22} + u_3\sigma_{23}) + P(u_1\sigma_{31} + u_2\sigma_{32} + u_3\sigma_{33})] + 0\left(\frac{1}{n^2}\right)$$

$E(\cup\,|\underline{x}) = U + \varphi_1 + \varphi_2$ (3.27)
where
$\varphi_1 = u_1a_1 + u_2a_2 + u_3a_3 + a_4 + a_5$ (3.28)
$\varphi_2 = \frac{1}{2}[(A\sigma_{11} + B\sigma_{21} + P\sigma_{31}).U_1 + (A\sigma_{12} + B\sigma_{22} + P\sigma_{32}).U_2 + (A\sigma_{13} + B\sigma_{23} +$
$P\sigma_{33})U_3]$(3.29)
evaluated at the MLE$\hat{\cup} = (\hat{\alpha}, \hat{\beta}, \hat{\gamma})$where
$a_1 = \sigma_{11} + \left(\frac{-\xi}{\beta} + \frac{\gamma}{\beta^2} - \frac{1}{\delta}\right)\frac{Y_{12}}{D} + \left(\frac{\xi-1}{\gamma} - \frac{1}{\beta}\right)\frac{Y_{13}}{D}$(3.30)
$a_2 = \sigma_{21} + \left(\frac{-\xi}{\beta} + \frac{\gamma}{\beta} - \frac{1}{\delta}\right)\frac{Y_{22}}{D} + \left(\frac{\xi-1}{\gamma} - \frac{1}{\beta}\right)\frac{Y_{23}}{D}$(3.31)
$a_3 = \sigma_{31} + \left(\frac{-\xi}{\beta} + \frac{\gamma}{\beta^2} - \frac{1}{\delta}\right)\frac{Y_{32}}{D} + \left(\frac{\xi-1}{\gamma} - \frac{1}{\beta}\right)\frac{Y_{33}}{D}$ (3.32)
$a_4 = \frac{Y_{12}}{D}U_{12} + \frac{Y_{13}}{D}U_{13} + \frac{Y_{23}}{D}U_{23}$(3.33)

$a_5 = \frac{1}{2D}(Y_{11}U_{11} + Y_{22}U_{22} + Y_{33}U_{33})$ (3.34)

3.Approximate Bayes Estimator under General Entropy loss function(GELF)

$\widehat{U}_{ABE} = \left[E_h\left(\frac{1}{\theta}\right)\right]^{-1}$(3.35)

Where;

$E_u(\theta|\underline{x}) = \frac{\int_\alpha\int_\beta\int_\gamma \frac{1}{\theta} G^*(\alpha,\beta,\gamma) d\alpha d\beta d\gamma}{\int_\alpha\int_\beta\int_\gamma G^*(\alpha,\beta,\gamma) d\alpha d\beta d\gamma}$(3.36)

The equation(3.36) isevaluated by method of Lindley approximation whose simplified from is equation no. replace θ by∪ (α, β, γ) in equation (3.27)

Special cases–

1.Approximate Bayes estimate of β

$\widehat{U}_{ABE} = \left[E_U\left(\frac{1}{\theta}\right)\right]^{-1}$(3.37)

$U = \frac{1}{\beta}$

$E = \left(\frac{1}{\beta}\right) + \varphi_1 + \varphi_2$

$$U_1 = U_{11} = U_{12} = U_{13} = 0$$

$U_3 = U_{31} = U_{32} = U_{33} = 0$

$U = \frac{1}{\beta}$; $U_2 = \frac{\partial}{\partial\beta}\left(\frac{1}{\beta}\right) = -\frac{1}{\beta^2}$; $U_{22} = \frac{\partial}{\partial\beta}\left(-\frac{1}{\beta^2}\right) = \frac{2}{\beta^3}$; $U_{23} = U_{21} = 0$

$\varphi_1 = -\frac{1}{\beta^2}\left(2\left(\frac{-\xi}{\beta} + \frac{\gamma}{\beta^2} - \frac{1}{\delta}\right)\frac{Y_{22}}{D} + \left(\frac{\xi-1}{\gamma} - \frac{1}{\beta}\right)\frac{Y_{23}}{D}\right)$

$\varphi_2 = -\frac{1}{2\beta^2}(A\sigma_{12} + B\sigma_{22} + P\sigma_{32})$

$E(U|\underline{x}) = \frac{1}{\beta} - \frac{1}{\beta^2}\left[\left(\frac{-\xi}{\beta} + \frac{\gamma}{\beta^2} - \frac{1}{\delta}\right)\frac{Y_{22}}{D} + \left(\frac{\xi-1}{\gamma} - \frac{1}{\beta}\right)\frac{Y_{23}}{D} + \frac{1}{2}(A\sigma_{12} + B\sigma_{22} + P\sigma_{32})\right]$

$E(U|\underline{x}) = \frac{1}{\beta} - \frac{1}{\beta^2}\Delta_5$

$\hat{\beta}_{ABE} = \beta[1 - \beta^{-1}\Delta_5]^{-1}$;at$(\hat{\alpha}_{ML}, \hat{\beta}_{ML}, \hat{\gamma}_{ML})$ (3.38)

Where

$$\Delta_5 = \left(\frac{-\xi}{\beta} + \frac{\gamma}{\beta^2} - \frac{1}{\delta} - \frac{1}{\beta}\right)\frac{Y_{22}}{D} + \left(\frac{\xi - 1}{\gamma} - \frac{1}{\beta}\right)\frac{Y_{23}}{D}$$
$$+ \frac{1}{2}\left[\frac{Y_{12}}{D}\left(\frac{Y_{11}}{D}\left(\frac{2n}{\gamma^3} - \left(\frac{1}{\gamma} + 1\right)\omega_{133}\right) + 2\frac{Y_{12}}{D}\left(\frac{1}{\gamma} + 1\right)\omega_{123} + 2\frac{Y_{13}}{D}\frac{\beta}{\gamma^2}\omega_{122}\right.\right.$$
$$\left.\left. - 2\frac{Y_{23}}{D}\frac{\omega_{14}}{\gamma^2} - 2\frac{Y_{22}}{D}\left(\frac{1}{\gamma} + 1\right)\omega_{113} - \frac{2}{\gamma^2}\frac{Y_{33}}{D}\omega_{11}\right)\right]$$
$$+ \frac{1}{2}\frac{Y_{23}}{D}\left[\frac{Y_{11}}{D}\left(\frac{1}{\gamma} + 1\right)\omega_{123} - \frac{4Y_{12}}{D}\left(\frac{1}{\gamma} + 1\right)\omega_{113} - \frac{2Y_{13}}{D}\frac{\omega_{14}}{\gamma^2}\right.$$
$$\left. + \left(\frac{Y_{22} + 2Y_{23}}{D}\right).\left(\frac{n}{\gamma^2\beta^2} - \frac{1}{\gamma^2}\delta_{12}\right) + \frac{Y_{33}}{D}\left(\frac{2}{\gamma^3}\left(\frac{n}{\beta} - \delta_{11}\right)\right)\right]$$
$$+ \frac{1}{2}\frac{Y_{31}}{D}\left[\frac{Y_{11}}{D}\frac{\beta}{\gamma^2}\omega_{122} - \frac{2Y_{12}}{D}\frac{\omega_{14}}{\gamma^4} - 4\frac{Y_{13}}{D}\omega_{11} + \frac{4Y_{23}}{D\gamma^3}\left(\frac{n}{\beta} - \delta_{11}\right)\right.$$
$$\left. + \frac{Y_{22}}{D}\left(\frac{n}{\gamma^2\beta^2} - \frac{\delta_{12}}{\gamma^2}\right) + \frac{Y_{33}}{D}\left(\frac{-2n}{\gamma^3} - \frac{6n\log\beta}{\gamma^4} + \frac{6\delta_{10}}{\gamma^4}\right)\right]$$

(3.39)

Simulations and Numerical Comparison

The simulations and numerical calculations are done by using R Language programming and results are presented in form of tables in table (1).

1. The Random variable of Generalized Compound Rayleigh Distribution is generated by R-Language programming by taking the values of the parameters α, β, γ, taken as $\alpha = 0.8$, $\beta = 0.6$ and $\gamma = 0.9$ in the equations[(3.2)-(3.4)] and equation(1.1).

2. Taking the different sizes of samples n=10(10)80 with complete sample, MLE's, the Approximate Bayes estimator ,and their respective MSE's (in parenthesis) are obtained by repeating the steps 500 times, are presented in the table from (1), and parameters of prior distribution a =3 and b =4.

3. Table (1) also presents the MLE of parameter of β and Approximate Bayes estimator ofβunder GELF (for α unknown) and their respective MSE's.

Table (1) Mean and MSE'S of β

($\alpha = 0.8$, $\beta = 0.6$ and $\gamma = 0.9$)

n	**10**	**20**	**30**	**40**	**50**	**60**	**70**	**80**
$\hat{\beta}_{ML}$	0.7900455	0.881254	0.881254	0.8987499	0.8996584	0.9990011	0.9988845	1.0107432
	[0.024412]	**[0.025874]**	**[0.098521]**	**[0.002354]**	**[0.004577]**	**[0.004663]**	**[0.001125]**	**[0.001021]**
$\hat{\beta}_{ABE}$	0.8845165	0.8845165	0.8787745	0.8874512	0.8889547	0.9700411	0.9859874	1.0357951
	[0.004521]	**[0.003208]**	**[0.022335]**	**[0.012547]**	**[0.001784]**	**[0.005412]**	**[0.032147]**	**[0.032587]**

References:

1. Burr, I.W. (1942): “Cumulative Frequency Distribution", Annals of Mathematical Statistics, Vol. 13, pp 215-232.
2. Calabria, R. and Pulcini, G. (1994): “An Engineering Approach to Bayes Estimation for the Weibull Distribution”, Microelectronics Reliability, Vol. 34 No-5, pp. 789-802.
3. Dubey(1968): A compound weibull distribution; Wiley online Library
4. Ferguson, Thomas S. (1967): “Mathematical Statistics: A Decision Theoretic Approach”. Academic press, New York.
5. Harris, E. and Shakarki, G. (1979): “Use of The Population Distribution to Improve Estimation of Individual Mean in Epidemiological Studies”, Journal of Chronicle Disease, Vol. 32, pp. 233–243.
6. Lindley, D.V. (1980): “Approximate Bayesian Method”. Trab. Esta., Vol. 31, pp 223-237.
7. Soliman, A.A. (2001): “Linex and Quadratic Approximate Bayesian Estimators Applied to Pareto Model”, Communication in Statistics - Simulation Vol. 30, No.1, pp 47-62.