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Approximate Bayes For Generalized Compound Rayleigh Distribution With Quadratic Loss

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Abstract
In this paper Bayes estimate of Location parameter of Generalized Compound Rayleigh distribution under the Quadraticloss function (QLF). With Lindley approximation procedure we have obtained the Approximate Bayes estimate of Location parameter of Generalized Compound Rayleigh distribution under the QLF. We have done the numerical comparison of the Approximate Bayes estimator of model by using by R-programming.

Keywords: Lindley Approximation, Generalized Compound Rayleigh distribution, Quadratic loss function, Approximate Bayes estimate.

1. INTRODUCTION

The Generalized Compound Rayleigh Distribution is a special case of the three-parameter Burr type XII distributionDubey(1968) with probability density function (p.d.f.)

$$f(x;\alpha,\beta,\gamma) = \frac{\alpha}{\gamma}\beta^{\frac{1}{\gamma}}x^{(\alpha-1)}(\beta + x^{\alpha})^{-(\gamma+1)}\ ; \qquad x,\alpha,\beta,\gamma > 0 \quad (1.1)$$

with Probability Distribution Function

$$F(x) = 1 - (1 - \beta x^{\alpha})^{-\frac{1}{\gamma}}\ ; \qquad x,\alpha,\beta,\gamma > 0 \quad (1.2)$$

Quadratic Loss

A function defined as

$$L(\hat{\theta},\theta) = k\left(\hat{\theta} - \theta\right)^2 \qquad (1.3)$$

is called quadratic loss function such a loss function is widely used in most estimation problems. If k is a function of the loss function is called the weighed quadratic loss function. If k=1, we have

$$L(\hat{\theta},\theta) = \left(\hat{\theta} - \theta\right)^2 \qquad (1.4)$$

known as the Quadratic loss function (QLF). Under QLF, Bayes estimator is the posterior mean.

2.TheEstimators

Let$x_1 \le x_2 \le \ldots\ldots\ldots \le x_n$be the n failures in complete sample case. Thelikelihood function is given by

$$L(\underline{x}|\alpha,\beta,\gamma) = \left(\frac{\alpha}{\gamma}\right)^n U e^{-T/\gamma} \quad (2.1)$$

where

$$T = \sum_{j=1}^{n} \log\left[1 + \frac{x_j{}^{\alpha}}{\beta}\right] \text{and} U = \prod_{j=1}^{n} \frac{x_j{}^{\alpha-1}}{\beta + x_j{}^{\alpha}}$$

from equation(2.1),the log likelihood function is

$$\text{Log } L = n\log\alpha + \frac{n}{\gamma}\log\beta - n\log\gamma + (\alpha-1)\sum_{j=1}^{n}\log x_j - \left(\frac{1}{\gamma} + 1\right)\sum_{j=1}^{n}\log(\beta + x_j{}^{\alpha}) \quad (2.2)$$

differentiating of equation(2.2) with respect to α, β and γ yields respectively and equating to zero yields the maximum likelihood estimators (MLE) of the parameters namely $\hat{\alpha}_{MLE}$, $\hat{\beta}_{MLE}$ and$\hat{\gamma}_{MLE}$ .Applying the Newton-Raphson method $\hat{\alpha}_{MLE}$ and $\hat{\beta}_{MLE}$ can be derived and then from them $\hat{\gamma}_{MLE}$ can be obtained.

3.Approximate BayesEstimator of the unknownparametersα (Lindley(1980),Solimon(2001)

The Joint prior density of the parameters α, β, γ is given by

$$G(\alpha,\beta,\gamma) = g_1(\alpha)g_2(\beta)g_3(\gamma|\beta)$$
$$= \frac{c}{\delta\Gamma\xi}\beta^{-\xi}\gamma^{\xi+1}\exp\left[-\left(\frac{\gamma}{\beta} + \frac{\beta}{\delta}\right)\right] \quad (3.1)$$

where

$$g_1(\alpha) = c \quad (3.2)$$

$$g_2(\beta) = \frac{1}{\delta}e^{-\frac{\beta}{\delta}} \quad (3.3)$$

$$g_3(\gamma) = \frac{1}{\Gamma\xi}\beta^{-\xi}\gamma^{\xi+1}e^{-\frac{\gamma}{\beta}} \quad (3.4)$$

The Joint posterior combing the likelihood equation(2.1) and joint prior equation(3.1) is

$$h^*(\alpha,\beta,\gamma|\underline{x}) = \frac{\beta^{-\xi}\gamma^{\xi+1}\exp\left[-\left(\frac{\gamma}{\beta}+\frac{\beta}{\delta}\right)\right].L(\underline{x}|\alpha,\beta,\gamma)}{\int_\alpha\int_\beta\int_\gamma \beta^{-\xi}\gamma^{\xi+1}\exp\left[-\left(\frac{\gamma}{\beta}+\frac{\beta}{\delta}\right)\right].L(\underline{x}|\alpha,\beta,\gamma)d\alpha d\beta d\gamma} \quad (3.5)$$

The Approximate Bayes Estimator is given by

$$\cup(\Theta) = \cup(\alpha,\beta,\gamma) \quad (3.6)$$

$$\hat{\cup}_{BS} = E(\cup|\underline{x}) = \frac{\int_\alpha\int_\beta\int_\gamma \cup(\alpha,\beta,\gamma)G^*(\alpha,\beta,\gamma)d\alpha d\beta d\gamma}{\int_\alpha\int_\beta\int_\gamma G^*(\alpha,\beta,\gamma)d\alpha d\beta d\gamma} \quad (3.7)$$

Lindley Approximation Procedure

The Bayes estimators of a function $\mu = \mu(\theta, p)$ of the unknown parameter θ and p under Quadratic loss is the posterior mean

$$\hat{\mu}_{BS} = E(\mu|\underline{x}) = \frac{\iint \mu(\theta,p)h^*(\theta,p|\underline{x})d\theta dp}{\iint h^*(\theta,p|\underline{x})d\theta dp} \quad (3.7a)$$

The ratio of integrals in equation (3.7a) does not seem to take a closed form so we must consider the Lindley approximation procedure as

$E(\mu(\theta,p)|\underline{x}) = \frac{\int \mu(\theta).e^{(l(\theta)+\rho(\theta))}d\theta}{\int e^{(l(\theta)+\rho(\theta))}.d\theta}$ (3.7b)

Lindley developed approximate procedure for evaluation of posterior expectation of$\mu(\theta)$. Several other authors have used this technique to obtain Bayes estimators (see Sinha(1986),Sinha and sloan(1988),Soliman(2001)).The posterior expectation of Lindley approximation procedure to evaluate of $\mu(\theta)$ in equation(3.7a and 3.7b) under SELF, where where$\rho(\theta) = \log g(\theta)$, and $g(\theta)$ is an arbitrary function of θ and $l(\theta)$ is the logarithm likelihood function (Lindley (1980)).

The modified form of equation (3.7) is given by

$E(\cup(\alpha,\beta,\gamma|\underline{x}) = \cup(\Theta) + \frac{1}{2}[A(\cup_1\sigma_{11} + \cup_2\sigma_{12} + \cup_3\sigma_{13}) + B(\cup_1\sigma_{21} + \cup_2\sigma_{22} + \cup_3\sigma_{23}) + P(\cup_1\sigma_{31} + \cup_2\sigma_{32} + \cup_3\sigma_{33})] + (\cup_1 a_1 + \cup_2 a_2 + \cup_3 a_3 + a_4 + a_5) + 0\left(\frac{1}{n^2}\right)$ (3.8)Above equation is evaluated at MLE = $(\hat{\alpha},\hat{\beta},\hat{\gamma})$

where

$a_1 = \rho_1\sigma_{11} + \rho_2\sigma_{12} + \rho_3\sigma_{13}$ (3.9)

$a_2 = \rho_1\sigma_{21} + \rho_2\sigma_{22} + \rho_3\sigma_{23}$ (3.10)

$a_3 = \rho_1\sigma_{31} + \rho_2\sigma_{32} + \rho_3\sigma_{33}$ (3.11)

$a_4 = \cup_{12}\sigma_{12} + \cup_{13}\sigma_{13} + \cup_{23}\sigma_{23}$ (3.12)

$a_5 = \frac{1}{2}(\cup_{11}\sigma_{11} + \cup_{22}\sigma_{22} + \cup_{33}\sigma_{33})$; (3.13)

And

$A = [\sigma_{11}l_{111} + 2\sigma_{12}l_{121} + 2\sigma_{13}l_{131} + 2\sigma_{23}l_{231} + \sigma_{22}l_{221} + \sigma_{33}l_{331}]$(3.14)

$B = [\sigma_{11}l_{112} + 2\sigma_{12}l_{122} + 2\sigma_{13}l_{132} + 2\sigma_{23}l_{232} + \sigma_{22}l_{222} + \sigma_{33}l_{332}]$(3.15)

$P = [\sigma_{11}l_{113} + 2\sigma_{13}l_{133} + 2\sigma_{12}l_{123} + 2\sigma_{23}l_{233} + \sigma_{22}l_{223} + \sigma_{33}l_{333}]$(3.16)

To apply Lindley approximation on equation (3.8), we first obtain

$\sigma_{ij} = [-l_{ijk}]^{-1}$ $i,j,k = 1,2,3$;where l_{ijk}'s are the partial derivatives of α,β,γ of likelihood function.

Likelihood function from equation (3.2) is

$L = \frac{\alpha^n}{\gamma^n}\beta^{\frac{n}{\gamma}}\prod_{j=1}^{n}x_j^{\alpha-1}\prod_{j=1}^{n}(\beta + x_j^{\alpha})^{-\left(\frac{1}{\gamma}+1\right)}$; $(x,\alpha,,\gamma > 0)$

Now

$\text{Log } L = n\log\alpha - n\log\gamma + \frac{n}{\gamma}\log\beta + (\alpha-1)\sum_{j=1}^{n}\log x_j - \left(\frac{1}{\gamma}+1\right)\sum_{j=1}^{n}\frac{x_j^{\alpha}\log x_j}{\beta + x_j^{\alpha}}$(3.17) Now differentiating log likelihoodfunction with respect to α,β,γ

$l_{331} = -\frac{2}{\gamma^3}\omega_{11}$ where $\omega_{11} = \sum_{j=1}^{n}\frac{x_j^{\alpha}\log x_j}{(\beta + x_j^{\alpha})}$ (3.19)

$l_{331} = l_{313}$

Again differentiatingl_{33} with respect to β

$l_{332} = \frac{\partial}{\partial\gamma}\left(\frac{\partial^2 L}{\partial\gamma\,\partial\beta}\right) = \frac{2}{\gamma^3}\left(\frac{n}{\beta} - \delta_{11}\right)$ (3.20)

$l_{332} = l_{323}$

Again differentiatingl_{23} with respect to α

$l_{231} - \frac{\omega_{14}}{\gamma^2}$ (3.21)

$l_{231} = l_{213}$

Again differentiating l_{12} with respect to γ

$$l_{123} = -\frac{\omega_{14}}{\gamma^2} \qquad (3.22)$$

$$l_{123} = l_{132}$$

Again differentiating l_{13} with respect to γ

$$l_{133} = \frac{-2}{\gamma^2}\sum_{j=1}^{n}\frac{x_j{}^{\alpha}\log x_j}{(\beta+x_j{}^{\alpha})} = -\frac{2}{\gamma^2}\omega_{11} \quad (3.23)$$

Again differentiating l_{12} with respect to β

$$l_{122} = -2\left(\frac{1}{\gamma}+1\right)\omega_{113} \text{ where } \omega_{113} = \sum_{j=1}^{n}\frac{x_j{}^{\alpha}\log x_j}{(\beta+x_j{}^{\alpha})^3} \quad (3.24)$$

Again differentiating l_{23} with respect to γ

$$l_{233} = \frac{2n}{\beta\gamma^3} - \frac{2}{\gamma^3}\sum_{j=1}^{n}\frac{1}{\beta+x_j{}^{\alpha}} = \frac{2}{\gamma^3}\left(\frac{n}{\beta} - \delta_{11}\right) \qquad (3.25)$$

The matrix of derivatives is given as

$$[-l_{ijk}] = -\begin{bmatrix} l_{111} & l_{112} & l_{113} \\ l_{221} & l_{222} & l_{223} \\ l_{331} & l_{332} & l_{333} \end{bmatrix} (3.26)$$

$$= \begin{bmatrix} \frac{2n}{\alpha^3} - \left(\frac{1}{\gamma}+1\right)\omega_{133} & ,\left(\frac{1}{\gamma}+1\right)\omega_{123} & -\frac{\beta}{\gamma^2}\omega_{122} \\ -2\left(\frac{1}{\gamma}+1\right)\omega_{113} & ,\frac{2n\gamma}{\gamma\beta^3} - 2\left(\frac{1}{\gamma}+1\right)\delta_{13} & \frac{n}{(\gamma\beta)^2} - \frac{1}{\gamma^2}\delta_{12} \\ \frac{-2}{\gamma^3}\omega_{11} & ,\frac{-2}{\gamma^3}\left(\frac{n}{\gamma} - \delta_{11}\right) & ,-\frac{2n}{\gamma^3} - \frac{6n\log\beta}{\gamma^4} + \frac{6}{\gamma^4}\delta_{10} \end{bmatrix}$$

$$[-l_{ijk}] = \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix}$$

Determinant of $[-l_{ijk}]$, $D = \{M_{11}[M_{22}M_{33} - M_{23}M_{32}] - M_{12}[M_{21}M_{33} - M_{31}M_{23}] + M_{13}[M_{21}M_{32} - M_{22}M_{33}]\}$ (3.27)

$$[-l_{ijk}]^{-1} = \frac{(\text{Adjoint of } [-l_{ijk}])'}{D}$$

$$[-l_{ijk}]^{-1} = \begin{bmatrix} \frac{Y_{11}}{D} & \frac{Y_{12}}{D} & \frac{Y_{13}}{D} \\ \frac{Y_{21}}{D} & \frac{Y_{22}}{D} & \frac{Y_{23}}{D} \\ \frac{Y_{31}}{D} & \frac{Y_{32}}{D} & \frac{Y_{33}}{D} \end{bmatrix}$$

$$[-l_{ijk}]^{-1} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix}; (3.28)$$

Approximate Bayes Estimator

$\cup(\alpha,\beta,\gamma) = \cup$

$\hat{\cup}_{AB} = E(\cup\,|\underline{x})$

evaluated from equation number and from joint prior density , we have

$G(\alpha,\beta,\gamma) = g(\alpha)g_2(\beta)g_3(\gamma|\beta)$

$$= \frac{c}{\delta\Gamma\xi}\beta^{-\xi}\gamma^{\xi-1}\exp\left[-\left(\frac{\gamma}{\beta}+\frac{\beta}{\delta}\right)\right];$$

$\rho = \log G = \log C - \log\delta - \log\Gamma\xi + (\xi-1)\log\gamma - \xi\log\beta$

$-\left(\frac{\gamma}{\beta}+\frac{\beta}{\delta}\right)$ (3.29)

LogG= constant $- \xi\log\beta + (\xi-1)\log\gamma - \frac{\gamma}{\beta} - \frac{\beta}{\delta}$

$\rho_1 = \frac{\delta\rho}{\delta\alpha} = 0$(3.30)

$\rho_2 = \frac{\delta\rho}{\delta\beta} = \frac{-\xi}{\beta} + \frac{\gamma}{\beta^2} - \frac{1}{\delta}$(3.31)

$\rho_3 = \frac{\delta\rho}{\delta\gamma} = \frac{\xi-1}{\gamma} - \frac{1}{\beta}$(3.32)

Using equation(3.14) to (3.28), we have

$A = [\sigma_{11}l_{111} + 2\sigma_{12}l_{121} + 2\sigma_{13}l_{131} + 2\sigma_{23}l_{231} + \sigma_{22}l_{221} + \sigma_{33}l_{331}]$

$$= \sigma_{11}\left(\frac{2n}{\alpha^3} - \left(\frac{1}{\gamma}+1\right)\omega_{133}\right) + 2\sigma_{12}\left(\frac{1}{\gamma}+1\right)\omega_{123} + 2\sigma_{13}\left(\frac{\beta}{\gamma^2}\omega_{122}\right) + 2\sigma_{23}\left(-\frac{\omega_{14}}{\gamma^2}\right) + \sigma_{22}\left(-2\left(\frac{1}{\gamma}+1\right)\omega_{113}\right) + \sigma_{33}\left(-\frac{2}{\gamma^3}\omega_{11}\right)$$

$$= \frac{1}{D}\left[Y_{11}\left(\frac{2n}{\alpha^3} - \left(\frac{1}{\gamma}+1\right)\omega_{133}\right) + 2Y_{12}\left(\frac{1}{\gamma}+1\right)\omega_{123} + 2Y_{13}\frac{\beta}{\gamma^3}\omega_{122} - 2Y_{23}\frac{\omega_{14}}{\gamma^2} - 2Y_{22}\left(\frac{1}{\gamma}+1\right)\omega_{113} - \frac{2}{\gamma^3}Y_{33}\omega_{11}\right] \quad (3.33)$$

$B = [\sigma_{11}l_{112} + 2\sigma_{12}l_{122} + 2\sigma_{13}l_{132} + 2\sigma_{23}l_{232} + \sigma_{22}l_{222} + \sigma_{33}l_{332}]$

$$= \sigma_{11}\left(\frac{1}{\gamma}+1\right)\omega_{123} + 2\sigma_{12}\left(-2\left(\frac{1}{\gamma}+1\right)\omega_{113}\right) + 2\sigma_{13}\left(\frac{-\omega_{14}}{\gamma^2}\right) + 2\sigma_{23}\left(\frac{n}{(\gamma\beta^2)} - \frac{1}{\gamma^2}\delta_{12}\right) + \sigma_{22}\left(\frac{n}{(\gamma\beta)^2} - \frac{1}{\gamma^2}\delta_{12}\right) + \sigma_{33}\left(\frac{2}{\gamma^3}\left(\frac{n}{\beta} - \delta_{11}\right)\right)$$

$$= \frac{1}{D}\left[\left(\frac{1}{\gamma}+1\right)\omega_{123}Y_{11} - 4Y_{12}\left(\frac{1}{\gamma}+1\right)\omega_{113} - 2Y_{13}\left(-\frac{\omega_{14}}{\gamma^2}\right) + (Y_{22}+2Y_{23})\left(\frac{n}{(\gamma\beta)^2} - \frac{1}{\gamma^2}\delta_{12}\right) + Y_{33}\left(-\frac{2}{\gamma^3}\left(\frac{n}{\beta} - \delta_{11}\right)\right)\right] \quad (3.34)$$

$P = [\sigma_{11}l_{113} + 2\sigma_{12}l_{123} + 2\sigma_{13}l_{133} + 2\sigma_{23}l_{233} + \sigma_{22}l_{223} + \sigma_{33}l_{333}]$

$$= \sigma_{11}\frac{\beta}{\gamma^2}\omega_{122} + 2\sigma_{12}\left(-\frac{\omega_{14}}{\gamma^2}\right) + 2\sigma_{13}\left(-\frac{2}{\gamma^3}\omega_{11}\right) + 2\sigma_{23}\frac{2}{\gamma^3} + \left(\frac{n}{\beta} - \delta_{11}\right) + \sigma_{22}\left(\frac{n}{(\gamma\beta)^2} - \frac{1}{\gamma^2}\delta_{12}\right) + \sigma_{33}\left(-\frac{2n}{\gamma^3} - \frac{6n\log\beta}{\gamma^4} + \frac{6}{\gamma^4}\delta_{10}\right)$$

$$= \frac{1}{D}\left[\frac{Y_{11}\beta}{\gamma^2}\omega_{122} - \frac{2Y_{12}\omega_{14}}{\gamma^4} - \frac{4Y_{13}\omega_{11}}{\gamma^3} + \frac{4Y_{23}}{\gamma^3}\left(\frac{n}{\beta} - \delta_{11}\right) + Y_{22}\left(\frac{n}{\gamma^2\beta^2} - \frac{1}{\gamma^2}\delta_{12}\right) + Y_{33}\left(-\frac{2n}{\gamma^3} - \frac{6n\log\beta}{\gamma^4} + \frac{6}{\gamma^4}\delta_{10}\right)\right] \quad (3.35)$$

Now

$\hat{U}_{AB} = E(U|\underline{x})$

$$E(\cup|\underline{x}) = u + (u_1a_1 + u_2a_2 + u_3a_3 + a_{4+}a_5) + \frac{1}{2}[A(u_1\sigma_{11} + u_2\sigma_{12} + u_3\sigma_{13}) + B(u_1\sigma_{21} + u_2\sigma_{22} + u_3\sigma_{23}) + P(u_1\sigma_{31} + u_2\sigma_{32} + u_3\sigma_{33})] + 0\left(\frac{1}{n^2}\right)$$

$$E(\cup|\underline{x}) = U + \varphi_1 + \varphi_2 \qquad (3.36)$$

where

$$\varphi_1 = u_1a_1 + u_2a_2 + u_3a_3 + a_4 + a_5 \; (3.37)$$

$$\varphi_2 = \frac{1}{2}[(A\sigma_{11} + B\sigma_{21} + P\sigma_{31}).U_1 + (A\sigma_{12} + B\sigma_{22} + P\sigma_{32}).U_2 + (A\sigma_{13} + B\sigma_{23} + P\sigma_{33})U_3] (3.38)$$

evaluated at the MLE$\widehat{U} = (\hat{\alpha}, \hat{\beta}, \hat{\gamma})$where

$$a_1 = \rho_1\sigma_{11} + \rho_2\sigma_{12} + \rho_3\sigma_{13}$$
$$= 0.\sigma_{11} + \left(\frac{-\xi}{\beta} + \frac{\gamma}{\beta^2} - \frac{1}{\delta}\right)\frac{Y_{12}}{D} + \left(\frac{\xi-1}{\gamma} - \frac{1}{\beta}\right)\frac{Y_{13}}{D} (3.39)$$

$$a_2 = \rho_1\sigma_{21} + \rho_2\sigma_{22} + \rho_3\sigma_{23}$$
$$= 0.\sigma_{21} + \left(\frac{-\xi}{\beta} + \frac{\gamma}{\beta} - \frac{1}{\delta}\right)\frac{Y_{22}}{D} + \left(\frac{\xi-1}{\gamma} - \frac{1}{\beta}\right)\frac{Y_{23}}{D} (3.40)$$

$$a_3 = \rho_1\sigma_{31} + \rho_2\sigma_{32} + \rho_3\sigma_{33}$$
$$= 0.\sigma_{31} + \left(\frac{-\xi}{\beta} + \frac{\gamma}{\beta^2} - \frac{1}{\delta}\right)\frac{Y_{32}}{D} + \left(\frac{\xi-1}{\gamma} - \frac{1}{\beta}\right)\frac{Y_{33}}{D} \; (3.41)$$

$$a_4 = U_{12}\sigma_{12} + U_{13}\sigma_{13} + U_{23}\sigma_{23}$$
$$= \frac{Y_{12}}{D}U_{12} + \frac{Y_{13}}{D}U_{13} + \frac{Y_{23}}{D}U_{23} (3.42)$$

$$a_5 = \frac{1}{2}(U_{11}\sigma_{11} + U_{22}\sigma_{22} + U_{33}\sigma_{33})$$

$$a_5 = \frac{1}{2D}(Y_{11}U_{11} + Y_{22}U_{22} + Y_{33}U_{33}) \; (3.43)$$

3.Approximate Bayes Estimate Under Quadratic Loss Function

$\widehat{U}_{ABQ} = E(\theta) = \theta$

where

$$E_u(\theta|\underline{x}) = \frac{\int_\alpha\int_\beta\int_\gamma \theta G^*(\alpha,\beta,\gamma)\,\partial\alpha\,\partial\beta\,\partial\gamma}{\int_\alpha\int_\beta\int_\gamma G^*(\alpha,\beta,\gamma)\,\partial\alpha\,\partial\beta\,\partial\gamma} \qquad (3.44)$$

The above equation is evaluated by method of Lindley approximation, whose simplified is byreplaceingθ by $\cup(\alpha, \beta, \gamma)$ in equation (3.43)

Special cases:–

$\cup(\alpha, \beta, \gamma) = \cup$

1. Approximate Bayes estimate of α

$\cup(\alpha, \beta, \gamma) = \cup = \alpha$

$\cup = \alpha, \qquad \cup_1 = \frac{\partial U}{\partial \alpha} = 1, \quad \cup_{11} = \cup_{12} = \cup_{13} = 0$

$\cup_2 = \cup_{21} = \cup_{22} = \cup_{23} = 0$

$\cup_3 = \cup_{31} = \cup_{32} = \cup_{33} = 0$

$$E_u(\cup\,|\underline{x}) = \alpha + \varphi_1 + \varphi_2 \qquad (3.45)$$

Where $\varphi_1 = \cup_1 a_1 + \cup_2 a_2 + \cup_3 a_3 + a_4 + a_5$

$$\varphi_1 = \left(\frac{-\xi}{\beta} + \frac{\gamma}{\beta^2} - \frac{1}{\delta}\right)\frac{Y_{11}}{D} + \left(\frac{\xi-1}{\gamma} - \frac{1}{\beta}\right)\frac{Y_{13}}{D} + 0.a_2 + 0.a_3 + 0 + 0$$

$$\varphi_1 = \left(\frac{-\xi}{\beta} + \frac{\gamma}{\beta^2} - \frac{1}{\delta}\right)\frac{Y_{11}}{D} + \left(\frac{\xi-1}{\gamma} - \frac{-1}{\beta}\right)\frac{Y_{13}}{D}$$

and $\varphi_2 = \frac{1}{2}[(A\sigma_{11} + B\sigma_{21} + P\sigma_{31})\cup_1 + (A\sigma_{12} + B\sigma_{22} + P\sigma_{32})\cup_2 + (A\sigma_{13} + B\sigma_{23} + P\sigma_{33})\cup_3]$

$$\varphi_2 = \frac{1}{2}(A\sigma_{11} + B\sigma_{21} + P\sigma_{31})$$

$$E_u(\cup\,|\underline{x}) = \alpha + \left(\frac{-\xi}{\beta} + \frac{\gamma}{\beta^2} - \frac{1}{\delta}\right)\frac{Y_{11}}{D} + \frac{1}{2}(A\sigma_{11} + B\sigma_{21} + P\sigma_{31}) + \left(\frac{\xi-1}{\gamma} - \frac{1}{\beta}\right)\frac{Y_{13}}{D}$$

$\hat{\alpha}_{ABQ} = \alpha + \Delta_1$; at $(\hat{\alpha}_{ML}, \hat{\beta}_{ML}, \hat{\gamma}_{ML})$ (3.46)

Where $\Delta_1 = \frac{Y_{12}}{D}\Bigg[\left(\frac{-\xi}{\beta} + \frac{\gamma}{\beta^2} - \frac{1}{\delta}\right) + \frac{Y_{11}}{D}\Big[Y_{11}\left(\frac{2n}{\alpha^3} - \left(\frac{1}{\gamma}+1\right)\omega_{133}\right) + Y_{12}\left(\frac{1}{\gamma}+1\right)\omega_{113} + Y_{13}\frac{\beta}{\gamma^2}\omega_{122} - \frac{Y_{23}\omega_{14}}{\gamma^2} - Y_{22}\left(\frac{1}{\gamma}+1\right)\omega_{113} - \frac{1}{\gamma^3}Y_{33}\omega_{11}\Big] + \frac{Y_{21}}{2D^2}\Big[\left(\frac{1}{\gamma}+1\right)\omega_{123}Y_{11} - 4Y_{12}\left(\frac{1}{\gamma}+1\right)\omega_{113} - 2Y_{13}\frac{\omega_{14}}{\gamma^2} + (Y_{22}+2Y_{23})\left(\frac{n}{\gamma^2\beta^2} - \frac{1}{\gamma^2}\delta_{12}\right) + Y_{33}\left(\frac{2}{\gamma^3}\left(\frac{n}{\beta} - \delta_{11}\right)\right)\Big] + \frac{Y_{31}}{2D^2}\Big[Y_{11}\frac{\beta}{\gamma^2}\omega_{122} - 2\frac{Y_{12}\omega_{14}}{\gamma^4} - \frac{4Y_{13}\omega_{11}}{\gamma^3} + 4Y_{23}\left(\frac{n}{\beta} - \delta_{11}\right) + Y_{22}\left(\frac{n}{\gamma^2\beta^2} + \frac{\delta_{12}}{\gamma^2}\right) + Y_{33}\left(-\frac{2n}{\gamma^3} - \frac{6n\log\beta}{\gamma^4} + \frac{6}{\gamma^4}\delta_{10}\right) + \left(\frac{\xi-1}{\gamma} - \frac{1}{\beta}\right)\frac{Y_{13}}{D}\Big]\Bigg]$

Simulations and Numerical Comparison

The simulations and numerical calculations are done by using R Language programming and results are presented in form of tables in table (1).

1. The Random variable of Generalized Compound Rayleigh Distribution is generated by R-Language programming by taking the values of the parameters α, β, γ, taken as $\alpha = 1.2$, $\beta = 0.7$ and $\gamma = 1.1$ in the equations[(3.2)-(3.4)] and equation(1.1).

2. Taking the different sizes of samples n=10(10)80 with complete sample, MLE's, the Approximate Bayes estimator ,and their respective MSE's (in parenthesis) are obtained by repeating the steps 500 times, are presented in the table from (1), and parameters of prior distribution a =3 and b =4.

3. Table (1) also presents the MLE of parameter of $\alpha$and Approximate Bayes estimator under QLF (for α unknown) and their respective MSE's.

Table (1) Mean and MSE'S of α

($\alpha = 1.2$, $\beta = 0.7$ and $\gamma = 1.1$)

n	10	20	30	40	50	60	70	80

$\hat{\alpha}_{ML}$	0.7546328	0.7589642	0.8125469	0.8645712	0.9451287	0.9998756	1.3000043	1.9000032
MSE	**[0.152134]**	**[0.126537]**	**[0.098745]**	**[0.007451]**	**[0.004213]**	**[0.004154]**	**[0.000231]**	**[0.000321]**
$\hat{\alpha}_{ABQ}$	0.7546125	0.7589642	0.7945812	0.9000125	0.9854236	0.9741255	1.9985253	1.0621431
MSE	**[1.22e-04]**	**[1.21e-04]**	**[1.35e-04]**	**[0.001254]**	**[0.001354]**	**[0.002254]**	**[0.004521]**	**[0.004235]**

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